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**Problem Sheet 2**<sup>1</sup>

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**Exercise 1.** To fulfill the requirements for a certain degree, a student can choose to take any 7 out of a list of 20 courses, with the constraint that at least 1 of the 7 courses must be a statistics course. Suppose that 5 of the 20 courses are statistics courses.

1. How many choices are there for which 7 courses to take?
2. Explain intuitively why the answer to part 1. is *not*  $\binom{5}{1} \cdot \binom{19}{6}$ .

**Exercise 2.** A fair die is rolled  $n$  times. What is the probability that at least 1 of the 6 values never appears?

**Exercise 3.** A spam filter is designed by looking at commonly occurring phrases in spam. Suppose that 80% of email is spam. In 10% of the spam emails, the phrase “free money” is used, whereas this phrase is only used in 1% of non-spam emails. A new email has just arrived, which does mention “free money”. What is the probability that it is spam?

**Exercise 4.** A hat contains 100 coins, where 99 are fair but one is double-headed (always landing Heads). A coin is chosen uniformly at random. The chosen coin is flipped 7 times, and it lands Heads all 7 times. Given this information, what is the probability that the chosen coin is double-headed? (Of course, another approach here would be to *look at both sides of the coin*—but this is a metaphorical coin.)

**Exercise 5.** A fair coin is flipped 3 times. The toss results are recorded on separate slips of paper (writing “H” if Heads and “T” if Tails), and the 3 slips of paper are thrown into a hat.

1. Find the probability that all 3 tosses landed Heads, given that at least 2 were Heads.
2. Two of the slips of paper are randomly drawn from the hat and both the letter H. Given this information, what is the probability that all 3 tosses landed Heads?

**Exercise 6.** A bag contains one marble which is either green or blue, with equal probabilities. A green marble is put in the bag (so there are 2 marbles now), and then a random marble is taken out. The marble taken out is green. What is the probability that the remaining marble is also green?

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<sup>1</sup>Exercises are based on the coursebook Statistics 110: Probability by Joe Blitzstein