Exercises and solutions: Chapter 1 only

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Exercise 1.1. We observed the eyes and hair colour of a sample of students in the University of Delaware[a]. The results are summarized in the next table.

Hair \Eyes	Brown	Blue	Hazel	Green
Black	68	20	15	5
Brown	119	84	54	29
Ginger	26	17	14	14
Blond	7	94	10	16

1. How many students participated in the study?

2. Create a table representing the frequency of seeing each hair-eyes colour combination.

3. What is the proportion of these students having black hair? and the proportion having blue eyes?

4. Consider black and brown to be dark colours and ginger, blond, blue, hazel, and green to be light colours. Amongst the students with dark hair, are there more with light or dark eyes?

[a]Data are taken from "Snee, R. D. (1974). Graphical display of two-way contingency tables. *The American Statistician*, 28, 9–12."

Solution 1.1.

1. $68 + 20 + 15 + 5 + 119 + 84 + 54 + 29 + 26 + 17 + 14 + 14 + 7 + 94 + 10 + 16 = 592$.

2. We need to normalize each entry in the table by the total number of participants:

Hair \Eyes	Brown	Blue	Hazel	Green
Black	68/592	20/592	15/592	5/592
Brown	119/592	84/592	54/592	29/592
Ginger	26/592	17/592	14/592	14/592
Blond	7/592	94/592	10/592	16/592

3. The proportion of these students with black hair is: $(68 + 20 + 15 + 5)/592 = 108/592 \approx 0.18$. With blue eyes: $(20 + 84 + 17 + 94)/592 = 215/592 \approx 0.36$.

4. There are $68 + 20 + 15 + 5 + 119 + 84 + 54 + 29 = 394$ students with dark hair and $68 + 119 = 187$ students with dark hair and dark eyes. So, the proportion of the students with dark hair that have dark eyes is $187/394 \approx 0.47$, whilst the proportion of the students with dark hair that have light eyes is $1 - \frac{187}{394} = \frac{207}{394} \approx 0.53$. There are more light than dark eyes amongst the students with dark hair.

Exercise 1.2. We observed the eyes and hair colour of a sample of students in the University of Delaware[a]. The results are summarized in the next table.

Hair \Eyes	Brown	Blue	Hazel	Green
Black	68	20	15	5
Brown	119	84	54	29
Ginger	26	17	14	14
Blond	7	94	10	16

1. How many students participated in the study?
2. Create a table representing the frequency of seeing each hair-eyes colour combination.
3. What is the proportion of these students having black hair? and the proportion having blue eyes?
4. Consider black and brown to be dark colours and ginger, blond, blue, hazel, and green to be light colours. Amongst the students with dark hair, are there more with light or dark eyes?

[a]Data are taken from "Snee, R. D. (1974). Graphical display of two-way contingency tables. *The American Statistician*, 28, 9–12."

Solution 1.2.

1. $68 + 20 + 15 + 5 + 119 + 84 + 54 + 29 + 26 + 17 + 14 + 14 + 7 + 94 + 10 + 16 = 592$.
2. We need to normalize each entry in the table by the total number of participants:

Hair \Eyes	Brown	Blue	Hazel	Green
Black	68/592	20/592	15/592	5/592
Brown	119/592	84/592	54/592	29/592
Ginger	26/592	17/592	14/592	14/592
Blond	7/592	94/592	10/592	16/592

3. The proportion of these students with black hair is: $(68 + 20 + 15 + 5)/592 = 108/592 \approx 0.18$. With blue eyes: $(20 + 84 + 17 + 94)/592 = 215/592 \approx 0.36$.
4. There are $68 + 20 + 15 + 5 + 119 + 84 + 54 + 29 = 394$ students with dark hair and $68 + 119 = 187$ students with dark hair and dark eyes. So, the proportion of the students with dark hair that have dark eyes is $187/394 \approx 0.47$, whilst the proportion of the students with dark hair that have light eyes is $1 - \frac{187}{394} = \frac{207}{394} \approx 0.53$. There are more light than dark eyes amongst the students with dark hair.

Exercise 1.3. We observed the eyes and hair colour of a sample of students in the University of Delaware[a]. The results are summarized in the next table.

Hair \Eyes	Brown	Blue	Hazel	Green
Black	68	20	15	5
Brown	119	84	54	29
Ginger	26	17	14	14
Blond	7	94	10	16

1. How many students participated in the study?
2. Create a table representing the frequency of seeing each hair-eyes colour combination.
3. What is the proportion of these students having black hair? and the proportion having blue eyes?
4. Consider black and brown to be dark colours and ginger, blond, blue, hazel, and green to be light colours. Amongst the students with dark hair, are there more with light or dark eyes?

