

Tip: To verify the Stokes theorem, proceed as follows:

1. Sketch the surface Σ , then compute $\text{curl } F(x, y, z)$.
2. Provide a parameterization $\sigma : \bar{A} \rightarrow \Sigma$ of the surface Σ and compute its normal vector. Add this vector to your sketch.
3. Express

$$\iint_{\Sigma} \text{curl } F \cdot ds$$

as a double integral where the bounds and the function to be integrated are explicitly indicated.

4. Write $\partial\Sigma$ as the union of simple regular curves; for each of them, provide a parameterization and indicate the circulation direction induced by the parameterization of Σ and the positive orientation of ∂A .
5. Express

$$\int_{\partial\Sigma} F \cdot dl$$

as a sum of integrals where the bounds and the functions to be integrated are explicitly indicated.

6. Verify the conclusion of the Stokes theorem for Σ and F .

Exercise 1 (Ex 7.2 page 89).

Verify Stokes' theorem for

$$\Sigma = \{(x, y, z) \in \mathbb{R}^3 : x^2 + y^2 = z^4, 0 \leq z \leq 1\} \text{ and } F(x, y, z) = (x^2y, z, x).$$

Exercise 2 (Ex 7.5 page 89).

Verify Stokes' theorem for

$$\Sigma = \{(x, y, z) \in \mathbb{R}^3 : x^2 + y^2 + z^2 = 4, x \geq 0, y \geq 0, 1 \leq z \leq \sqrt{3}\}$$

$$\text{and } F(x, y, z) = (0, z^2, 0).$$

Exercise 3 (Ex 7.7 page 89).

Verify the Stokes theorem for $F(x, y, z) = (0, x^2, 0)$ and Σ the triangle of vertices $(1, 0, 0)$, $(2, 2, 0)$ and $(1, 1, 0)$.

Exercise 4 (Ex 7.6 page 89).

Verify the Stokes theorem for $F(x, y, z) = (0, 0, y + z^2)$ and

$$\Sigma = \left\{ (x, y, z) \in \mathbb{R}^3 : x^2 + y^2 + z^2 = 4; x, y, z \geq 0; 0 \leq \arccos \frac{z}{2} \leq \arctan \frac{y}{x} \leq \frac{\pi}{2} \right\}.$$