EPFL

XXX

Ens: Leonid Monin
Analysis I - Section
Fall 2025
1 hour

999999

Do not turn the page before the start of the exam. This document is double-sided, has 4 pages, the last ones possibly blank. Do not unstaple.

- Place your student card on your table.
- **No other paper materials** are allowed to be used during the exam.
- Using a **calculator** or any electronic device is not permitted during the exam.
- For the **multiple choice** questions, we give :
  $+3$ points if your answer is correct,
  0 points if you give no answer or more than one,
  -1 points if your answer is incorrect.
- For the **true/false** questions, we give :
  $+1$ points if your answer is correct,
  0 points if you give no answer or more than one,
  -1 points if your answer is incorrect.
- Use a **black or dark blue ballpen** and clearly erase with **correction fluid** if necessary.
- If a question is wrong, the teacher may decide to nullify it.

Respectez les consignes suivantes | Read these guidelines | Beachten Sie bitte die unten stehenden Richtlinien

choisir une réponse | select an answer Antwort auswählen	ne PAS choisir une réponse | NOT select an answer NICHT Antwort auswählen	Corriger une réponse | Correct an answer Antwort korrigieren

ce qu'il ne faut **PAS** faire | what should **NOT** be done | was man **NICHT** tun sollte

First part: multiple choice questions

For each question, mark the box corresponding to the correct answer. Each question has **exactly one** correct answer.

Question 1 : The series

$$\sum_{n=1}^{\infty} \frac{e^{\lambda n}}{n^{1+\lambda}}$$

converges if and only if $\lambda \in I$, where I is the set

☐ $]-\infty, -1[$ ☐ $]-\infty, 0[$ ☐ $[-1, +\infty[$ ☐ $]-\infty, 0]$

Question 2 : Let $(u_n)_{n\geq 0}$ be the sequence defined by $u_0 = \sqrt{3}$ and for $n \geq 1$, $u_n = \sqrt{3u_{n-1}}$. Then

☐ $\lim_{n\to\infty} u_n = \sqrt{3}$ ☐ $\lim_{n\to\infty} u_n = 0$ ☐ $\lim_{n\to\infty} u_n = 3$ ☐ $(u_n)_{n\geq 0}$ diverge

Question 3 : Let $A \subset \mathbb{R}$ be the set defined by

$$A = \left\{ x \in \mathbb{R}^* \ : \ \frac{1}{x} \geq 2 \right\}.$$

Then

☐ $\inf A = 2$ ☐ $\inf A = \frac{1}{2}$

☐ A is not bounded from below ☐ $\inf A = 0$

Question 4 : Consider the equation

$$\frac{|z|}{z} = \frac{z^2}{4(\cos\left(\frac{\pi}{3}\right) + \mathrm{i}\sin\left(\frac{\pi}{3}\right))}.$$

Among the complex numbers given below, which one is a solution of the equation ?

☐ $z = \sqrt[3]{4}(\cos\left(\frac{13\pi}{12}\right) + \mathrm{i}\sin\left(\frac{13\pi}{12}\right))$ ☐ $z = \sqrt[3]{4}(\cos\left(\frac{\pi}{9}\right) + \mathrm{i}\sin\left(\frac{\pi}{9}\right))$

☐ $z = 2(\cos\left(\frac{7\pi}{9}\right) + \mathrm{i}\sin\left(\frac{7\pi}{9}\right))$ ☐ $z = 2(\cos\left(\frac{7\pi}{12}\right) + \mathrm{i}\sin\left(\frac{7\pi}{12}\right))$

Question 5 : Let $a_0 \in \mathbb{R}$ and $(a_n)_{n\geq 0}$ a sequence of real numbers satisfying the following recurrence relation for $n \geq 1$

$$a_n = \frac{a_{n-1}}{2} + \frac{1}{2}.$$

Then:

☐ if $a_0 < 1$ the sequence is decreasing

☐ if $a_0 = 0$ the sequence is convergent

☐ if $a_0 < 0$, $\lim_{n\to\infty} a_n = -\infty$

☐ if $a_0 > 1$ the sequence is increasing

Question 6 : Let $(a_n)_{n\geq 0}$ be a sequence such that $\lim_{n\to\infty} a_n = 2$, and let $(b_n)_{n\geq 0}$ be the sequence given by

$$b_n = 1 + a_n \cos\left(n\frac{\pi}{2}\right) , \qquad n \geq 0 .$$

Then

- ☐ $\liminf_{n\to\infty} b_n = 3$ and $\limsup_{n\to\infty} b_n = 3$
- ☐ $\liminf_{n\to\infty} b_n = -2$ and $\limsup_{n\to\infty} b_n = 2$
- ☐ $\liminf_{n\to\infty} b_n = -1$ and $\limsup_{n\to\infty} b_n = 3$
- ☐ $\liminf_{n\to\infty} b_n = 1$ and $\limsup_{n\to\infty} b_n = 3$

Question 7 : Consider the sequence

$$x_n = e^{2\sqrt{n^2+1}-n}.$$

Then,

- ☐ $\lim_{n\to\infty} x_n = 1$
- ☐ $\lim_{n\to\infty} x_n = +\infty$
- ☐ $\lim_{n\to\infty} x_n = e$
- ☐ $\lim_{n\to\infty} x_n = 0$

Question 8 : The series $\sum_{k=2}^{\infty} \frac{(-1)^k}{\sqrt{k^3-k}}$

- ☐ does not converge and does not converge absolutely
- ☐ does not converge but converges absolutely
- ☐ converges but does not converge absolutely
- ☐ converges and converges absolutely

Second part: true/false questions

For each question, mark the box (without erasing) TRUE if the statement is **always true** and the box FALSE if it is **not always true** (i.e., it is sometimes false).

Question 9 : Let $A, B \subset \mathbb{R}$ be two bounded nonempty sets and $c \in \mathbb{R}$. Then,

$$\sup\{x + c : x \in A\} - \sup\{x + c : x \in B\} = \sup A - \sup B.$$

☐ TRUE ☐ FALSE

Question 10 : Let $(a_n)_{n\geq 0}$ and $(b_n)_{n\geq 0}$ be two sequences surch that for all $n \in \mathbb{N}$, $0 < a_n < b_n$. If the series $\sum_{n=0}^{\infty} a_n$ diverges, then the series $\sum_{n=0}^{\infty} \frac{1}{b_n}$ converges.

☐ TRUE ☐ FALSE

Question 11 : The roots of the polynomial $z^4 + z^3 - 2z^2 + 2z + 4$ are $\{-2, -1, \frac{1}{4}, 1 + i\}$.

☐ TRUE ☐ FALSE

Question 12 : Let $(a_n)_{n\geq 0}$ be a bounded sequence such that for all $n \in \mathbb{N}$, $a_n > 3$. Then $\liminf_{n\to\infty} a_n > 3$.

☐ TRUE ☐ FALSE

Question 13 : Let $\lambda \in \mathbb{R}^*$ and $(a_n)_{n\geq 0}$ be the sequence defined by

$$a_n = \left(\frac{\lambda + n}{\lambda n}\right)^n.$$

Then, for every $\lambda \in \mathbb{R}^*$ such that (a_n) converges, one has

$$\lim_{n\to\infty} a_n = 0.$$

☐ TRUE ☐ FALSE