

MINI SOLUTIONS for Homework 6

Ex 6.3 (Different methods for computing determinants)

Compute the determinant of each of the following matrices in three ways: Once using cofactor expansion across a row, once using cofactor across a column, and once with row reduction.

$$A = \begin{pmatrix} 2 & 0 & 0 \\ 2 & 1 & 2 \\ 0 & 3 & 4 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 0 \\ 0 & 8 & 0 \end{pmatrix}$$

Solution:

$$\det(A) = -4, \quad \det(B) = 96$$

Ex 6.4 (Determinants based on another determinant)

Let

$$A = \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix}$$

and assume that $\det(A) = 7$. Compute the determinants of the following matrices

$$B = \begin{pmatrix} a+d & b+e & c+f \\ d & e & f \\ g & h & i \end{pmatrix}, \quad C = \begin{pmatrix} a & b & c \\ 2d+a & 2e+b & 2f+c \\ g & h & i \end{pmatrix}.$$

Solution:

$$\det(B) = 7, \quad \det(C) = 14$$

Ex 6.5 (More determinants)

Compute the determinants of the following matrices (You may use your preferred method or try to practice different methods.)

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 0 & 0 \\ 2 & 1 & 1 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & 1 & 0 & -1 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 2 & 3 & 1 \end{pmatrix}, \quad C = \begin{pmatrix} 10 & 5 & 10 & 5 \\ 6 & 9 & 0 & -3 \\ 3 & 0 & 0 & 3 \\ 1 & 0 & 1 & 1 \end{pmatrix}$$

Solution:

$$\det(A) = 1, \quad \det(B) = 2, \quad \det(C) = 0.$$

Ex 6.7 (Determinants and volume)

(a) Calculate the volume of the parallelepiped with the following vertices:

$$\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 4 \\ 0 \end{pmatrix}, \begin{pmatrix} -2 \\ -5 \\ 2 \end{pmatrix}, \begin{pmatrix} -1 \\ 2 \\ -1 \end{pmatrix}, \begin{pmatrix} -1 \\ -1 \\ 2 \end{pmatrix}, \begin{pmatrix} 0 \\ 6 \\ -1 \end{pmatrix}, \begin{pmatrix} -3 \\ -3 \\ 1 \end{pmatrix}, \begin{pmatrix} -2 \\ 1 \\ 1 \end{pmatrix}.$$

(b) Calculate the area of the triangle whose vertices are the points $(1, 2), (2, 4), (3, 3) \in \mathbb{R}^2$.

Solution:

(a) 15.

(b) $3/2$.