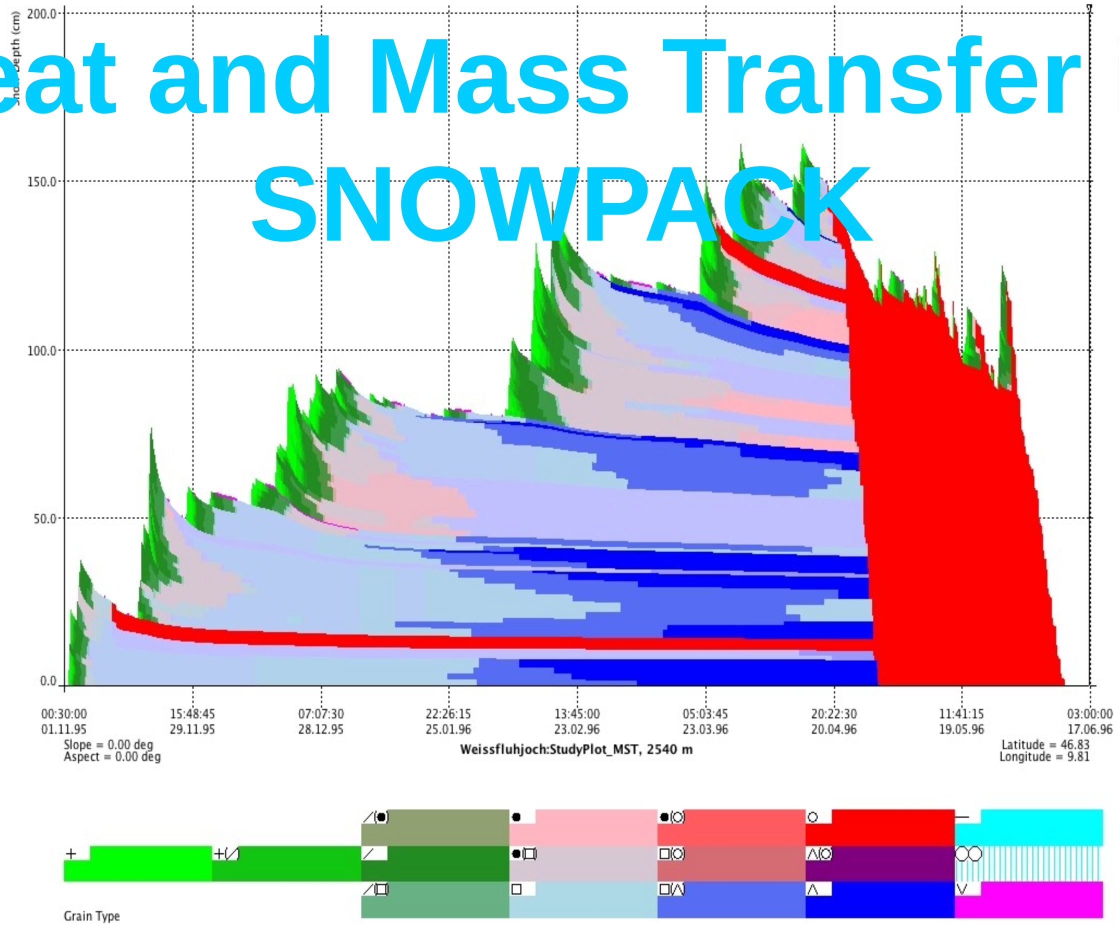
Heat and Mass Transfer in SNOWPACK



Outline

- The purpose of snow modelling
- The modeling principle
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- Adams-Sato Thermal Conductivity
- SNOWPACK Applications

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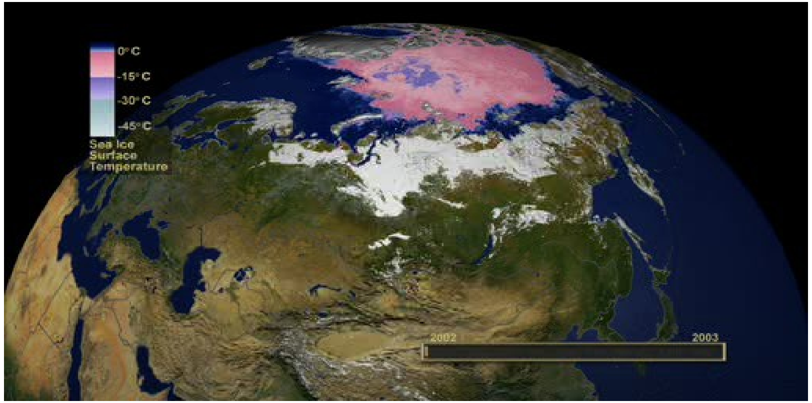
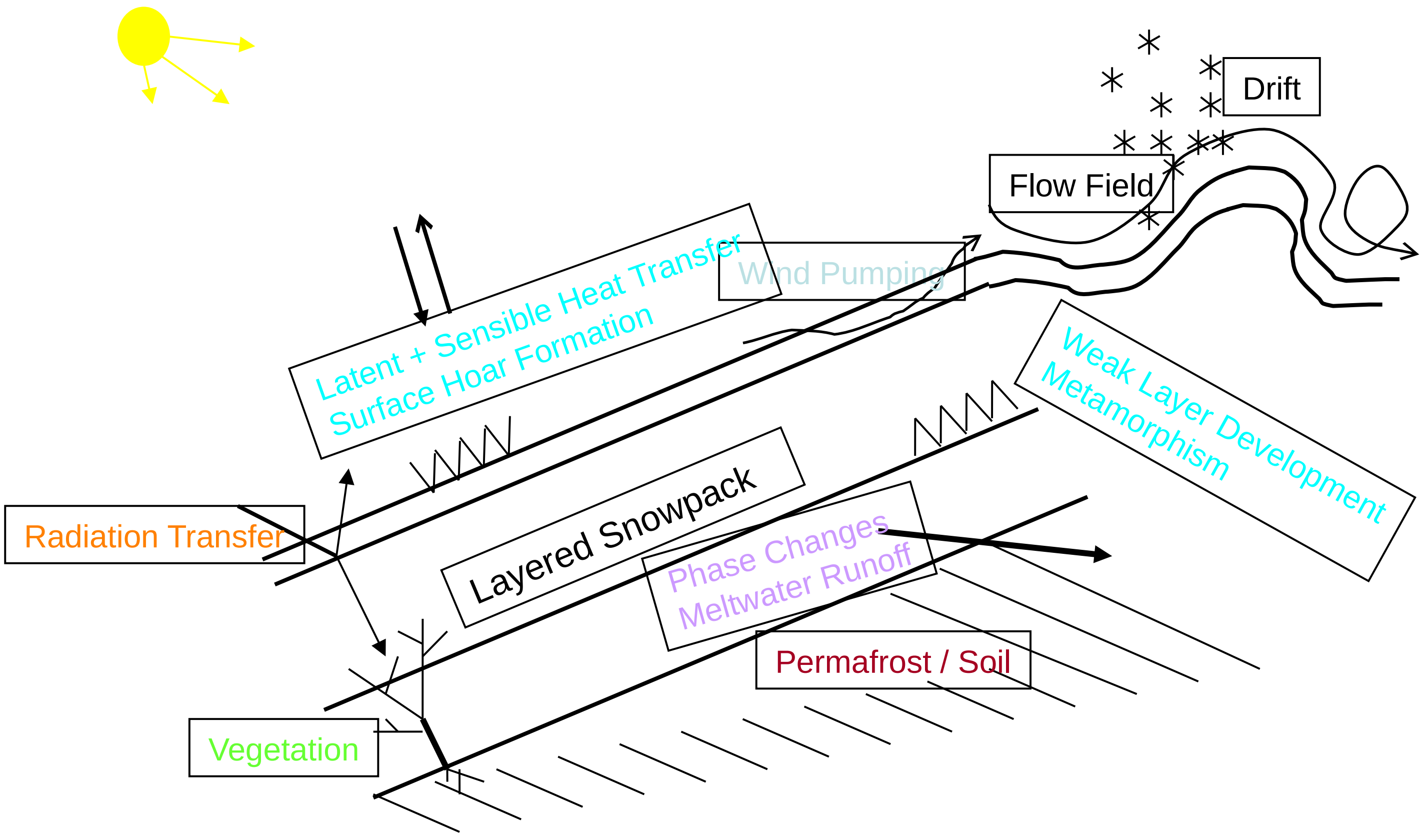
What snow modelling is good for?

