

EPFL

Probability and Statistics for SIC
2017–2018, Spring semester

Probability and Statistics: Test

16 April 2018

Duration: The test starts at 16:15 and ends at 18:00.

Surname:

Forename:

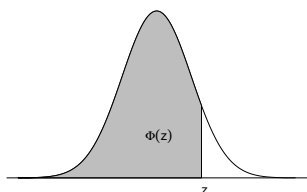
SCIPER number:

Exercise	Points	Indicative marks
1		10 points
2		13 points
3		11 points
4		10 points
Total :		44 points

REMARKS:

- Personal documents may not be brought into the test.
- A simple calculator may be used.
- Answers may be given in English or in French.
- Explain your reasoning! An unjustified answer will be treated as incorrect.
- Write your answers in the exam scripts. If you need more space, use the blank pages at the end of the script, or ask for another blank page and staple it into the script.
- The assistants will reply to questions only if there is a typo. If you find a question unclear, explain how you understand it when giving your solution.
- Do NOT unstaple pages.

Distribution normale standard $\Phi(z)$



Pour $z < 0$ on utilise symmetrie: $P(Z \leq z) = \Phi(z) = 1 - \Phi(-z)$, $z \in \mathbb{R}$.

z	0	1	2	3	4	5	6	7	8	9
0.0	.50000	.50399	.50798	.51197	.51595	.51994	.52392	.52790	.53188	.53586
0.1	.53983	.54380	.54776	.55172	.55567	.55962	.56356	.56750	.57142	.57535
0.2	.57926	.58317	.58706	.59095	.59483	.59871	.60257	.60642	.61026	.61409
0.3	.61791	.62172	.62552	.62930	.63307	.63683	.64058	.64431	.64803	.65173
0.4	.65542	.65910	.66276	.66640	.67003	.67364	.67724	.68082	.68439	.68793
0.5	.69146	.69497	.69847	.70194	.70540	.70884	.71226	.71566	.71904	.72240
0.6	.72575	.72907	.73237	.73565	.73891	.74215	.74537	.74857	.75175	.75490
0.7	.75804	.76115	.76424	.76730	.77035	.77337	.77637	.77935	.78230	.78524
0.8	.78814	.79103	.79389	.79673	.79955	.80234	.80511	.80785	.81057	.81327
0.9	.81594	.81859	.82121	.82381	.82639	.82894	.83147	.83398	.83646	.83891
1.0	.84134	.84375	.84614	.84850	.85083	.85314	.85543	.85769	.85993	.86214
1.1	.86433	.86650	.86864	.87076	.87286	.87493	.87698	.87900	.88100	.88298
1.2	.88493	.88686	.88877	.89065	.89251	.89435	.89617	.89796	.89973	.90147
1.3	.90320	.90490	.90658	.90824	.90988	.91149	.91309	.91466	.91621	.91774
1.4	.91924	.92073	.92220	.92364	.92507	.92647	.92786	.92922	.93056	.93189
1.5	.93319	.93448	.93574	.93699	.93822	.93943	.94062	.94179	.94295	.94408
1.6	.94520	.94630	.94738	.94845	.94950	.95053	.95154	.95254	.95352	.95449
1.7	.95543	.95637	.95728	.95818	.95907	.95994	.96080	.96164	.96246	.96327
1.8	.96407	.96485	.96562	.96638	.96712	.96784	.96856	.96926	.96995	.97062
1.9	.97128	.97193	.97257	.97320	.97381	.97441	.97500	.97558	.97615	.97670
2.0	.97725	.97778	.97831	.97882	.97932	.97982	.98030	.98077	.98124	.98169
2.1	.98214	.98257	.98300	.98341	.98382	.98422	.98461	.98500	.98537	.98574
2.2	.98610	.98645	.98679	.98713	.98745	.98778	.98809	.98840	.98870	.98899
2.3	.98928	.98956	.98983	.99010	.99036	.99061	.99086	.99111	.99134	.99158
2.4	.99180	.99202	.99224	.99245	.99266	.99286	.99305	.99324	.99343	.99361
2.5	.99379	.99396	.99413	.99430	.99446	.99461	.99477	.99492	.99506	.99520
2.6	.99534	.99547	.99560	.99573	.99585	.99598	.99609	.99621	.99632	.99643
2.7	.99653	.99664	.99674	.99683	.99693	.99702	.99711	.99720	.99728	.99736
2.8	.99744	.99752	.99760	.99767	.99774	.99781	.99788	.99795	.99801	.99807
2.9	.99813	.99819	.99825	.99831	.99836	.99841	.99846	.99851	.99856	.99861
3.0	.99865	.99869	.99874	.99878	.99882	.99886	.99889	.99893	.99896	.99900
3.1	.99903	.99906	.99910	.99913	.99916	.99918	.99921	.99924	.99926	.99929
3.2	.99931	.99934	.99936	.99938	.99940	.99942	.99944	.99946	.99948	.99950
3.3	.99952	.99953	.99955	.99957	.99958	.99960	.99961	.99962	.99964	.99965
3.4	.99966	.99968	.99969	.99970	.99971	.99972	.99973	.99974	.99975	.99976
3.5	.99977	.99978	.99978	.99979	.99980	.99981	.99981	.99982	.99983	.99983
3.6	.99984	.99985	.99985	.99986	.99986	.99987	.99987	.99988	.99988	.99989
3.7	.99989	.99990	.99990	.99990	.99991	.99991	.99992	.99992	.99992	.99992
3.8	.99993	.99993	.99993	.99994	.99994	.99994	.99994	.99995	.99995	.99995
3.9	.99995	.99995	.99996	.99996	.99996	.99996	.99996	.99996	.99997	.99997

