Irrigation and Drainage Engineering

(Soil Water Regime Management)

(ENV-549)

4ETCS, Master option

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Platform of Hydraulic Constructions

Project: flooding in Nouakchott (Mauritania) – Groundwater modelling and numerical solutions for steady state

Water budget

- Input:
  - Precipitation
  - Groundwater recharge = definition
  - Injection
- Output:
  - Evapotranspiration
  - Pumping
  - Groundwater flow

Groundwater flow

- Groundwater = continuous variable (a value at a time depends on the value at the previous time)
- Determination of groundwater flow by measuring groundwater depth in observation wells (single point measurement)
- From groundwater depth measurements ➔ piezometric map
- Groundwater flow from the highest hydraulic heads to the lowest ➔ diffusivity equation

Diffusion equation

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- Continuity equation

$$\frac{\partial \rho}{\partial t} + \rho \nabla \cdot \vec{U} = 0$$

  - 2D situation → $w = 0$
  - Steady state → $\frac{\partial}{\partial t} = 0$

$$\vec{U} = (u, v, w)$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

- Darcy equation

$$\vec{U_i} = -K_i \frac{dh}{di}$$

$$\frac{\partial}{\partial x}\left(-K_x \frac{\partial h}{\partial x}\right) + \frac{\partial}{\partial y}\left(-K_y \frac{\partial h}{\partial y}\right) = 0$$

Directionally homogeneous and isotropic

$$K_x \frac{\partial^2 h}{\partial x^2} + K_y \frac{\partial^2 h}{\partial y^2} = 0$$

Fully homogeneous and isotropic $(K_x = K_y)$

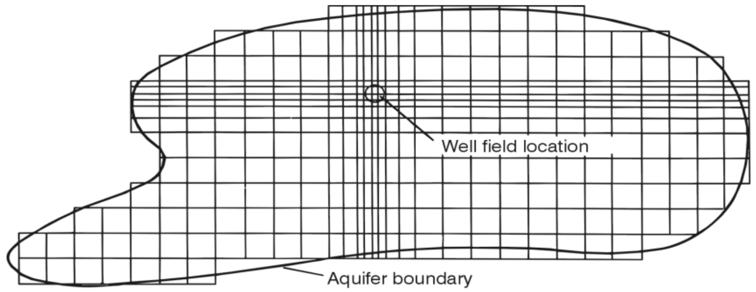
$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} = \nabla^2 h = 0$$

Laplace equation

Numerical modelling

- Simplifying assumptions for the diffusion equation
- Spatial discretization (cells of a grid) and temporal discretization (time steps)

➔ Finite difference to estimate the partial derivative in each cell (one equation per cell)