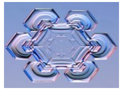
- Snow in the weather and climate models
- Snow mechanics and avalanches
- Industrial and touristic applications of snow
- Snow hydrology



Bringing the Processes together in a model

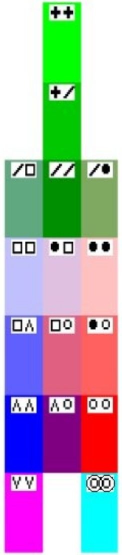


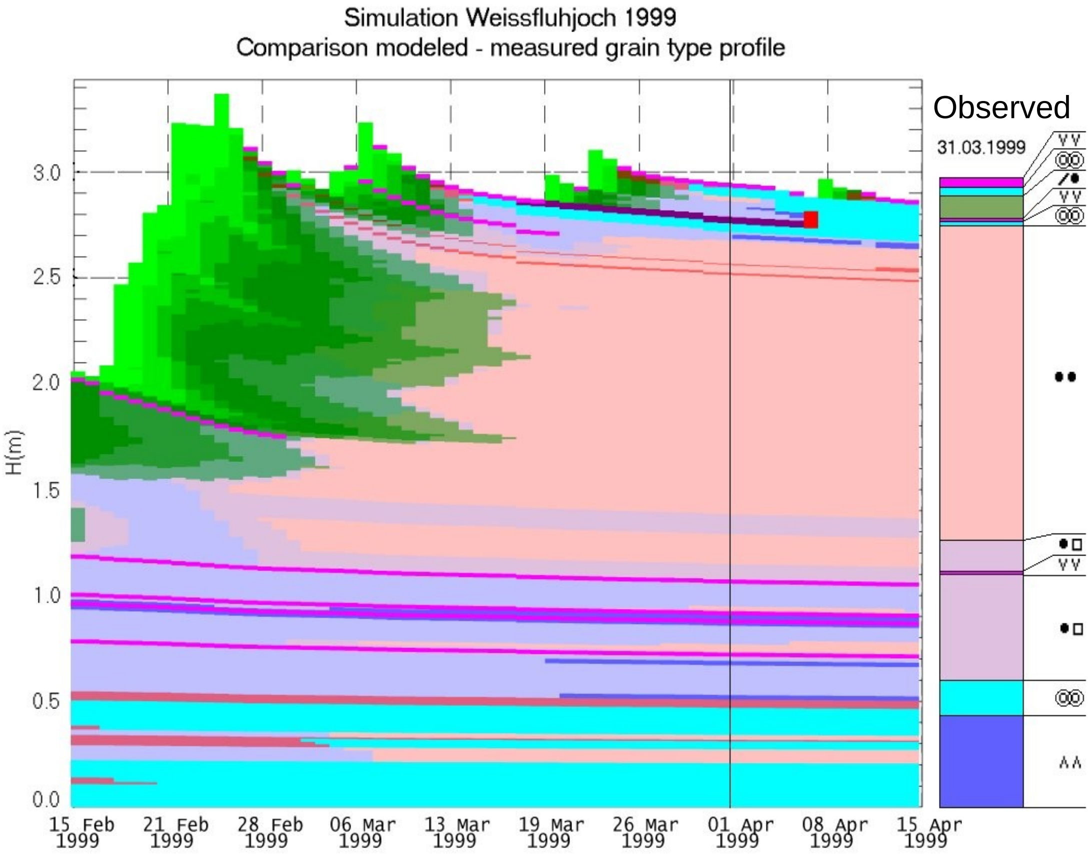
Grain type classification – snow has many forms




Class	*Symbol*	*Code*	*Colour*[1]	Web colour name	*RGB*[2] (0–255)
Precipitation Particles	+	PP		Lime	0 / 255 / 0
Machine Made snow	⊚	MM		Gold	255 / 215 / 0
Decomposing and Fragmented precipitation particles	⁄	DF		ForestGreen	34 / 139 / 34
Rounded Grains	●	RG		LightPink	255 / 182 / 193
Faceted Crystals	□	FC		LightBlue	173 / 216 / 230
Depth Hoar	∧	DH		Blue	0 / 0 / 255
Surface Hoar	∨	SH		Fuchsia	255 / 0 / 255
Melt Forms	○	MF		Red	255 / 0 / 0
	⦾⦾	MFcr			
Ice Formations	▬	IF		Cyan/Aqua	0 / 255 / 255

Integrated weather leads to a layered snow cover





SNOWPACK simulated snow types (left) show similar layers as observed (column on the right) end of march. Discrepancies can be explained with model weaknesses and subjective judgement of observers of the "manual" profile.

This profile is semi-stable: thin but pronounced weak layers are buried below thick snow – potentially large avalanches can be triggered as was the case in the avalanche winter of 1999.