Exercise 1. A coin that shows a head with probability $p \in (0, 1)$ is thrown twice in succession, and the outcomes are recorded.

- (a) Write down the sample space for this experiment, and give probabilities for the outcomes.
- (b) Give the probabilities of the events A_1 , ‘the first toss shows a head’, A_2 , ‘the second toss shows a head’, and B , ‘there is exactly one head’.
- (c) If the coin is fair, are the events A_1 , A_2 and B (i) pairwise independent? (ii) mutually independent?
- (d) Give the probability of A_1 given B . Hence explain how you could obtain the outcome of tossing a fair coin, regardless of the value of p .

Exercise 2. Negative daily percent returns X on a financial asset have an $\mathcal{N}(0, 4)$ distribution with probability 0.99 on ordinary days, but the market panics with probability 0.01, and then they have density function

$$f(x) = \frac{8}{x^3}, \quad x > 2.$$

- (a) Find $\Pr(X > 4)$.
- (b) Given that $X > 4$, what is the probability that this is a panic day?
- (c) Compute the mean and variance of X on a panic day. Discuss your findings.
- (d) On a panic day, find the conditional probability that $X > y$, given that $X > x$, where $y > x$. Hence find the conditional expectation of X on a panic day, given that $X > x$.

Exercise 3. A probability course has a midterm test and a final exam, whose respective marks X and Y have joint probability density function

$$f_{X,Y}(x, y) = \begin{cases} c(x + y), & 0 \leq x, y \leq 3, \\ 0, & \text{otherwise.} \end{cases}$$

- (a) For which value of c is $f_{X,Y}$ a density function?
- (b) Find the probability that the sum of the marks is less than 3.
- (c) Give the marginal density of a mark at the test. Are the marks independent?
- (d) Give the conditional density of Y given that $X = x$, and hence compute $E(Y | X = 2)$.

Exercise 4. A document is equally likely to be in any of three boxes. A search of the j th box will reveal the document (if it is there) with probability p_j ($j = 1, 2, 3$), where $0 < p_1, p_2, p_3 < 1$. Given the location of the document, the outcomes of each search are independent.

- (a) What is the probability that the document is in the first box, given that I have searched the first box once and not found it?
- (b) What is the probability that the document is in the first box, given that I have searched all three boxes once and not found it?
- (c) If I decide to search the boxes in order $1, 2, 3, 1, 2, \dots$ until I find the document, what is my expected number of searches?

Hint: $\sum_{r=1}^{\infty} ru^{r-1} = 1/(1-u)^2$, for $|u| < 1$.