

EPFL

Probability and Statistics for SIC
2015–2016, Spring semester

Probability and Statistics: Test

18 April 2016

Duration: The test starts at 16:15 and ends at 18:00.

Surname:

Forename:

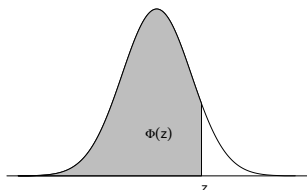
SCIPER number:

Exercise	Points	Indicative marks
1		8 / 6.5 points
2		5 points
3		13 points
4		10 points
Total :		36 / 34.5 points

REMARKS:

- Personal documents may not be brought into the test.
- A simple calculator may be used.
- Answers may be given in English or in French.
- Explain your reasoning! An unjustified answer will be treated as incorrect.
- Write your answers in the exam scripts. If you need more space, use the blank pages at the end of the script, or ask for another blank page and staple it into the script.
- The assistants will reply to questions only if there is a typo. If you find a question unclear, explain how you understand it when giving your solution.
- Do NOT unstaple pages.

Standard normal distribution function $\Phi(z)$



For $z < 0$, use symmetry: $P(Z \leq z) = \Phi(z) = 1 - \Phi(-z)$, $z \in \mathbb{R}$.

z	0	1	2	3	4	5	6	7	8	9
0.0	.50000	.50399	.50798	.51197	.51595	.51994	.52392	.52790	.53188	.53586
0.1	.53983	.54380	.54776	.55172	.55567	.55962	.56356	.56750	.57142	.57535
0.2	.57926	.58317	.58706	.59095	.59483	.59871	.60257	.60642	.61026	.61409
0.3	.61791	.62172	.62552	.62930	.63307	.63683	.64058	.64431	.64803	.65173
0.4	.65542	.65910	.66276	.66640	.67003	.67364	.67724	.68082	.68439	.68793
0.5	.69146	.69497	.69847	.70194	.70540	.70884	.71226	.71566	.71904	.72240
0.6	.72575	.72907	.73237	.73565	.73891	.74215	.74537	.74857	.75175	.75490
0.7	.75804	.76115	.76424	.76730	.77035	.77337	.77637	.77935	.78230	.78524
0.8	.78814	.79103	.79389	.79673	.79955	.80234	.80511	.80785	.81057	.81327
0.9	.81594	.81859	.82121	.82381	.82639	.82894	.83147	.83398	.83646	.83891
1.0	.84134	.84375	.84614	.84850	.85083	.85314	.85543	.85769	.85993	.86214
1.1	.86433	.86650	.86864	.87076	.87286	.87493	.87698	.87900	.88100	.88298
1.2	.88493	.88686	.88877	.89065	.89251	.89435	.89617	.89796	.89973	.90147
1.3	.90320	.90490	.90658	.90824	.90988	.91149	.91309	.91466	.91621	.91774
1.4	.91924	.92073	.92220	.92364	.92507	.92647	.92786	.92922	.93056	.93189
1.5	.93319	.93448	.93574	.93699	.93822	.93943	.94062	.94179	.94295	.94408
1.6	.94520	.94630	.94738	.94845	.94950	.95053	.95154	.95254	.95352	.95449
1.7	.95543	.95637	.95728	.95818	.95907	.95994	.96080	.96164	.96246	.96327
1.8	.96407	.96485	.96562	.96638	.96712	.96784	.96856	.96926	.96995	.97062
1.9	.97128	.97193	.97257	.97320	.97381	.97441	.97500	.97558	.97615	.97670
2.0	.97725	.97778	.97831	.97882	.97932	.97982	.98030	.98077	.98124	.98169
2.1	.98214	.98257	.98300	.98341	.98382	.98422	.98461	.98500	.98537	.98574
2.2	.98610	.98645	.98679	.98713	.98745	.98778	.98809	.98840	.98870	.98899
2.3	.98928	.98956	.98983	.99010	.99036	.99061	.99086	.99111	.99134	.99158
2.4	.99180	.99202	.99224	.99245	.99266	.99286	.99305	.99324	.99343	.99361
2.5	.99379	.99396	.99413	.99430	.99446	.99461	.99477	.99492	.99506	.99520
2.6	.99534	.99547	.99560	.99573	.99585	.99598	.99609	.99621	.99632	.99643
2.7	.99653	.99664	.99674	.99683	.99693	.99702	.99711	.99720	.99728	.99736
2.8	.99744	.99752	.99760	.99767	.99774	.99781	.99788	.99795	.99801	.99807
2.9	.99813	.99819	.99825	.99831	.99836	.99841	.99846	.99851	.99856	.99861
3.0	.99865	.99869	.99874	.99878	.99882	.99886	.99889	.99893	.99896	.99900
3.1	.99903	.99906	.99910	.99913	.99916	.99918	.99921	.99924	.99926	.99929
3.2	.99931	.99934	.99936	.99938	.99940	.99942	.99944	.99946	.99948	.99950
3.3	.99952	.99953	.99955	.99957	.99958	.99960	.99961	.99962	.99964	.99965
3.4	.99966	.99968	.99969	.99970	.99971	.99972	.99973	.99974	.99975	.99976
3.5	.99977	.99978	.99978	.99979	.99980	.99981	.99981	.99982	.99983	.99983
3.6	.99984	.99985	.99985	.99986	.99986	.99987	.99987	.99988	.99988	.99989
3.7	.99989	.99990	.99990	.99990	.99991	.99991	.99992	.99992	.99992	.99992
3.8	.99993	.99993	.99993	.99994	.99994	.99994	.99994	.99995	.99995	.99995
3.9	.99995	.99995	.99996	.99996	.99996	.99996	.99996	.99996	.99997	.99997