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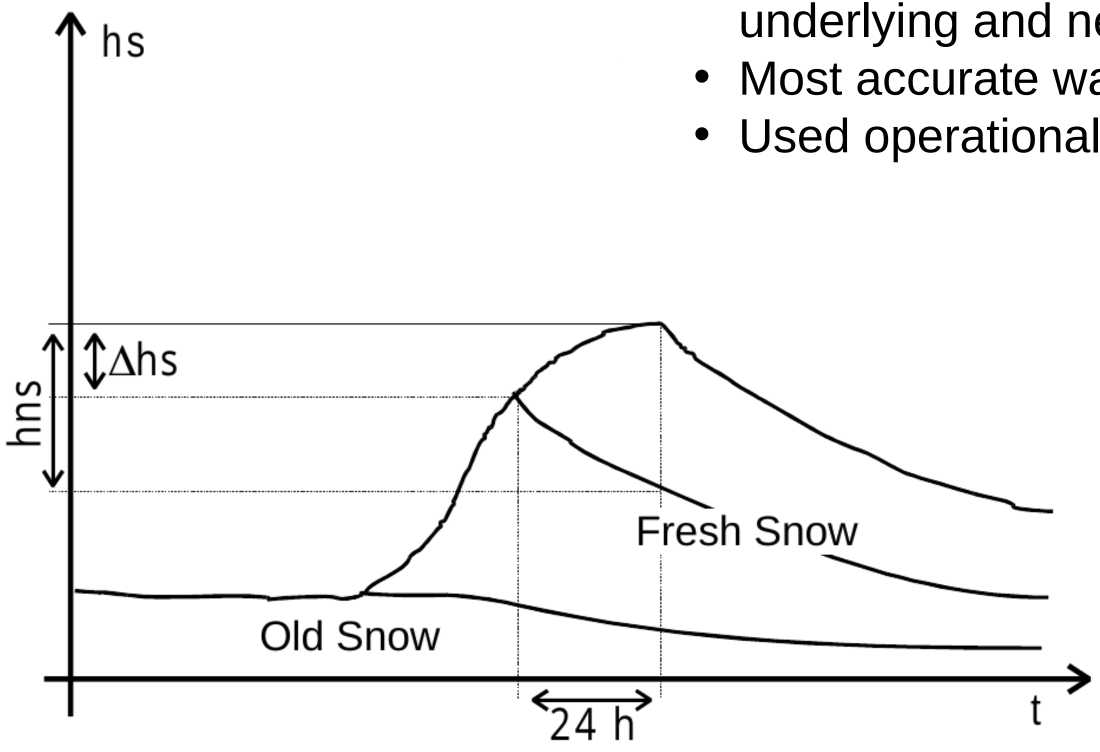
Determination of snowfall from snow depth

- Based on snow depth measurements
- The model provides the settling/compaction of underlying and newly deposited snow layers
- Most accurate way to measure snowfall
- Used operationally at IMIS stations

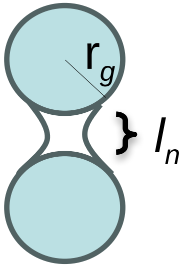


Formulation of strain rate ($\dot{\varepsilon}$) in SNOWPACK using viscosity:

$$\dot{\varepsilon} = -\sigma_s / \eta \qquad [s^{-1}]$$

σ_s: Load through weight of snow(Pa)
ρ: Density of Snow (kg m^{-3})
η: Viscosity (Pa s)
w: Vertical velocity of snow (m s^{-1})
r_g: Radius of grains (m)
l_n: Length of necks / bonds between grains (m)
T_C: Temperature (°C)
Z: wall normal direction (m)

$$\dot{\varepsilon} = \frac{dw}{dz} \simeq \frac{\Delta w}{\Delta z} \simeq \frac{l_n \dot{\varepsilon}_n}{\Delta z} \simeq \frac{l_n \dot{\varepsilon}_n}{2 r_g + l_n}$$



Empirical Approximation for Fresh Snow:

$$\eta = 0.007\, \rho^{(4.75 - T_C / 40)}$$

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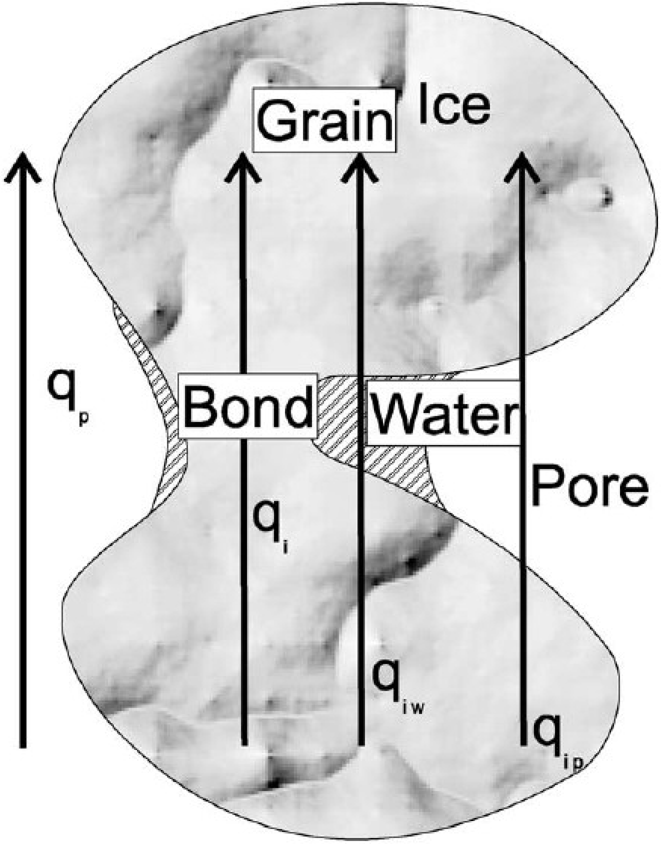
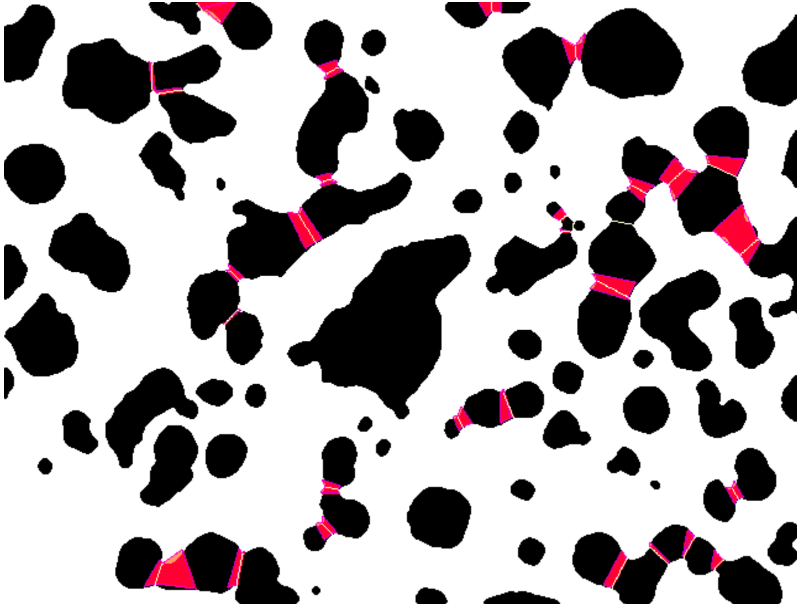