[a] Data are taken from "Snee, R. D. (1974). Graphical display of two-way contingency tables. *The American Statistician*, 28, 9–12."

Solution 1.3.

1. $68 + 20 + 15 + 5 + 119 + 84 + 54 + 29 + 26 + 17 + 14 + 14 + 7 + 94 + 10 + 16 = 592$.
2. We need to normalize each entry in the table by the total number of participants:

Hair \Eyes	Brown	Blue	Hazel	Green
Black	68/592	20/592	15/592	5/592
Brown	119/592	84/592	54/592	29/592
Ginger	26/592	17/592	14/592	14/592
Blond	7/592	94/592	10/592	16/592

3. The proportion of these students with black hair is: $(68 + 20 + 15 + 5)/592 = 108/592 \approx 0.18$. With blue eyes: $(20 + 84 + 17 + 94)/592 = 215/592 \approx 0.36$.
4. There are $68 + 20 + 15 + 5 + 119 + 84 + 54 + 29 = 394$ students with dark hair and $68 + 119 = 187$ students with dark hair and dark eyes. So, the proportion of the students with dark hair that have dark eyes is $187/394 \approx 0.47$, whilst the proportion of the students with dark hair that have light eyes is $1 - \frac{187}{394} = \frac{207}{394} \approx 0.53$. There are more light than dark eyes amongst the students with dark hair.

Exercise 1.4. In straight poker, a player receives 5 cards from a deck of 52 cards: 4 colours, 13 values (2,3,4,5,6,7,8,9,10,J,Q,K,A) for each colours. We suppose that all possible hands have the same probability. The A can also play the role of a 1 in sequences of successive values. Compute the probabilities of the following hands.

1. Royal straight flush (from 10 to A in the same colour).
2. Straight flush (5 cards of the same colour with successive values, which do not form a royal straight flush).
3. Four of a Kind (4 cards with the same value).
4. Full house (3 with the same value + 2 cards with the same value).
5. Flush (5 cards of the same colour which do not form a straight flush nor a royal straight flush).
6. Straight (5 cards with successive values which do not form a straight flush nor a royal straight flush).
7. Three of a kind (3 cards with the same value, but not a Four of a Kind and not a Full house).
8. Two pair (2 pairs of cards with the same value which do not form a Four of a Kind, and 1 card with a different value than the two pairs).
9. One pair (2 cards with the same value and 3 cards with distinct values, different from the pair).
10. High card (5 cards of distinct values which do not have all the same colour, and which do not form a straight).

Solution 1.4. First, there are $\binom{52}{5}$ possible hands in total (pick 5 cards amongst 52 different cards).

1. The values being fixed, we only have 4 choices for the colour. We obtain a probability of $4/\binom{52}{5} \approx 0,000154\%$.
2. There are 4 choices for the colour and 9 choices for the smallest value (1 to 9). We obtain a probability of $4 \cdot 9/\binom{52}{5} \approx 0,00139\%$.
3. We need to choose the value of the Four of a Kind (13 choices), and the value (12 choices) as well as the colour (4 choices) of the 5th card. We obtain a probability of $13 \cdot 12 \cdot 4/\binom{52}{5} \approx 0,0240\%$.
4. For the 3 cards with the same value, we have 13 choices for the value, and 4 choices for the colour (3 colours to pick amongst 4). For the 2 cards of the same value, we have 12 choices for the value (has to be different than the three cards), and 6 choices for the colour (2 colours amongst 4). We obtain a probability of $13 \cdot 4 \cdot 12 \cdot 6/\binom{52}{5} \approx 0,144\%$.

5. There are 4 choices for the colour. For the values, we can consider the $\binom{13}{5} = 1287$ possible arrangements, and remove the 10 giving rise to a straight or a straight flush. We obtain a probability of $1277 \cdot 4/\binom{52}{5} \approx 0,197\%$.

6. There are 10 choices for the smallest value (1 to 10) and 4^5 choices for the colours. Amongst these, we need to remove the 4 arrangements giving a royal straight flush, and the 36 giving a straight flush. We obtain a probability of $(10 \cdot 4^5 - 40)/\binom{52}{5} \approx 0,393\%$.

7. There are 13 choices for the value of the Three of a Kind, and 4 choices for the colour of these 3 cards (3 colours amongst 4). There are $\binom{12}{2} = 66$ choices for the values of the 2 other cards, and 4^2 choices for their colours. We obtain a probability of $13 \cdot 4 \cdot 66 \cdot 16/\binom{52}{5} \approx 2,113\%$.

8. There are $\binom{13}{2} = 78$ choices for the values of the two pairs, and $\binom{4}{2}^2 = 36$ choices for their colours. Then, there are 11 choices for the value of the 5th card (has to be different from the two pairs) and 4 choices for its colour. We obtain a probability of $78 \cdot 36 \cdot 11 \cdot 4/\binom{52}{5} \approx 4,754\%$.

