

Exercise 1 (Ex 17.7 page 271).

Find the function $y : \mathbb{R} \rightarrow \mathbb{R}$ solution of:

$$\begin{aligned} \forall x \in \mathbb{R} & : y''(x) + 5y'(x - \pi) - y(x) = \cos x - 3 \sin(2x) + 2, \\ & : y(x + 2\pi) = y(x). \end{aligned}$$

Exercise 2.

Compute:

$$\int_0^{+\infty} \frac{t}{(t^2 + 4)^2} \sin\left(\frac{t}{2}\right) dt.$$

Hint: The Fourier transform of the function f defined by $f(x) = xe^{-\omega|x|}$ with $\omega > 0$ is given by:

$$\mathfrak{F}(f)(\alpha) = \frac{4\omega}{i\sqrt{2\pi}} \frac{\alpha}{(\alpha^2 + \omega^2)^2}.$$

Exercise 3 (Exemple 17.7 page 269).

Find a solution y of the following equation by using the properties of the Fourier transform (convolution):

$$y(x) + 3 \int_{-\infty}^{+\infty} e^{-|t|} y(x - t) dt = e^{-|x|}, \quad \forall x \in \mathbb{R}.$$

Hint: The Fourier transform of the function f defined by $f(x) = \frac{e^{-\omega|x|}}{\omega}$ with $\omega > 0$ is given by:

$$\mathfrak{F}(f)(\alpha) = \sqrt{\frac{2}{\pi}} \frac{1}{\alpha^2 + \omega^2}.$$

Exercise 4 (Ex 17.11 page 272).

Find a solution y of the following equation by using the properties of the Fourier transform (convolution and differentiation):

$$3y(x) + \int_{-\infty}^{+\infty} [y''(t) - y(t)] f(x-t) dt = g(x), \quad \forall x \in \mathbb{R}.$$

where $f(x) = e^{-|x|}$, and $g(x) = xe^{-x^2}$.

Hint: The Fourier transforms of functions f and g are respectively given by:

$$\mathfrak{F}(f)(\alpha) = \sqrt{\frac{2}{\pi}} \frac{1}{\alpha^2 + 1} \quad \text{and} \quad \mathfrak{F}(g)(\alpha) = \frac{-i\alpha}{2\sqrt{2}} e^{-\frac{\alpha^2}{4}}.$$