Processes:

- Conduction: Transmission of molecular kinetic energy
- Vapor Transport and phase change
- Advection through externally forced flow
- Convection triggered by buoyancy



Energy Conservation (Temperature Equation)

Formulation in SNOWPACK: $\theta_i + \theta_w + \theta_a = 1$

$$\rho_s = \rho_i \theta_i + \rho_w \theta_w + \rho_a \theta_a$$

θ: Volumetric Fraction (-)
ρ: Density (kg m^{-3})
s,i,w,a,v : Subscripts for Snow, Ice, Water, Air, Vapour

Bulk Temperature Equation (without convection):

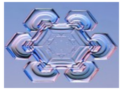
$$\rho_s c_p \frac{\partial T_s}{\partial t} - \frac{\partial}{\partial z}\left(k_{eff} \frac{\partial T_s}{\partial z}\right) = [Q_{pc}] + [Q_{mm}] + Q_{sw} \; ;$$

$$[Q] = Wm^{-3} \text{ Volumetric Heat Source}$$

$$\rho_s c_p = \rho_i c_i \theta_i + \rho_w c_w \theta_w + \rho_a c_a \theta_a$$

T_S: Temperature of Snow (K)
c_p: Heat Capacity (J kg^{-1} K^{-1})
k_{eff}: Effective Thermal Conductivity (W m^{-1} K^{-1})

Coupled Equations including Water Vapor




Why do we want to simulate vapor transport explicitly?

- Because it has an influence on density profiles and microstructure via the specific surface area.

$$\theta_a \frac{\partial \rho_v}{\partial t} + \frac{\partial}{\partial z}(\theta_a J_v) = M_{mm}$$

$$J_v = -D_{\mathrm{eff},a} \frac{\partial \rho_v}{\partial z}$$

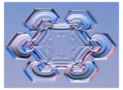
J_v: Flux of Water Vapor (kg m^{-2} s^{-1})
$D_{eff,\ a}$: Effective Diffusion Coefficient (m^2 s^{-1})
a_S : Specific surface area of snow (m^{-1})
d_p: Grain diameter (m)
h_m: Mass transfer coefficient (m s^{-1})
ρ_{vs}: *Saturation vapor density* (kg m^{-3})

$$M_{mm} = h_m a_s(\rho_{vs} - \rho_v)$$

M_{mm}= Q_{mm} / L: Mass Change (kg m^{-3} s^{-1})
L: Latent Heat of Sublimation (J kg^{-1})

$$a_s = \frac{6\theta_i}{d_p}$$

Boundary Conditions




Temperature:

Upper: 1) Neumann $$k\frac{\partial T_s}{\partial z} = q_{lw} + q_{sh} + q_{lh} + q_{rr}$$

Or 2) Dirichlet $$T_s = T_{ss}$$

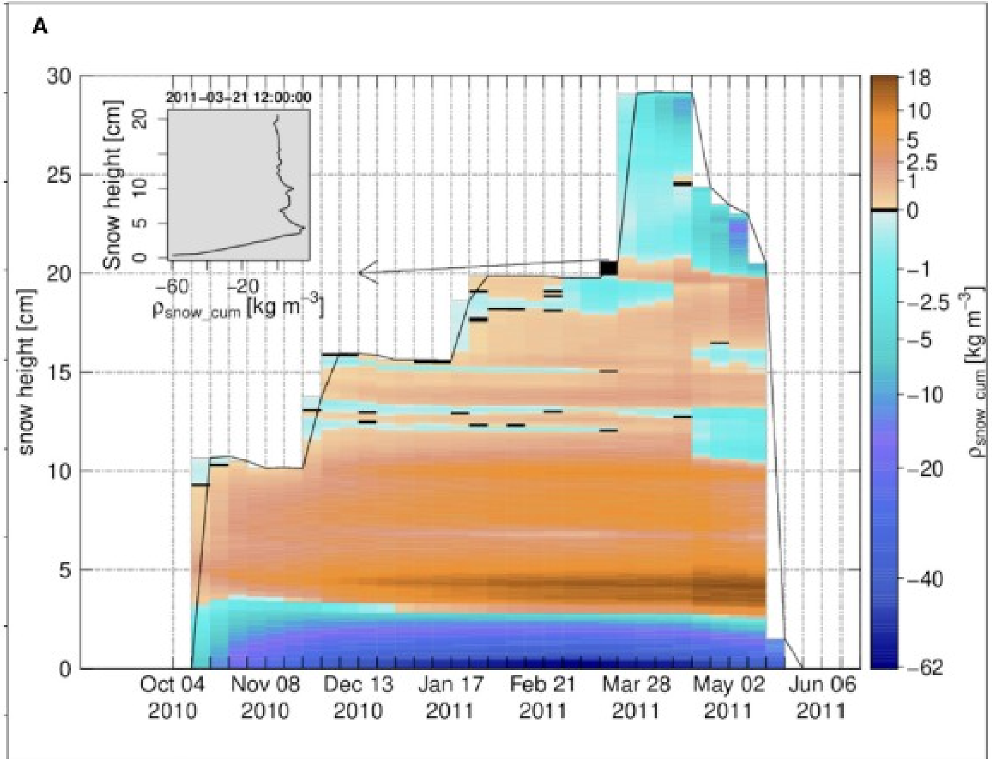
Lower: 1) Neumann $$k\frac{\partial T_s}{\partial z} = q_G$$

Or 2) Dirichlet $$T_s = T_G$$

q: Flux of Energy (W m^{-2})
lw,sh,lh,rr,ss: Subscripts for longwave, sensible heat, latent heat, rain, snow surface

Water Vapor: Neumann $$D\frac{\partial \rho_v}{\partial z} = \frac{q_{lh}}{L}\ (upper) = 0\ (lower)$$

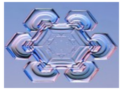
Results of coupled SNOWPACK simulations



Cumulative density change by water vapor transport in Siberia (Samoylov) for the winter 2010 (Jafari et al., 2020).