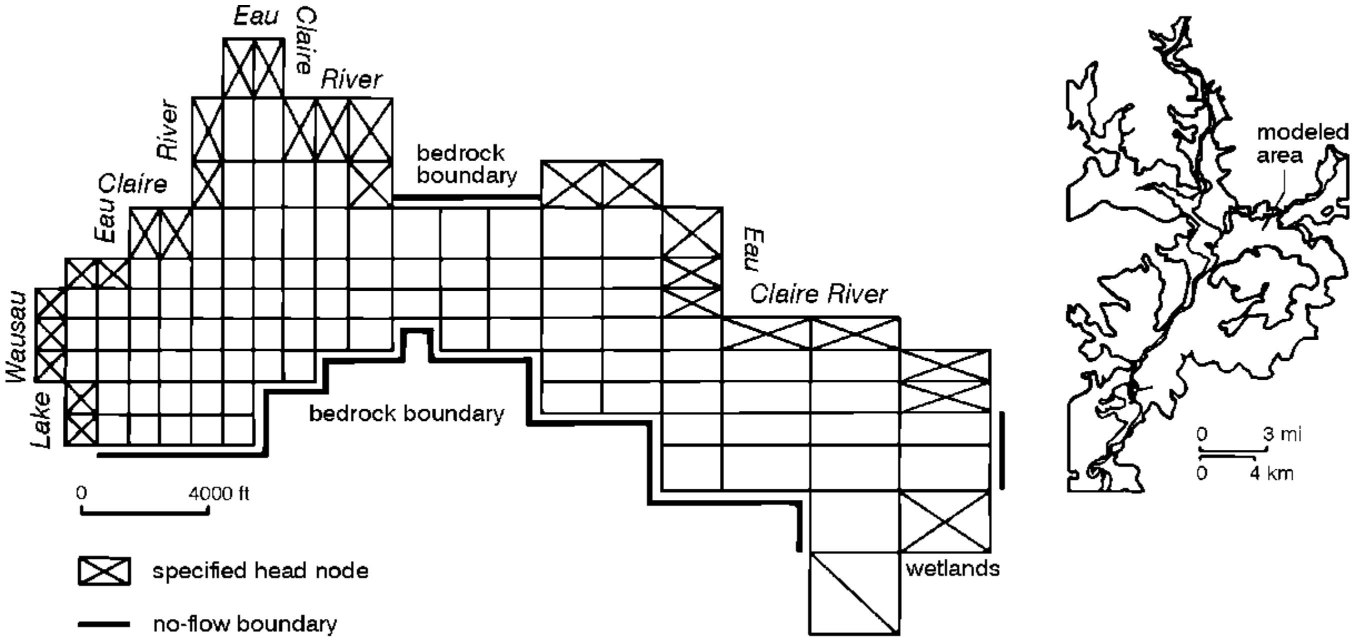
Fetter, 1994

More on numerical solutions vs analytical solutions?
➔ Bear, J., & Cheng, A. H.-D. (2010). Modeling Groundwater Flow and Contaminant Transport. Springer Netherlands. https://doi.org/10.1007/978-1-4020-6682-5

Needed information for simulation

- Hydraulic conductivity (or transmissivity) in each cell
- Pumping and groundwater recharge in each cell
- Aquifer geometry and spatial discretization
- Boundary conditions (constant head;

  specified flow, including no flow)



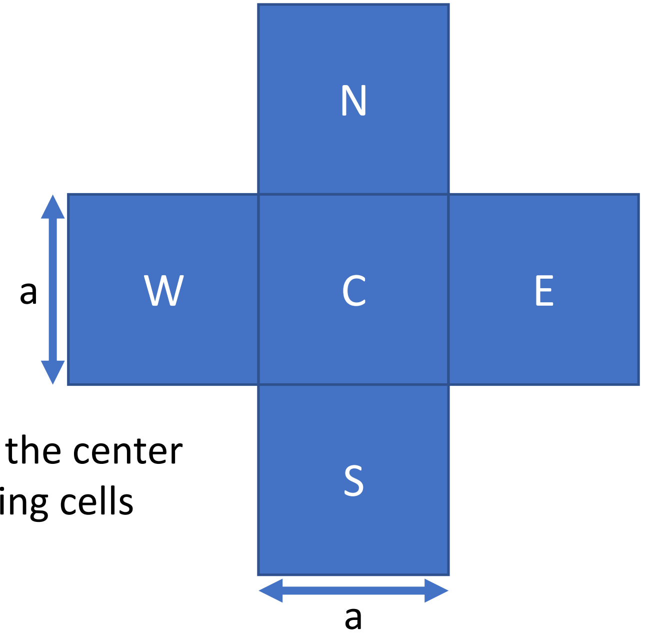
Anderson et Woessner, 1991

Numerical solution

- Finite difference solution $U_{Ci} = -K_{Ci}\frac{\Delta h_{Ci}}{L} = K_{Ci}\frac{\Delta h_{iC}}{L}$

Hp: unconfined water table, 2D horizontal flow, homogeneous and isotropic soil

➔ Darcy law on a square-shaped grid $U_{Ci} = -K_{Ci}\frac{\Delta h_{Ci}}{L} = K_{Ci}\frac{\Delta h_{iC}}{L}$



➔ Flow rate $Q_{Ci} = U_{Ci}\, a\, b = K_{Ci}\frac{\Delta h_{iC}}{L}\, a\, b = T_{Ci}\frac{\Delta h_{iC}}{L}\, a$

$T_{Ci} = K_{Ci}\, b$ Transmissivity

L is the distance between the center of two neighbouring cells
b is the aquifer thickness

