Exercise 1 (Ex 2.1 page 17)**.**

1. Let $\Gamma = \left\{(x, y) \in \mathbb{R}^2 : y = f(x),\ x \in [a, b]\right\}$. Show that:

$$\text{length}(\Gamma) = \int_a^b \sqrt{1 + \left(f'(t)\right)^2}\,\mathrm{d}t.$$

2. Deduce the length of the curve:

$$\Gamma = \left\{(x, y) \in \mathbb{R}^2 :\ y = \cosh x,\ x \in [0, 1]\right\}.$$

3. Let $\Gamma = \left\{(x, y) \in \mathbb{R}^2 :\ x(t) = r(t)\cos t;\ y(t) = r(t)\sin t,\ t \in [a, b]\right\}$. Calculate the length of Γ in terms of r.

Exercise 2 (Ex 2.4 page 18)**.**
Calculate $\int_\Gamma f\,\mathrm{d}l$ when $f(x, y, z) = x^2 + y^2 + \sqrt{2z}$, and:

$$\Gamma = \left\{\gamma(t) = \left(\cos t,\ \sin t,\ \frac{1}{2}t^2\right) : t \in [0, 1]\right\}.$$

Hint: $\int \sqrt{x^2 + a^2}\,\mathrm{d}x = \frac{x}{2}\sqrt{x^2 + a^2} + \frac{a^2}{2}\ln\left(x + \sqrt{x^2 + a^2}\right).$

Exercise 3 (Ex 2.2 page 17)**.**
We consider $F(x, y) = \left(xy,\ y^2 - x\right)$ and:

$$\Gamma_1 = \{(t, t) : t \in [0, 1]\},\ \Gamma_2 = \left\{(t, e^t) : t \in [0, 1]\right\},\ \Gamma_3 = \left\{\left(\sqrt{t}, t^2\right) : t \in [1, 2]\right\}.$$

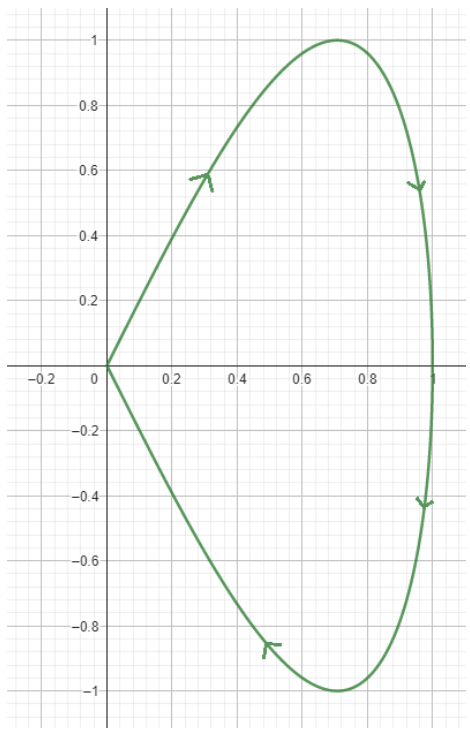
Calculate $\int_{\Gamma_i} F \cdot \mathrm{d}l$ for $i = 1, 2, 3$.

Exercise 4 (Ex 2.3 page 17)**.**
Calculate $\int_\Gamma F \cdot \mathrm{d}l$ when:

1. $\Gamma = \{(x, y, z) \in \mathbb{R} : x^2 + y^2 = 1,\ z = 0\}, \quad F(x, y, z) = (x, z, y);$
2. $\Gamma = \{(x, y, z) \in \mathbb{R} : y = e^x,\ z = x,\ x \in [0, 1]\}, \quad F(x, y, z) = (x, y, z).$

Exercise 5 (Ex 2.6 page 18)**.**
Let $F(x, y) = (x + y, -x)$ and $\Gamma = \left\{(x, y) \in \mathbb{R}^2 : y^2 + 4x^4 - 4x^2 = 0,\ x \geq 0\right\}$ parameterized by $\gamma(t) = (\sin t, \sin(2t))$ with $t \in [0, \pi]$.

1. Show that γ is indeed a parameterization of Γ.
2. Calculate $\int_\Gamma F \cdot \mathrm{d}l$.



Exercise 6 (Ex 2.5 page 18)**.**
Let Γ be a simple regular curve in $\mathbb{R}^3$, joining A and B.
Using Newton's Law (Force = mass $\times$ acceleration) , calculate the necessary work to move a particle of constant mass from A to B along Γ.