Looking at net effect of vapor transport through a thin snow cover; leads to a "weak" foot with reduced densities; reduces surface densities as well; appears to initiate a positive feed-back loop, where denser layers collect more additional mass and less dense layers continue to loose mass independent of how the snow depth evolves;

Results of coupled SNOWPACK simulations



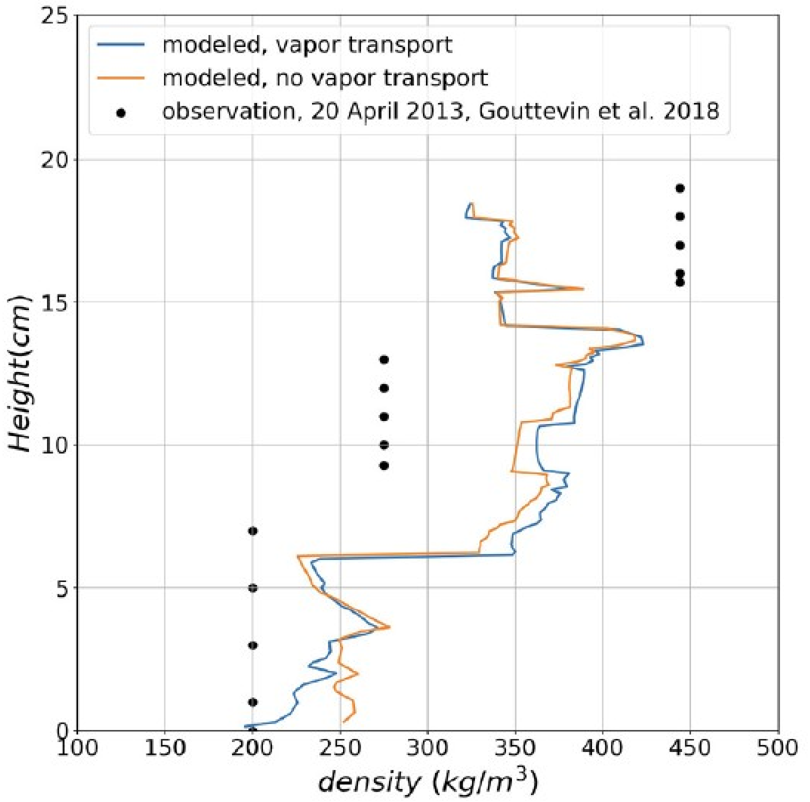
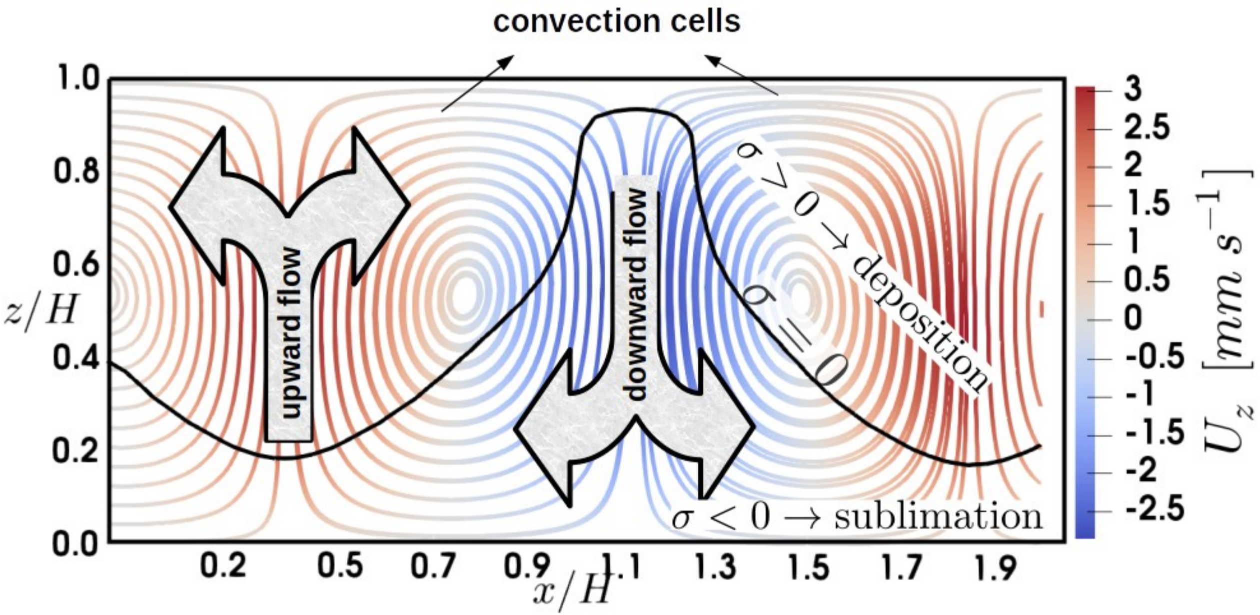



FIGURE 13 | The comparison of simulated snow density profile with measured one for Samoylov on 20 April 2013.

Comparison with measured profiles shows still large errors of the simulation with only minor corrections through the vapor transport (Jafari et al., 2020). In particular, the observed high-density layer at the surface is not reproduced; high density snow at the surface is typical for polar snow but its physics are not fully understood / implemented in snow models. These high-density layers are called wind-slabs and probably caused by wind-transported snow, which deposits at higher density and strength.



Process:

- Convection

σ : Supersaturation

Natural convection can occur in natural snow covers; first simulations of convection in snow by Mahdi Jafari (Jafari et al., 2022); Convection is flow motion caused by buoyancy, e.g. differences in density caused by differences in temperature; Need to have large temperature gradients and high porosity;

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Melt – Refreeze Phase Change

Formulation in SNOWPACK:

- Calculate „Hypothetical“ Temperature
- Determine Mass of Phase Change and associated Energy
- Add Energy to Temperature Equation(s)

$$\Delta T = T_s - T_{melt}$$

$$\Delta \theta_w = \frac{c_s \theta_i \rho_i \Delta T}{L_f \rho_w}$$

$$\Delta \theta_i = \frac{\rho_w \Delta \theta_w}{\rho_i}$$

$$Q_{pc} = \Delta \theta_i \rho_i L_f$$

$$L_f = 334 \frac{kJ}{kg}$$

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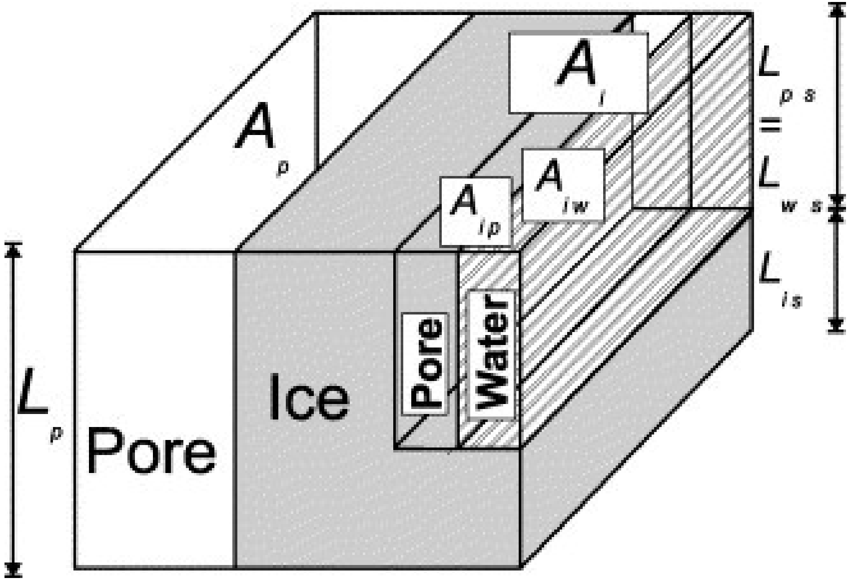
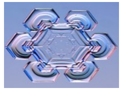
Effective Conductivity Model



The thermal conductivity in SNOWPACK is highly parameterized but based on the principle that snow is a mixture of ice, air and potentially water. Since the pore space is not simply dry air but moist air with water vapor that moves along a temperature and therefore vapor gradient, the thermal conductivity of the pore space must be augmented to consider the effect of the moving vapor and the associated phase changes at the grain boundaries. This augmented thermal conductivity is called k_{ap} in the next slide and its derivation is in the write-up on moodle.