➔ Continuity equation (steady state) $\sum_i Q_{Ci} = 0$

$$\sum_i Q_{Ci} = 0 \;\rightarrow\; \sum_i T_{Ci}\frac{\Delta h_{iC}}{L}\, a = 0 \;\xrightarrow[L=a]{\text{for square cells}}\; \sum_i T_{Ci}\,\Delta h_{iC} = 0$$

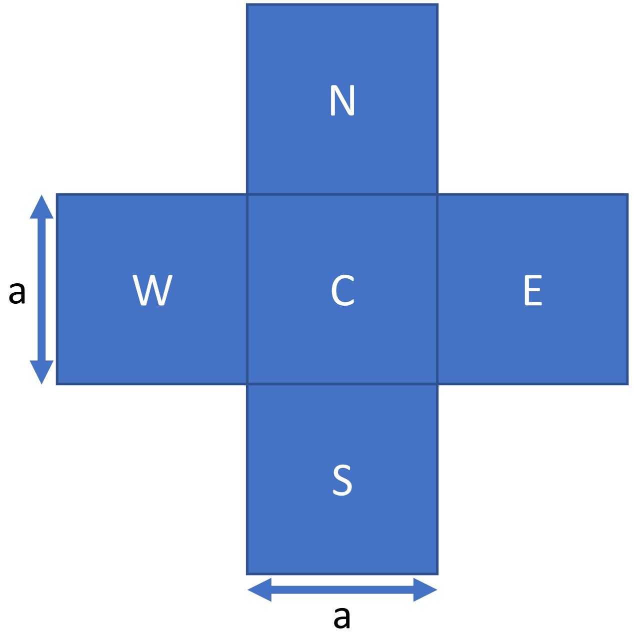
Steady state

$$\sum_i T_{Ci}\,\Delta h_{iC} = 0$$

$$T_{CN}(h_C - h_N) + T_{CS}(h_C - h_S) + T_{CE}(h_C - h_E) + T_{CW}(h_C - h_W) = 0$$

$$h_C = \frac{T_{CN}h_N + T_{CS}h_S + T_{CE}h_E + T_{CW}h_W}{T_{CN} + T_{CS} + T_{CE} + T_{CW}}$$

$$h_C = \frac{\sum T_{Ci}h_i}{\sum T_{Ci}}$$



For T = constant → $$h_C = \frac{T\sum h_i}{4\,T} = \frac{\sum h_i}{4}$$

Steady state

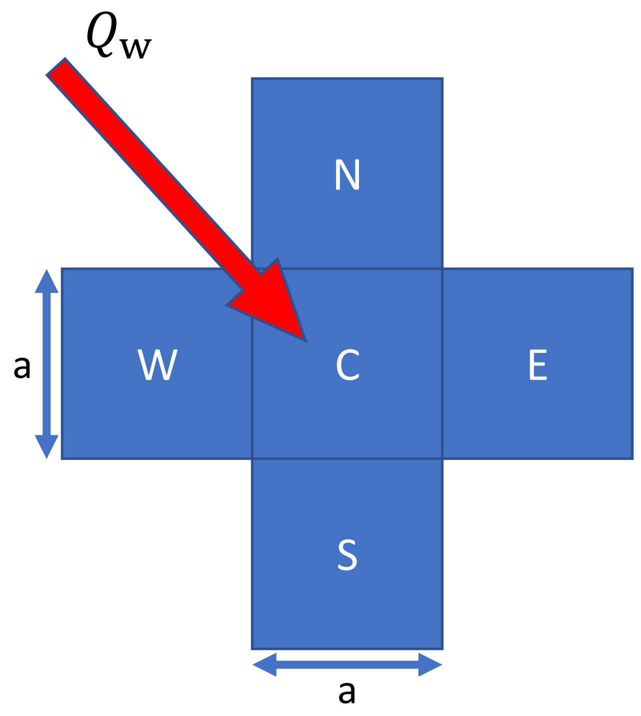
External flow in cell C (recharge/injection → $Q_w > 0$, pumping → $Q_w < 0$)

