**MATH-111(en)** FALL 2025
**Linear Algebra** developed by Annina Iseli
taught by Andrei Negut

# MINI SOLUTIONS for Homework 5

Do NOT use determinants to solve the problems on this Homework. Next weeks exercises will be full of problems about determinants. This week, I want you to practice other methods.

### Ex 5.2 (Different methods for computing the inverse matrix)

Let $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$. Compute $A^{-1}$ using $\begin{cases} \text{(a) the formula for the inverse of a } 2 \times 2 \text{ matrix,} \\ \text{(b) row reduction.} \end{cases}$

**Solution:**

$$A^{-1} = \begin{pmatrix} -2 & 1 \\ \frac{3}{2} & -\frac{1}{2} \end{pmatrix}$$

### Ex 5.3 (More inverse matrix calculations)

Compute the inverses of the following matrices:

$$(a) \begin{pmatrix} 1 & 2 \\ -1 & 1 \end{pmatrix} \qquad (b) \begin{pmatrix} 1 & 1 & 2 \\ -1 & 0 & 1 \\ 1 & -2 & 1 \end{pmatrix} \qquad (c) \begin{pmatrix} 1 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ 3 & 2 & 1 & 0 \\ 4 & 3 & 2 & 1 \end{pmatrix}$$

**Solution:**

(a)

$$\begin{pmatrix} \frac{1}{3} & -\frac{2}{3} \\ \frac{1}{3} & \frac{1}{3} \end{pmatrix}$$

(b)

$$\frac{1}{8} \begin{pmatrix} 2 & -5 & 1 \\ 2 & -1 & -3 \\ 2 & 3 & 1 \end{pmatrix}$$

(c)

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ -2 & 1 & 0 & 0 \\ 1 & -2 & 1 & 0 \\ 0 & 1 & -2 & 1 \end{pmatrix}$$

### Ex 5.5 (Inverting a linear transformation)

Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be the following linear transformation:

$$T(x) = \begin{pmatrix} x_1 + 2x_2 \\ 2x_1 - 3x_3 \\ x_2 + x_3 \end{pmatrix}.$$

Prove that $T$ is invertible and give a formula that defines the inverse transformation $T^{-1}$ of $T$.

**Solution:**

$$T^{-1}(y) = \begin{pmatrix} -3 & 2 & 6 \\ 2 & -1 & -3 \\ -2 & 1 & 4 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = \begin{pmatrix} -3y_1 + 2y_2 + 6y_3 \\ 2y_1 - y_2 - 3y_3 \\ -2y_1 + y_2 + 4y_3 \end{pmatrix}.$$