Exercise 1. You are invited to play in a game show, with the following rules: three cards marked with the numbers 1, 2, 3 are placed in three identical unmarked envelopes, which are then shuffled. You then choose an envelope and open it. You must then answer the number of questions written on the card inside the envelope. If you answer $r > 0$ of the questions correctly, you win $\$2^r \times 1000$. You answer each question correctly with probability 0.5, independently of the other questions.

- (a) Give the sample space for this random experiment, and give the probabilities of the different outcomes.
- (b) Find the probability that you win nothing.
- (c) Find the probability that you win \$2000.
- (d) Given that you have won at least \$1000, give the probability of winning at least \$4000.

Exercise 2. An attempt to download a file from the internet fails with probability $1 - p$, but even if the download is successful, the file is corrupted with probability $1 - q$. The outcomes of successive downloads are independent, and I keep on trying until I have successfully downloaded an uncorrupted file.

- (a) Give the probability of downloading a non-corrupted file at a first attempt.
- (b) Give the distribution of the waiting time to the first successful download, and say what the expected number of downloads is.
- (c) Give the probability that the first successful download is of a corrupt file.

Exercise 3. Each weekday morning I take the bus at 7.45 in order to catch the train at 8.00. The bus journey should take 10 minutes, but on one weekday each week, on average, it takes an additional X minutes, where $X \sim N(3, 1)$. I miss my train if the bus is more than 4 minutes late.

- (a) What is the probability that I will catch the train tomorrow?
- (b) If the bus is already 3 minutes late, what is the probability that I will miss the train?
- (c) I caught the train this morning. What is the probability that the bus was late?
- (d) At what time do I expect to arrive at the station?

Exercise 4. A lake is fed by two rivers, and their daily flows X and Y (millions of litres) into the lake have joint probability distribution

$$\Pr(X \leq x, Y \leq y) = \begin{cases} (1 - e^{-x})(1 - e^{-y}), & x, y > 0, \\ 0, & \text{otherwise.} \end{cases}$$

- (a) Find the joint probability density function of X and Y . Are these flows independent?
- (b) Find the marginal density of X , and give the probabilities that X exceeds 3 million litres, and that both X and Y exceed 3 million litres.
- (c) An alert is sounded if the total inflow in a given day, $X + Y$, exceeds 6 million litres. Find the probability that no alert is sounded today. How does this compare to $\Pr(X > 3, Y > 3)$? Explain.