Adams - Sato:

$$k_{eff} = \frac{n_{ca}}{n_{cl}}\left[\frac{\pi^2 r_b k_i N_3}{32} + \frac{k_i k_{ap} A_{ip}}{L_{is} k_{ap} + L_{ps} k_i} + \frac{k_{ap} A_p}{L_p}\right]$$

$n_{ca/cl}$: Number of grains per unit area/length
r_b: Bond radius
N_3: Coordination number
L_x: Length scale
L: Latent heat of sublimation
A: Area / Cross section
i,w,a,p,s: Subscripts for ice, water, air, pore space, snow
V: Volume
D_p: Diffusion coefficient of water vapor in air

$$k_{ap} = k_a + L D_p \frac{\partial p}{\partial T} \approx k_a + \frac{L^2 D_p}{T(\Delta V)}$$

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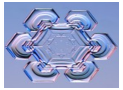
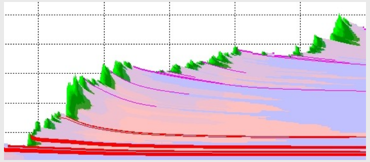
Avalanche Warning




Input



SNOWPACK data

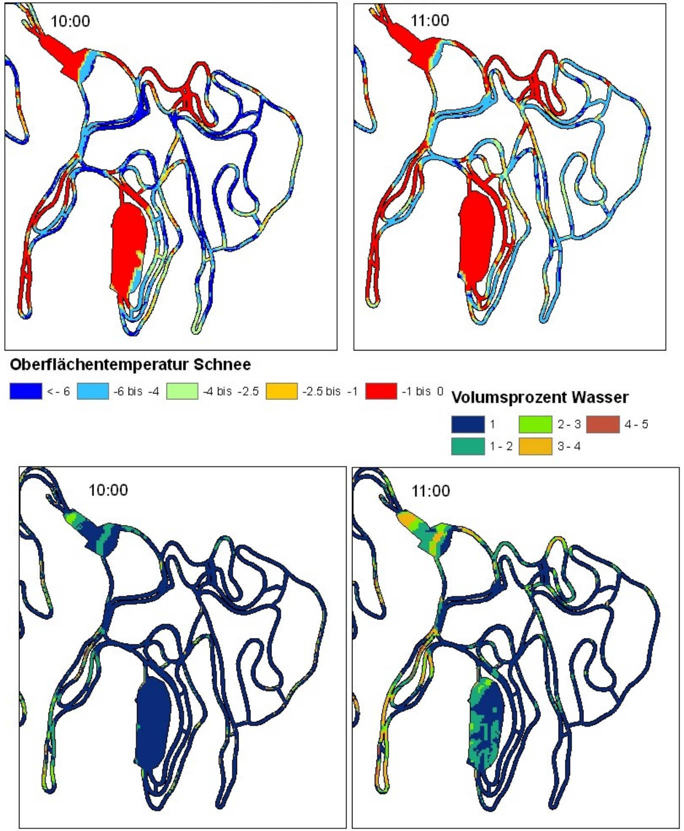
→ Statistical methods →

Output

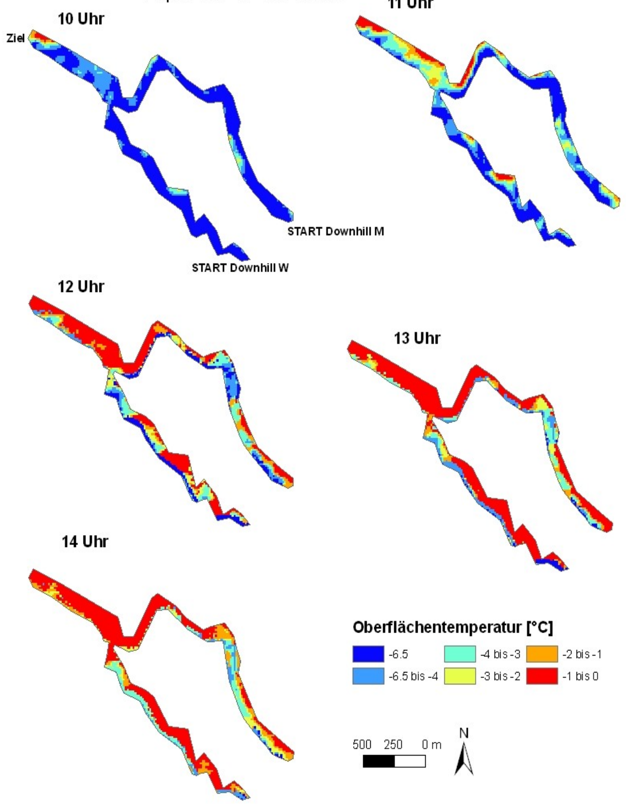
- Low
- Moderate
- Considerable
- High
- **Very High**

Based on numerical SNOWPACK output, statistical or machine learning models can be trained to predict avalanche danger or snow cover stability; This has first been demonstrated by Schirmer et al., J. of Glaciology (193), 2009; A similar approach is in operation today at SLF;

