MATH-111(en)
Linear Algebra

FALL 2025
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Graded Homework 3

1 December 2025

Rules:

- Keep your solution concise but complete (i.e. justify your steps).
- Submit (either LaTeX or scan/photo of legible handwriting) on Moodle by December 12.
- Aim to spend 30 minutes in total.
- Address questions to your graders: Omar Zakariya and Damien Freddy Jacques.

Problem 1

Let $v_1, \dots, v_n \in \mathbb{R}^n$ be linearly independent vectors.
Let A be a diagonalizable matrix in $\mathbb{R}^{n\times n}$ so that the vectors $v_1, \dots, v_n$ are eigenvectors of A for the eigenvalues $\alpha_1, \dots, \alpha_n$ respectively.
Let B be a diagonalizable matrix in $\mathbb{R}^{n\times n}$ so that the vectors $v_1, \dots, v_n$ are eigenvectors of B for the eigenvalues $\beta_1, \dots, \beta_n$ respectively.

Prove that $A - B$ is diagonalizable and satisfies $\det(A-B) = (\alpha_1 - \beta_1)\cdots(\alpha_n - \beta_n)$.

Problem 2

Let $A \in \mathbb{R}^{n\times n}$ and let 0 be the $n\times n$ zero matrix.

Show that if A is diagonalizable and $A^2 = 0$, then $A = 0$.

BE AWARE : "Grade" just means that we provide you with feedback / corrections on your submission. It does not mean that you will receive a numeric grade. This is an exercise and does not count towards the final grade of this course. Participation is not mandatory.

For better feedback, we will assign letters to each graded problem:
A = *good solution, only minor mistakes or imperfect (but still clear) notation.*
B = *your solution catches relevant aspects but also has considerable flaws or gaps.*
C = *your solution was mostly wrong and/or there were many substantial mistakes or gaps.*