9. There are 13 choices for the value of the pair and $\binom{4}{2} = 6$ choices for the colours. Then, there are $\binom{12}{3} = 220$ choices for the values of the 3 other cards, and 4^3 choices for their colours. We obtain a probability of $13 \cdot 6 \cdot 220 \cdot 64/\binom{52}{5} \approx 42,26\%$.

10. There are $\binom{13}{5} = 1287$ choices for the values of the cards, and $4^5 = 1024$ choices for their colours. To this, we have to remove the 10200 straights, the 36 straight flush, the 4 royal straight flush, and the 5108 flush. We obtain a probability of $(1287 \cdot 1024 - 15348)/\binom{52}{5} \approx 50,12\%$.

Exercise 1.5. Consider a room with 23 people. What is the probability that two people in the room have the same birthday?
To simplify, we assume that every year has 365 days, and that the births are evenly distributed amongst these days.

Solution 1.5. The probability that two people are born the same day is 1 minus the probability that they are all born different days. The total number of possible birthdays for each of the 23 people is 365^{23} (chose a day per person). The number of birthdays so that all 23 people are born different days is

$$365 \cdot 364 \cdot \dots \cdot 343 = \frac{365!}{342!},$$

as there are 365 possibilities for the first person, 364 possibilities for the second (has to be different from the first), 363 for the third (has to be different from the first and second), and so on. The wanted probability is thus

$$1 - \frac{365 \cdot 364 \cdot \dots \cdot 343}{365^{23}} \approx 0.507.$$

More than 50% chances!

Exercise 1.6. During a wedding, the 60 guests are seated uniformly at random in the 60 available seats. There are 10 round tables with 6 seats each. The following questions are all independent from one another.

1. What is the probability that the 5 members of family A. are seated at the same table?

2. The car of family B. broke down. The 7 members of this family did not make it to the wedding. Given that the 53 other guests are all present, what is the probability that at most one person is missing at each table?

3. What is the probability that Mr. and Mrs. C. are seated one next to the other at the same table?

4. Given that 3 men and 3 women are seated at the third table, what is the probability that there are no two men seated next to one another?

Solution 1.6.

1. *First approach:* The first member sits anywhere. The second ends up at the same table as the first with probability $5/59$ (5 seats left at the table over 59 remaining seats); the third with probability $4/58$ (same reasoning); the fourth with probability $3/57$ (same reasoning); the last with probability $2/56$ (same reasoning). The probability of the event is therefore $5!55!/59! = 5/455126 \approx 0.001\%$.
   Second approach: Number of favourable cases: $\binom{10}{1}$ choices for the table, $\binom{6}{5}$ choices for the seats occupied at that table by the members of family A.. Number of seating options for the 5 members of family A.: $\binom{60}{5}$ (pick 5 seats amongst 60). The probability is thus $10 \cdot 6/\binom{60}{5} = 5/455126$.

2. *First approach:* We chose the first seat arbitrarily; the probability that the second seat chosen is not at the same table is $54/59$ (54 seats not at the table over 59 remaining seats); the probability that the third is not at one of the two already chosen table is $48/58$ (same reasoning); and so on. We find a probability of $54 \cdot 48 \cdot 42 \cdot 36 \cdot 30 \cdot 24/(59!/53!) = 31104/357599 \approx 8,7\%$.
   Second approach: Number of favourable cases: $\binom{10}{7}$ for the table with one missing person; $\binom{6}{1}^7$ for the empty seat at each of the tables with a missing person. Total number of cases: $\binom{60}{7}$ (7 empty seats to be picked in the 60 seats). We find a probability of $31104/357599$.

3. *First approach:* Mr. C. sits on an arbitrary seat. The probability that his wife is seated next to him is $2/59 \approx 3.4\%$ (two seats are next to Mr. C. over the 59 seats that are not Mr. C. seat).
   Second approach: Number of favourable cases: $\binom{10}{1}$ for the choice of the table; $\binom{6}{1}$ for the first seat at the table (the other one being on the right of the first seat). Total number of cases: $\binom{60}{2}$ (two seats to be picked in the 60 possible ones). we find a probability of $10 \cdot 6/(60 \cdot 59/2) = 2/59$.

4. *First approach:* The first man sits anywhere at the table; the second has a probability $2/5$ to chose one of the two seats permitting the wanted arrangement; the third has a probability $1/4$ of choosing the last compatible seat. We obtain

a probability of $1/10 = 10\%$.
Second approach: Number of favourable cases: there are 2 choices for the 3 seats attributed to men. Total number of cases: there are $\binom{6}{3}$ ways to seat the 3 men at the table. We find a probability of $2/(6!/3!3!) = 1/10$.