













**Ens: Leonid Monin**  
**Analysis I - Section**  
**Fall 2025**  
**1 hour**

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**Do not turn the page before the start of the exam. This document is double-sided, has 4 pages, the last ones possibly blank. Do not unstaple.**

- Place your student card on your table.
- **No other paper materials** are allowed to be used during the exam.
- Using a **calculator** or any electronic device is not permitted during the exam.
- For the **multiple choice** questions, we give :
  - +3 points if your answer is correct,
  - 0 points if you give no answer or more than one,
  - 1 points if your answer is incorrect.
- For the **true/false** questions, we give :
  - +1 points if your answer is correct,
  - 0 points if you give no answer or more than one,
  - 1 points if your answer is incorrect.
- Use a **black or dark blue ballpen** and clearly erase with **correction fluid** if necessary.
- If a question is wrong, the teacher may decide to nullify it.

Respectez les consignes suivantes   Read these guidelines   Beachten Sie bitte die unten stehenden Richtlinien		
choisir une réponse   select an answer Antwort auswählen	ne PAS choisir une réponse   NOT select an answer NICHT Antwort auswählen	Corriger une réponse   Correct an answer Antwort korrigieren
  		 
ce qu'il ne faut <b>PAS</b> faire   what should <b>NOT</b> be done   was man <b>NICHT</b> tun sollte		
     		

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### First part: multiple choice questions

For each question, mark the box corresponding to the correct answer. Each question has **exactly one** correct answer.

**Question 1 :** The series

$$\sum_{n=1}^{\infty} \frac{e^{\lambda n}}{n^{1+\lambda}}$$

converges if and only if  $\lambda \in I$ , where  $I$  is the set

- $] -\infty, -1[$         $] -\infty, 0[$         $[-1, +\infty[$         $] -\infty, 0]$

**Question 2 :** Let  $(u_n)_{n \geq 0}$  be the sequence defined by  $u_0 = \sqrt{3}$  and for  $n \geq 1$ ,  $u_n = \sqrt{3u_{n-1}}$ . Then

- $\lim_{n \rightarrow \infty} u_n = \sqrt{3}$         $\lim_{n \rightarrow \infty} u_n = 0$         $\lim_{n \rightarrow \infty} u_n = 3$         $(u_n)_{n \geq 0}$  diverge

**Question 3 :** Let  $A \subset \mathbb{R}$  be the set defined by

$$A = \left\{ x \in \mathbb{R}^* : \frac{1}{x} \geq 2 \right\}.$$

Then

- $\inf A = 2$         $\inf A = \frac{1}{2}$   
  $A$  is not bounded from below        $\inf A = 0$

**Question 4 :** Consider the equation

$$\frac{|z|}{z} = \frac{z^2}{4(\cos(\frac{\pi}{3}) + i \sin(\frac{\pi}{3}))}.$$

Among the complex numbers given below, which one is a solution of the equation ?

- $z = \sqrt[3]{4}(\cos(\frac{13\pi}{12}) + i \sin(\frac{13\pi}{12}))$         $z = \sqrt[3]{4}(\cos(\frac{\pi}{9}) + i \sin(\frac{\pi}{9}))$   
  $z = 2(\cos(\frac{7\pi}{9}) + i \sin(\frac{7\pi}{9}))$         $z = 2(\cos(\frac{7\pi}{12}) + i \sin(\frac{7\pi}{12}))$

**Question 5 :** Let  $a_0 \in \mathbb{R}$  and  $(a_n)_{n \geq 0}$  a sequence of real numbers satisfying the following recurrence relation for  $n \geq 1$

$$a_n = \frac{a_{n-1}}{2} + \frac{1}{2}.$$

Then:

- if  $a_0 < 1$  the sequence is decreasing  
 if  $a_0 = 0$  the sequence is convergent  
 if  $a_0 < 0$ ,  $\lim_{n \rightarrow \infty} a_n = -\infty$   
 if  $a_0 > 1$  the sequence is increasing

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**Question 6 :** Let  $(a_n)_{n \geq 0}$  be a sequence such that  $\lim_{n \rightarrow \infty} a_n = 2$ , and let  $(b_n)_{n \geq 0}$  be the sequence given by

$$b_n = 1 + a_n \cos\left(n \frac{\pi}{2}\right), \quad n \geq 0.$$

Then

- $\liminf_{n \rightarrow \infty} b_n = 3$  and  $\limsup_{n \rightarrow \infty} b_n = 3$         $\liminf_{n \rightarrow \infty} b_n = -2$  and  $\limsup_{n \rightarrow \infty} b_n = 2$   
  $\liminf_{n \rightarrow \infty} b_n = -1$  and  $\limsup_{n \rightarrow \infty} b_n = 3$         $\liminf_{n \rightarrow \infty} b_n = 1$  and  $\limsup_{n \rightarrow \infty} b_n = 3$

**Question 7 :** Consider the sequence

$$x_n = e^{2\sqrt{n^2+1}-n}.$$

Then,

- $\lim_{n \rightarrow \infty} x_n = 1$         $\lim_{n \rightarrow \infty} x_n = +\infty$   
  $\lim_{n \rightarrow \infty} x_n = e$         $\lim_{n \rightarrow \infty} x_n = 0$

**Question 8 :** The series  $\sum_{k=2}^{\infty} \frac{(-1)^k}{\sqrt{k^3 - k}}$

- does not converge and does not converge absolutely       does not converge but converges absolutely  
 converges and converges absolutely       converges but does not converge absolutely

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## Second part: true/false questions

For each question, mark the box (without erasing) TRUE if the statement is **always true** and the box FALSE if it is **not always true** (i.e., it is sometimes false).

**Question 9 :** Let  $A, B \subset \mathbb{R}$  be two bounded nonempty sets and  $c \in \mathbb{R}$ . Then,

$$\sup\{x + c : x \in A\} - \sup\{x + c : x \in B\} = \sup A - \sup B.$$

TRUE     FALSE

**Question 10 :** Let  $(a_n)_{n \geq 0}$  and  $(b_n)_{n \geq 0}$  be two sequences such that for all  $n \in \mathbb{N}$ ,  $0 < a_n < b_n$ . If the series  $\sum_{n=0}^{\infty} a_n$  diverges, then the series  $\sum_{n=0}^{\infty} \frac{1}{b_n}$  converges.

TRUE     FALSE

**Question 11 :** The roots of the polynomial  $z^4 + z^3 - 2z^2 + 2z + 4$  are  $\{-2, -1, \frac{1}{4}, 1 + i\}$ .

TRUE     FALSE

**Question 12 :** Let  $(a_n)_{n \geq 0}$  be a bounded sequence such that for all  $n \in \mathbb{N}$ ,  $a_n > 3$ . Then  $\liminf_{n \rightarrow \infty} a_n > 3$ .

TRUE     FALSE

**Question 13 :** Let  $\lambda \in \mathbb{R}^*$  and  $(a_n)_{n \geq 0}$  be the sequence defined by

$$a_n = \left( \frac{\lambda + n}{\lambda n} \right)^n.$$

Then, for every  $\lambda \in \mathbb{R}^*$  such that  $(a_n)$  converges, one has

$$\lim_{n \rightarrow \infty} a_n = 0.$$

TRUE     FALSE