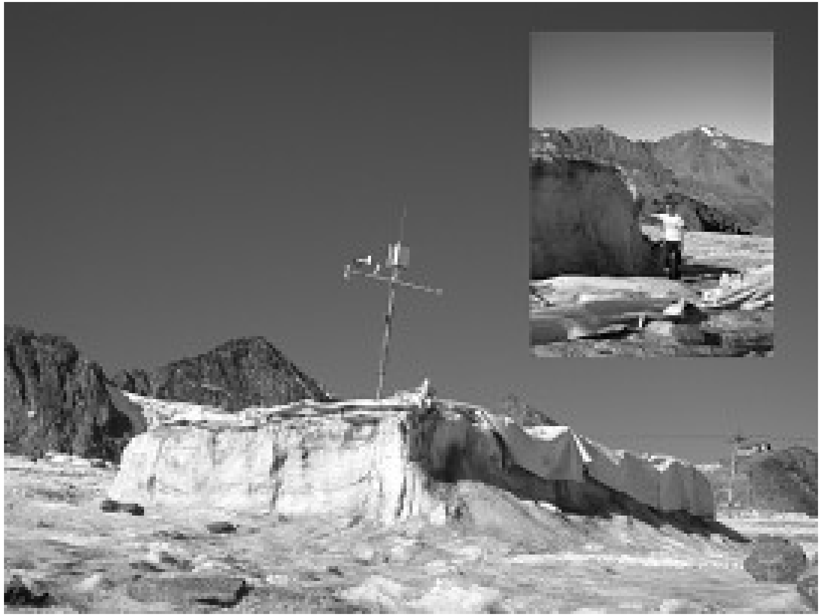
**Morgenprognose Callaghan:
Modellierte Oberflächentempratur Schnee und Schneefeuchte 02-17**



Morgenprognose: Oberflächentemperaturen Schnee
Alpin für 17-02-2010

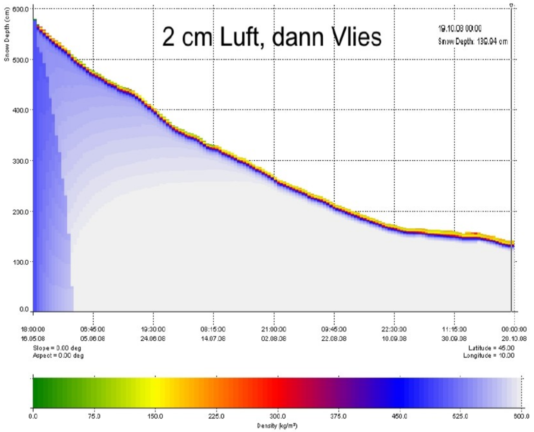


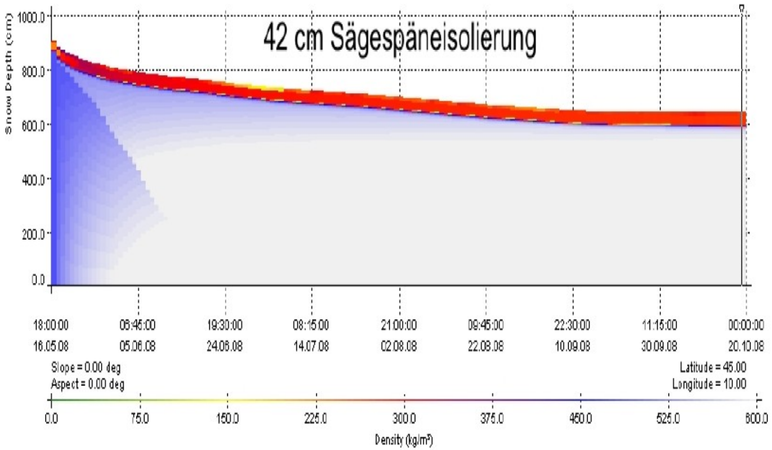
Glacier Protection and Snow Farming

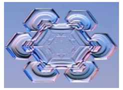


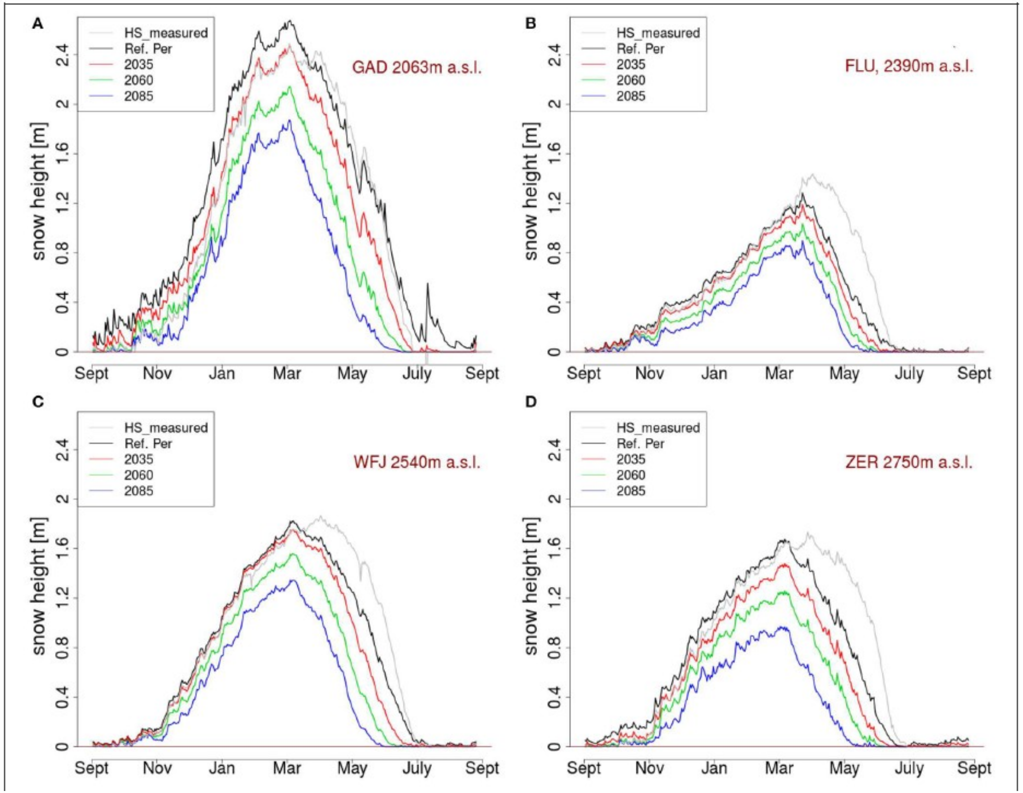
By changing the surface energy balance, snow and ice can be protected from summer melt; At high elevations, it is most effective to change albedo (left), at lower elevations you need to protect against high temperatures (right); SNOWPACK simulations help to find the best cover and explain observed effects (Olefs and Lehning, CRST, 2010); snow farming is a way to allow winter tourism in early winter when demand is high;