$$\sum_j Q_j = 0 \quad \rightarrow \quad \sum_{i=N,S,W,E} Q_{iC} + Q_\mathrm{w} = 0$$

$$Q_{iC} = T_{iC}\, \Delta h_{iC}$$

$$\sum_{i=N,S,W,E} T_{iC}\, \Delta h_{iC} + Q_\mathrm{w} = 0$$

$$h_C = \frac{\sum T_{iC}\, h_i + Q_\mathrm{w}}{\sum T_{iC}}$$



- Recharge / injection → $Q_w > 0$
- Pumping / extraction → $Q_w < 0$

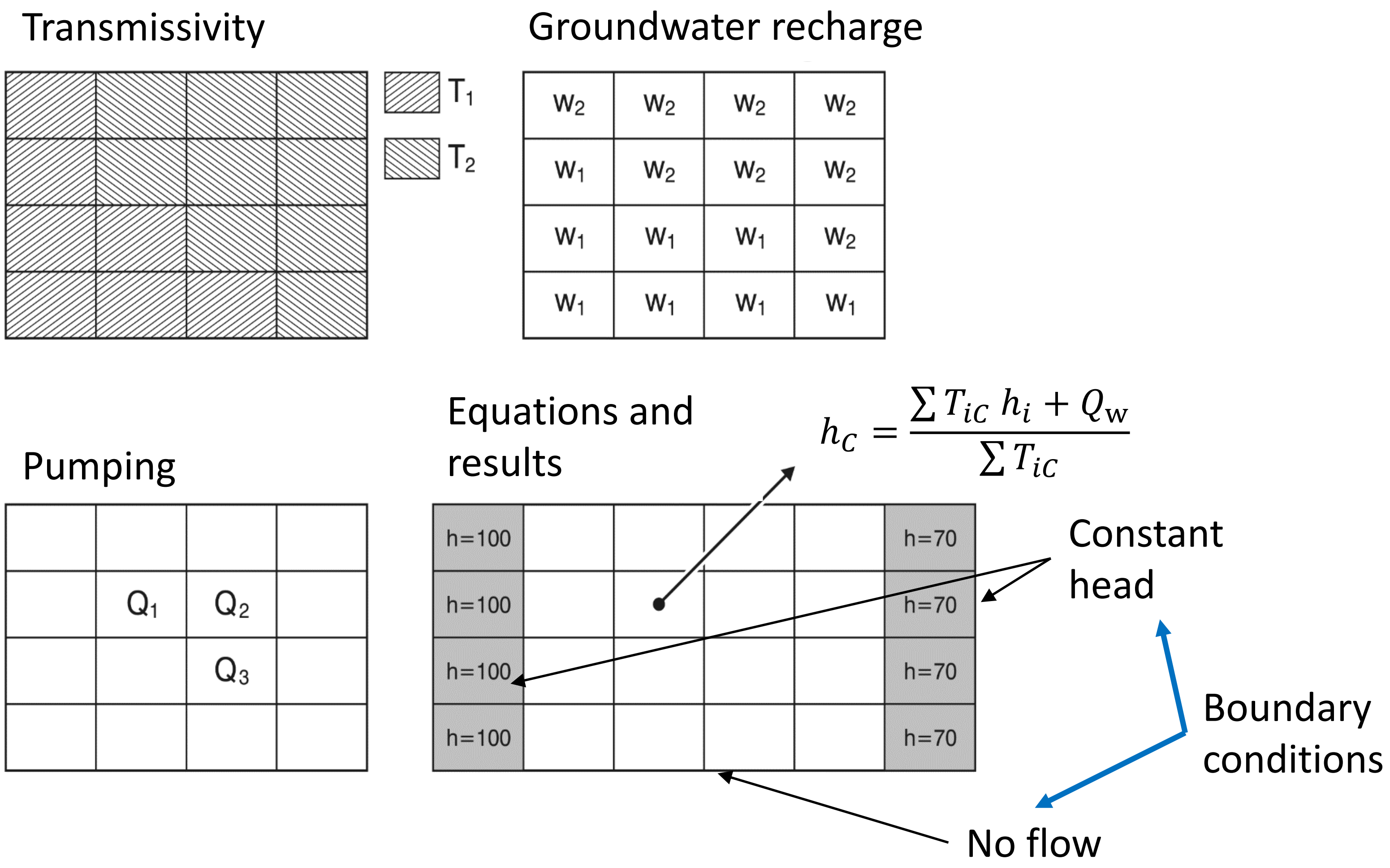
Steady state

- In Excel??

