Analysis I (English)
Fall Semester 2025–2026

Analysis 1 - Exercise Set 3

Remember to check the correctness of your solutions whenever possible.

To solve the exercises you can use only the material you learned in the course.

1. Let $[x]$ denote the integral part of a number $x \in \mathbb{R}$.
   Prove that, for every $x \in \mathbb{R}$, $[x] = -[-x]$.

2. Let $a, b \in \mathbb{R}$. Prove that $||a| - |b|| \leq |a - b|$ and $||a| - |b|| \leq |a + b|$

3. Compute $\sup S$ and $\inf S$ where $S \subseteq \mathbb{R}$ is defined as

   (a) $S := \bigcup_{n=1}^{\infty} [-1 + \frac{1}{n}, 1 - \frac{1}{n}]$. Does S admit maximum and/or minimum?

   (b) $S := \bigcap_{n=1}^{\infty} (-1 - \frac{1}{n}, 1 + \frac{1}{n})$. Does S admit maximum and/or minimum?

4. Compute $\min S$ where $S \subseteq \mathbb{N}$ is defined as

   (a) $S := \{n \in \mathbb{N} : \sqrt{n} > 17\}$

   (b) $S := \{n \in \mathbb{N} : \sum_{i=1}^{n} i \geq 17\}$

   (c) $S := \{n \in \mathbb{N} : \sum_{i=1}^{n} 2^{-i} > 1.7\}$

5. Compute $\max S$ where $S \subseteq \mathbb{Z}$ is defined as

   (a) $S = \{n \in \mathbb{Z} \mid n \neq 0 \text{ and } n + \frac{20}{n} < 9\}$

   (b) $S = \{n \in \mathbb{Z} \mid (\sqrt{3})^n \leq 10^{17}\}$.

   (c) $S = \{n \in \mathbb{Z} \mid \alpha^n \leq C\}$ where $\alpha > 1$ and $C > 1$ are constants. [You must discuss how $\max S$ varies, when α and C vary.]

6. For the following complex numbers z compute the real and imaginary part, the complex conjugate $\bar{z}$, the absolute value $|z|$, the argument (also called phase) $\arg(z)$ and the inverse z^{-1}:

$$z = \frac{1}{2} + \frac{\sqrt{3}}{2}i; \quad z = 16i; \quad z = 2 + 3i - 3e^{i\frac{\pi}{2}}; \quad z = e^{-5\pi i} + i.$$

7. Write the following complex numbers in the form $x + iy$.

   (a) i^{17}

   (b) $\frac{4-i}{3-2i}$

   (c) $2i(i-1) + \left(\overline{\sqrt{3} + i}\right)^3 + (1+i)\overline{(1+i)}$