Snow Farming Calculations



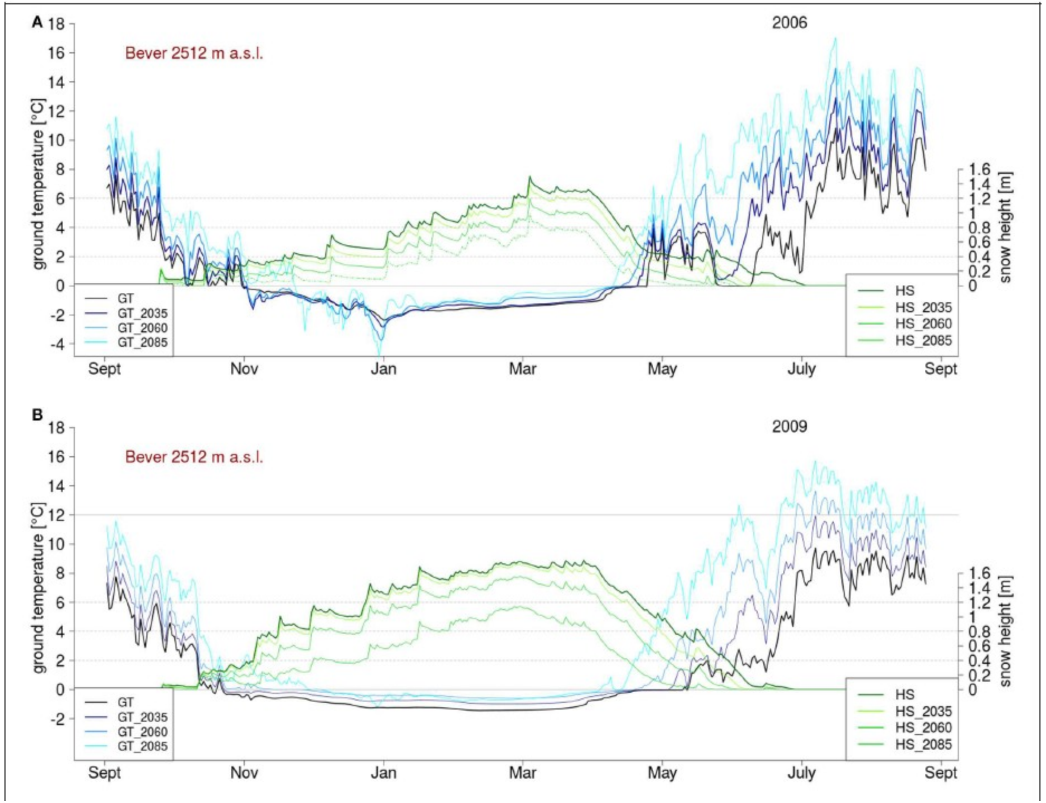




Different snow climates react differently on climate warming in the future (Bender et al., 2020).

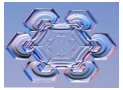
Mean snow heights (HS) for the average of 12 years, for the reference period (black line) and the three climate change periods (red, green, blue) for Gadmen (GAD2, A), Fluela (FLU2, B), Weissfluhjoch (WFJ2, C), and Zermatt (ZER2, D). Average year follows the snow season, so September to September is chosen, elevation of the stations is indicated. Gray lines show measured snow height, black lines modeled snow height for reference period (2004–2016), followed by red (2035–2060), green (2060–2085), blue (2085–2100).

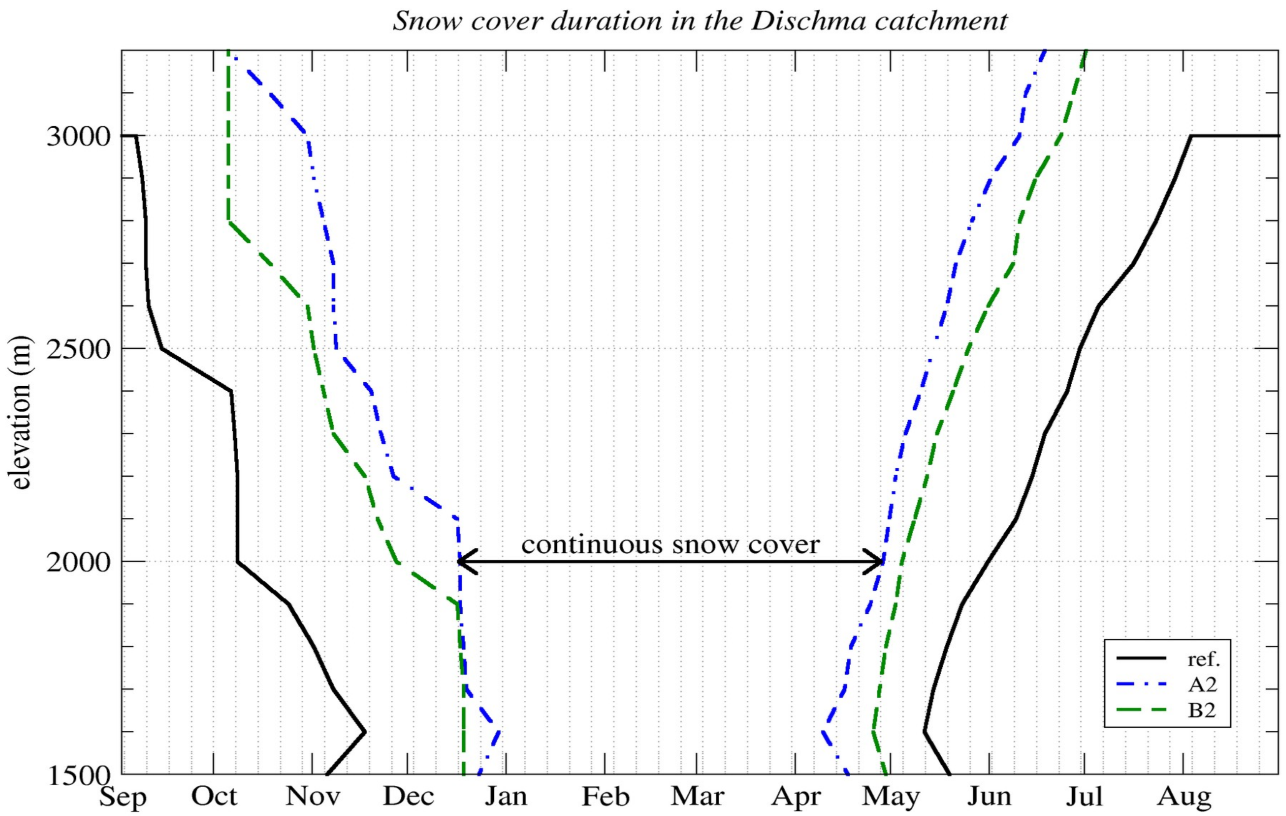


Different snow climates cause different soil temperature behaviour (Bender et al., 2020).

Example of a snow-poor year (A, 2006) where a cooling of the ground in early winter can be seen and a snow-rich year (B, 2009) with no cooling effect for the Bever-station in Grison. Snow height (HS) is in greenish colors and ground temperature (GT) in blue.

Climate Change: how much snow will remain?

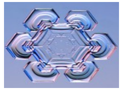




The length of the winter will decrease at all elevations, while maximum snow depth will decrease mainly at lower elevations (not shown). The model serves to make quantitative predictions based on climate change scenarios, which are produced by larger-scale climate models. In the Dischma catchment, no year-round snow will be supported in the future with known consequences for the fate of the remaining glacier.

Bavay et al., HP, 2009

References

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