MATH-111(en) FALL 2025

Linear Algebra

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MINI SOLUTIONS for Homework 4

Ex 4.4 (Some matrix products)

Let

$$A = \begin{pmatrix} 4 & -5 & 3 \\ 5 & 7 & -2 \\ -3 & 2 & -1 \end{pmatrix}, \quad B = \begin{pmatrix} 7 & 0 & -1 \\ -1 & 5 & 2 \end{pmatrix}, \quad C = \begin{pmatrix} -1 & 5 \\ 4 & -3 \\ 1 & 0 \end{pmatrix}.$$

Compute AC, BC and CB.

Solution:

$$AC = \begin{pmatrix} -21 & 35 \\ 21 & 4 \\ 10 & -21 \end{pmatrix}, \quad BC = \begin{pmatrix} -8 & 35 \\ 23 & -20 \end{pmatrix} \quad \text{and} \quad CB = \begin{pmatrix} -12 & 25 & 11 \\ 31 & -15 & -10 \\ 7 & 0 & -1 \end{pmatrix}.$$

Ex 4.5 (When do these matrices commute?)

Consider the matrices

$$A = \begin{pmatrix} 3 & -4 \\ -5 & 1 \end{pmatrix} \quad \text{and } B = \begin{pmatrix} 7 & 4 \\ 5 & k \end{pmatrix}.$$

For which values of k does the equality $AB = BA$ hold?

Solution:

$AB = BA$ if and only if $k = 9$.

Ex 4.6 (More matrix products)

Consider the matrices:

$$A = \begin{pmatrix} 7 & 0 \\ -1 & 5 \\ -1 & 2 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & 4 \\ -4 & 0 \end{pmatrix}, \quad C = \begin{pmatrix} 7 \\ -3 \end{pmatrix}, \quad D = \begin{pmatrix} 8 & 2 \end{pmatrix}.$$

If they are defined, compute the matrices

$$AB, CA, CD, DC, DBC, BDB, A^TA \text{ and } AA^T.$$

For those that are not defined, explain why.

Solution:

$$AB = \begin{pmatrix} 7 & 28 \\ -21 & -4 \\ -9 & -4 \end{pmatrix}, \; CD = \begin{pmatrix} 56 & 14 \\ -24 & -6 \end{pmatrix}, \; DC = [50], \; DBC = [-96].$$

$$A^TA = \begin{pmatrix} 51 & -7 \\ -7 & 29 \end{pmatrix}, \qquad AA^T = \begin{pmatrix} 49 & -7 & -7 \\ -7 & 26 & 11 \\ -7 & 11 & 5 \end{pmatrix}.$$

Ex 4.7 (Multiplication by diagonal matrices)
Consider the matrices

$$A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 4 & 5 \end{pmatrix} \quad \text{and} \quad D = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 4 \end{pmatrix}.$$

1. Compute AD and DA and explain how the rows and columns of A change when one multiplies A by D from the right and from the left.

2. Find all the diagonal matrices M of dimension 3×3 such that $AM = MA$.

Solution:

1. $$AD = \begin{pmatrix} 2 & 3 & 4 \\ 2 & 6 & 12 \\ 2 & 12 & 20 \end{pmatrix} \quad \text{and} \quad DA = \begin{pmatrix} 2 & 2 & 2 \\ 3 & 6 & 9 \\ 4 & 16 & 20 \end{pmatrix}.$$

2. $M = \lambda I_3$, $\lambda \in \mathbb{R}$ where I_3 is the 3×3 identity matrix.

Ex 4.8 (Upper triangular matrices)

1. Compute $\begin{pmatrix} 1 & a \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & b \\ 0 & 1 \end{pmatrix}$ for $a, b \in \mathbb{R}$.

2. Let $A = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$, and compute the following matrices: A^8, A^TA, AA^T.

3. Find a matrix B such that $\begin{pmatrix} 1 & 5 \\ 0 & 1 \end{pmatrix} \cdot B = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$.

Solution:

1. $$\begin{pmatrix} 1 & a \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & b \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & a+b \\ 0 & 1 \end{pmatrix}$$

2. $$A^8 = \begin{pmatrix} 1 & 8 \\ 0 & 1 \end{pmatrix}, \qquad A^TA = \begin{pmatrix} 1 & 1 \\ 1 & 2 \end{pmatrix}, \qquad AA^T \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}.$$

3. $$B = \begin{pmatrix} 1 & -5 \\ 0 & 1 \end{pmatrix}.$$

Ex 4.10 (A matrix equation)
Find a solution X for the matrix equation

$$\begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{pmatrix} X = \begin{pmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \\ 3 & 5 & 7 \end{pmatrix}.$$

Solution:

$$X = \begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 2 & 3 & 4 \end{pmatrix}$$

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