EPFL - Fall 2025	Dr. Pablo Antolin
Analysis III (for IC) – MATH-203(d) – IN SC	**Exercises**
Series 11	November 27 2025

Hint: For the following exercises, we suggest to:

1. *Start by sketching the graph of f and the graph of f' over at least two periods.*
2. *Verify that the function f is a piecewise C^1 function.*

Exercise 1 (Ex 14.5 page 220)**.**

1. Compute the Fourier series of the 2π-periodic and odd function f which is defined by $f(x) = x(\pi - x)$ for $x \in [0, \pi]$.
2. With the help of question 1 and Parseval's identity, deduce the sum of the series:
$$\sum_{k=1}^{\infty} \frac{1}{(2k-1)^6}.$$

Exercise 2 (Ex 14.6 page 220)**.**
With the help of Parseval's identity, show that:

$$\int_{-\pi}^{\pi} \cos^4 x \, dx = \frac{3}{4}\pi.$$

Exercise 3 (Ex 14.7 page 220)**.**

1. Compute the Fourier series of the following 2π-periodic function defined by:
$$f(x) = |x| \quad \text{for} \quad x \in [-\pi, \pi[.$$
2. Deduce the sums of the following series:
$$\sum_{k=0}^{\infty} \frac{1}{(2k+1)^4} \quad \text{and} \quad \sum_{k=1}^{\infty} \frac{1}{k^4} \,.$$

Exercise 4.
Let $f : [0, \pi] \to \mathbb{R}$ be the function defined by $f(x) = x$.

1. Compute the Fourier sine and cosine series denoted here $F_c f$ for the cosine form and $F_s f$ for the sinus form.
2. Compare $F_c f$, $F_s f$, and f over $[0, \pi]$.
3. Deduce the values for the two following sums:

$$\sum_{n=0}^{+\infty} \frac{(-1)^n}{2n+1} \qquad \text{and} \qquad \sum_{n=0}^{+\infty} \frac{1}{(2n+1)^2}\,.$$

4. With the help of Parseval's identity, determine the value of the following sum:

$$\sum_{n=0}^{+\infty} \frac{1}{(2n+1)^4}.$$

Exercise 5.
Let f be the 2π-periodic function defined in the Exercise 1 (14.5). Let's define

$$F(x) = \int_0^x f(y)\,\mathrm{d}y, \quad x \in \mathbb{R}.$$

1. Is F a 2π-periodic function? Justify your answer.
2. Find the Fourier series of F with the use of the Fourier series of f (express also the constant term a_0 of the series of F explicitly).