# Algèbre linéaire — Série 9

*CGC, EL, MX* — Réponses

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**1.** $\dim W = 2$

base: $\{(1,0,1),(0,1,3)\}$

$(7,-1,4),(-5,10,25) \in W$

**2.** **a)** $A_T = \begin{bmatrix} 1 & 3 & -5 & 7 \\ 1 & -1 & 0 & 1 \\ 0 & 3 & -1 & 0 \end{bmatrix}$

**b)** $A_T^{\mathcal{B}'\mathcal{B}} = \begin{bmatrix} 1 & 4 & -1 & 6 \\ 1 & 0 & 0 & 1 \\ -2 & -1 & 3 & -5 \end{bmatrix}$

**3.** **a)** $A_T = \begin{bmatrix} 3 & 1 & 0 \\ 0 & 3 & 2 \\ 0 & 0 & 3 \end{bmatrix}, \qquad \operatorname{rang}(A_T) = 3$

**b)** $\dim(\operatorname{Ker} T) = 0, \qquad \operatorname{Ker}(T) = \{0\}$

$\dim(\operatorname{Im} T) = 3, \qquad \operatorname{Im}(T) = \mathbb{P}_2$

**c)** $A_T^{\mathcal{B}} = \begin{bmatrix} 3 & 1 & -1 \\ 0 & 3 & 2 \\ 0 & 0 & 3 \end{bmatrix}$

**4.** $\begin{bmatrix} 1 & 2 \\ 0 & -1 \\ 2 & -2 \\ -1 & 2 \end{bmatrix}$

**5.** $\vec{u} \in \operatorname{Im} T$ et $\vec{u} \notin \operatorname{Ker} T$

**6.** **a)** F **b)** F **c)** V

**7.** $\dim W = 2$

base: $\{(1,0,2),(0,1,1)\}$

$(1,3,5), (1,5,7), (9,-8,10) \in W$

**9.** **a)** oui **b)** oui **c)** non

**10.** **a)** $A_T = \begin{bmatrix} 1 & -2 & 3 & -4 \\ 1 & 0 & 0 & -2 \\ 0 & 1 & 1 & 0 \end{bmatrix}$

**b)** $A_T^{\mathcal{B}'\mathcal{B}} = \begin{bmatrix} -1 & 0 & 0 & 1 \\ 0 & 2 & -3 & 2 \\ 1 & -1 & 4 & -3 \end{bmatrix}$

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**11.** **a)** $A_T = \begin{bmatrix} 2 & -2 & 4 \\ 5 & -4 & 7 \\ 3 & -2 & 3 \\ 1 & -1 & 2 \end{bmatrix}$, $\quad \operatorname{rang}(A_T) = 2$

**b)** $\dim(\operatorname{Ker} T) = 1$, $\quad \operatorname{Ker} T = \operatorname{Vect}\{(1,3,1)\}$

$\dim(\operatorname{Im} T) = 2$, $\quad \operatorname{Im} T = \operatorname{Vect}\{(2,5,3,1), (4,7,3,2)\}$

**12.** **a)** $A_T = \begin{bmatrix} 0 & 1 & 2 & 0 \\ 0 & 0 & 2 & 6 \\ 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$, $\quad \operatorname{rang}(A_T) = 3$

**b)** $\dim(\operatorname{Ker} T) = 1$, $\quad \operatorname{Ker} T = \mathbb{P}_0$

$\dim(\operatorname{Im} T) = 3$, $\quad \operatorname{Im} T = \mathbb{P}_2$

**c)** $A_T^{\mathcal{B}} = \begin{bmatrix} 0 & 1 & 0 & -5 \\ 0 & 0 & 2 & 8 \\ 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

**13.** **a)** $A_T = \begin{bmatrix} 0 & 0 & 2 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & -2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

**b)** $\dim(\operatorname{Ker} T) = 0$, $\quad \operatorname{Ker} T = \left\{ \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \right\}$

$\dim(\operatorname{Im} T) = 4$, $\quad \operatorname{Im} T = M_{2,2}(\mathbb{R})$

**14.** **b)** $A_T = \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 2 \end{bmatrix}$

**c)** $\dim(\operatorname{Ker} T) = 1$, $\quad$ base: $\left\{ \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \right\}$

$\dim(\operatorname{Im} T) = 3$, $\quad$ base: $\left\{ \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \right\}$