EPFL
Probability and Statistics for SIC
2016–2017, Spring semester

Probability and Statistics: Test

24 April 2017

Duration: The test starts at 16:15 and ends at 18:00.

Surname:

Forename:

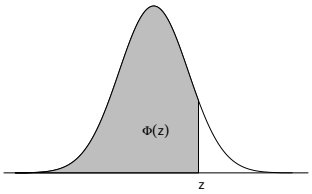
SCIPER number:

Exercise	Points	Indicative marks
1		8 points
2		10 points
3		13 points
4		9 points
Total :		40 points

REMARKS:

- Personal documents may not be brought into the test.
- A simple calculator may be used.
- Answers may be given in English or in French.
- Explain your reasoning! An unjustified answer will be treated as incorrect.
- Write your answers in the exam scripts. If you need more space, use the blank pages at the end of the script, or ask for another blank page and staple it into the script.
- The assistants will reply to questions only if there is a typo. If you find a question unclear, explain how you understand it when giving your solution.
- Do NOT unstaple pages.

Standard normal distribution function $\Phi(z)$



Fot $z < 0$, use symmetry: $P(Z \leq z) = \Phi(z) = 1 - \Phi(-z)$, $z \in \mathbb{R}$.

z	0	1	2	3	4	5	6	7	8	9
0.0	.50000	.50399	.50798	.51197	.51595	.51994	.52392	.52790	.53188	.53586
0.1	.53983	.54380	.54776	.55172	.55567	.55962	.56356	.56750	.57142	.57535
0.2	.57926	.58317	.58706	.59095	.59483	.59871	.60257	.60642	.61026	.61409
0.3	.61791	.62172	.62552	.62930	.63307	.63683	.64058	.64431	.64803	.65173
0.4	.65542	.65910	.66276	.66640	.67003	.67364	.67724	.68082	.68439	.68793
0.5	.69146	.69497	.69847	.70194	.70540	.70884	.71226	.71566	.71904	.72240
0.6	.72575	.72907	.73237	.73565	.73891	.74215	.74537	.74857	.75175	.75490
0.7	.75804	.76115	.76424	.76730	.77035	.77337	.77637	.77935	.78230	.78524
0.8	.78814	.79103	.79389	.79673	.79955	.80234	.80511	.80785	.81057	.81327
0.9	.81594	.81859	.82121	.82381	.82639	.82894	.83147	.83398	.83646	.83891
1.0	.84134	.84375	.84614	.84850	.85083	.85314	.85543	.85769	.85993	.86214
1.1	.86433	.86650	.86864	.87076	.87286	.87493	.87698	.87900	.88100	.88298
1.2	.88493	.88686	.88877	.89065	.89251	.89435	.89617	.89796	.89973	.90147
1.3	.90320	.90490	.90658	.90824	.90988	.91149	.91309	.91466	.91621	.91774
1.4	.91924	.92073	.92220	.92364	.92507	.92647	.92786	.92922	.93056	.93189
1.5	.93319	.93448	.93574	.93699	.93822	.93943	.94062	.94179	.94295	.94408
1.6	.94520	.94630	.94738	.94845	.94950	.95053	.95154	.95254	.95352	.95449
1.7	.95543	.95637	.95728	.95818	.95907	.95994	.96080	.96164	.96246	.96327
1.8	.96407	.96485	.96562	.96638	.96712	.96784	.96856	.96926	.96995	.97062
1.9	.97128	.97193	.97257	.97320	.97381	.97441	.97500	.97558	.97615	.97670
2.0	.97725	.97778	.97831	.97882	.97932	.97982	.98030	.98077	.98124	.98169
2.1	.98214	.98257	.98300	.98341	.98382	.98422	.98461	.98500	.98537	.98574
2.2	.98610	.98645	.98679	.98713	.98745	.98778	.98809	.98840	.98870	.98899
2.3	.98928	.98956	.98983	.99010	.99036	.99061	.99086	.99111	.99134	.99158
2.4	.99180	.99202	.99224	.99245	.99266	.99286	.99305	.99324	.99343	.99361
2.5	.99379	.99396	.99413	.99430	.99446	.99461	.99477	.99492	.99506	.99520
2.6	.99534	.99547	.99560	.99573	.99585	.99598	.99609	.99621	.99632	.99643
2.7	.99653	.99664	.99674	.99683	.99693	.99702	.99711	.99720	.99728	.99736
2.8	.99744	.99752	.99760	.99767	.99774	.99781	.99788	.99795	.99801	.99807
2.9	.99813	.99819	.99825	.99831	.99836	.99841	.99846	.99851	.99856	.99861
3.0	.99865	.99869	.99874	.99878	.99882	.99886	.99889	.99893	.99896	.99900
3.1	.99903	.99906	.99910	.99913	.99916	.99918	.99921	.99924	.99926	.99929
3.2	.99931	.99934	.99936	.99938	.99940	.99942	.99944	.99946	.99948	.99950
3.3	.99952	.99953	.99955	.99957	.99958	.99960	.99961	.99962	.99964	.99965
3.4	.99966	.99968	.99969	.99970	.99971	.99972	.99973	.99974	.99975	.99976
3.5	.99977	.99978	.99978	.99979	.99980	.99981	.99981	.99982	.99983	.99983
3.6	.99984	.99985	.99985	.99986	.99986	.99987	.99987	.99988	.99988	.99989
3.7	.99989	.99990	.99990	.99990	.99991	.99991	.99992	.99992	.99992	.99992
3.8	.99993	.99993	.99993	.99994	.99994	.99994	.99994	.99995	.99995	.99995
3.9	.99995	.99995	.99996	.99996	.99996	.99996	.99996	.99996	.99997	.99997

