

MATH-111(en)
Linear Algebra

FALL 2025
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Graded Homework 2

3 November 2025

Rules:

- Keep your solution concise but complete (i.e. justify your steps).
 - Submit (either LaTeX or scan/photo of legible handwriting) on Moodle by November 14.
 - Aim to spend 30 minutes in total.
 - Address questions to your graders: Eliota Braha and Omar Zakariya.
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Problem 1

Let $A \in \mathbb{R}^{m \times n}$ be a matrix such that its reduced echelon form has exactly k zero rows. Determine the dimension of $\text{Ker}(A)$ and of $\text{Col}(A)$ in terms of m , n and k .

Problem 2

Let $f : \mathbb{P}_3 \rightarrow \mathbb{R}^4$ be the linear function defined by

$$f(1) = \begin{pmatrix} 1 \\ -1 \\ 1 \\ 1 \end{pmatrix}, \quad f(x) = \begin{pmatrix} -1 \\ 2 \\ 2 \\ 2 \end{pmatrix}, \quad f(x^2) = \begin{pmatrix} 0 \\ 1 \\ 3 \\ 3 \end{pmatrix}, \quad f(x^3) = \begin{pmatrix} -3 \\ 5 \\ 3 \\ 3 \end{pmatrix}$$

(a) Find a basis for $\text{Ker}(f)$.

(b) Determine whether $\begin{pmatrix} 0 \\ 0 \\ 2 \\ -1 \end{pmatrix}$ is in $\text{Im}(f)$.

BE AWARE : “Grade” just means that we provide you with feedback / corrections on your submission. It does not mean that you will receive a numeric grade. This is an exercise and does not count towards the final grade of this course. Participation is not mandatory.

For better feedback, we will assign letters to each graded problem:

A = good solution, only minor mistakes or imperfect (but still clear) notation.

B = your solution catches relevant aspects but also has considerable flaws or gaps.

C = your solution was mostly wrong and/or there were many substantial mistakes or gaps.