➔One equation for each cell

➔System with N equations

➔Iterative computing



More on groundwater modelling?
➔ Anderson, M. P., Woessner, W. W., & Hunt, R. J. (2015). Applied Groundwater Modeling: Simulation of Flow and Advective Transport (Second Edition). Academic Press, Elsevier Inc.

Model precision

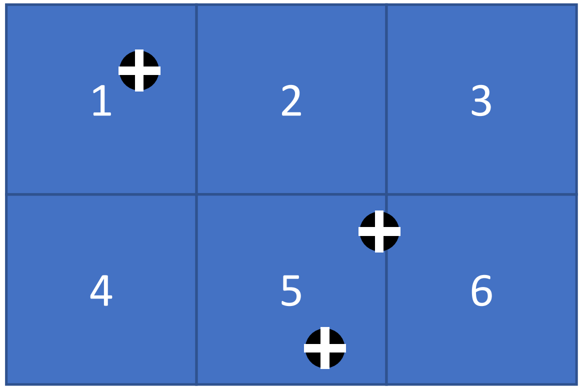
What do we use to compute the precision of a simulation?

➔ The comparison of the simulated and observed variable(s) of interest

Which data for the Nouakchott project?

➔ Simulated hydraulic head (groundwater level) and measured hydraulic heads

  - ➔ Simulated hydraulic head = 1 value for each cell of the simulated area (hydraulic head constant within each cell)
  - ➔ Measured hydraulic head = 3 observation wells + 1



Objective functions

Objective functions ➔ indication on the performance of the model

« how well did the model perform? »

➔ Comparison between simulated and observed variables

$$ME = \frac{1}{n}\sum_{t=1}^{n}(\text{sim}_t - \text{obs}_t)$$

$$AME = \frac{1}{n}\sum_{t=1}^{n}|\text{sim}_t - \text{obs}_t|$$

With
n the number of measurements
Sim the simulated value
Obs the observed value

An objetive function is optimized ⇔ maximization or minimization depending on the formula

Name	Description	Formula*
DRMS	Daily Root Mean Squared Error	$\sqrt{\frac{1}{n}\sum_{t=1}^{n}(d_t - o_t(\theta))^2}$
TMVOL	Total Mean Monthly Volume Squared Error	$\sum_{i=1}^{n_{\text{month}}}\left(\frac{1}{n_{\text{day}}(i)}\sum_{t=1}^{n_{\text{day}}(i)}(d_t - o_t(\theta))\right)^2$
ABSERR	Mean Absolute Error	$\frac{1}{n}\sum_{t=1}^{n}\|d_t - o_t(\theta)\|$
ABSMAX	Maximum Absolute Error	$\max_{1\le t\le n}\|d_t - o_t(\theta)\|$
NS	Nash-Sutcliffe Measure	$1 - \frac{\frac{1}{n}\sum_{t=1}^{n}[d_t - o_t(\theta)]^2}{\frac{1}{n}\sum_{t=1}^{n}(d_t - \bar{d})^2}$
BIAS	Bias (mean daily error)	$\frac{1}{n}\sum_{t=1}^{n}(d_t - o_t(\theta))$
PDIFF	Peak Difference	$\max_{1\le t\le n}\{d_t\} - \max_{1\le t\le n}\{o_t(\theta)\}$
RCOEF	First Lag Autocorrelation	$\frac{\frac{1}{n}\sum_{t=1}^{n}(d_t - o_t(\theta))(d_{t+1} - o_{t+1}(\theta))}{\sigma_d\sigma_{o(\theta)}}$
NSC	Number of Sign Changes	(Count the number of times the sequence of residuals changes sign)

*Minimize with respect to θ.

Vijai Gupta et al. (1998)