Exercise 1. Two fair dice, one red and one green, are thrown, and the numbers on their top faces are noted.

(a) Write down the sample space for this experiment, and give probabilities for the outcomes.

(b) Give the probabilities of the events A, 'the total score is less than 5', and B, 'the score on the green die is 2 or less'.

(c) Give the probability of B given A.

(d) Find the expected total score, conditional on A.

Exercise 2. Aramis has challenged Porthos to a duel. Porthos can choose either that they will fight with swords or with pistols, and he does this by tossing a fair coin. Porthos will win a swordfight with probability 0.4. If they fight with pistols, they shoot at each other alternately until one of them is hit, with Aramis shooting first; on each attempt, Porthos hits Aramis with probability 0.6, and Aramis hits Porthos with probability 0.4.

(a) Write down the sample space for this experiment.

(b) If they fight with pistols, what is the probability that Porthos will win?

(c) You are told that Aramis won. What is the probability that they had a swordfight?

(d) Give an expression for the expected number of shots fired, if they fight with pistols.

Exercise 3. The daily power generated by a photovoltaic installation is normally distributed, with mean 30 kWh and standard deviation 3 kWh on a sunny day, and mean 20 kWh and standard deviation 4 kWh on a dull day. Days are sunny or dull independently, with probability 0.3 of being sunny.

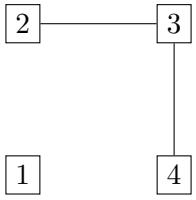
(a) What is the probability of generating between 25 kWh and 35 kWh on an unspecified day?

(b) The forecast this week is for five sunny days, followed by two dull days. If the forecast is correct, give the probability that the system will generate at least 200 kWh this week.

(c) In (b), given that 130 kWh were generated in the first four days and the weather forecast is correct, give the probability that the system will generate at least 220 kWh this week.

(d) What is the expected power generated on an unspecified day? What is its variance?

Exercise 4. In the network shown below, let $X_{i,j} = 1$ indicate that there is a link between nodes i and j, with $X_{i,j} = 0$ otherwise ($1 \leq i < j \leq 4$). Let $X_{2,3} = X_{3,4} = 1$ throughout.



Suppose that $X_{1,2}$, $X_{1,3}$, $X_{1,4}$, $X_{2,4}$ are independent Bernoulli variables with success probability $p = 1/2$, let S denote the number of links in addition to those shown above, and let T denote the number of triangles, i.e.,

$$S = X_{1,2}+X_{1,3}+X_{1,4}+X_{2,4}, \quad T = X_{1,2}X_{2,3}X_{3,1}+X_{1,2}X_{2,4}X_{4,1}+X_{1,3}X_{3,4}X_{4,1}+X_{2,3}X_{3,4}X_{4,2}.$$

(a) How many possible configurations are there? Complete the table below and hence find the joint probability mass function of S and T.

s	0	1	1	2	2	3	3	3	4
t	0	0	1	0	1	0	1	2	4
Number of configurations		3			5	0			1

(b) Find the marginal mass functions of S and of T. Are they independent?

(c) Compute the expected number of links when there is one triangle in the network.