Snow Energy Balance

Outline:

- Radiation (mini review)
- Radiative / optical properties of snow and ice (in more detail)
- Energy transfer and relevant heat fluxes
- Turbulent heat fluxes and atmospheric boundary layer
- Snowpack energy balance
- Effects of topography and vegetation on the energy balance (optional)

Snow energy balance: Rationale

Why is precise knowledge of snow thermodynamics important?

- Thermodynamic state of the snowpack
- Timing and spatial distribution of melt
- Additional forcing for snow metamorphism
- Water movement in snow and melt water release
- Atmospheric stability and boundary layer processes
- Long-term effects on local climate

Snowpack energy & mass balance

- Accumulation period
  - Energy balance is negative
  - Mass balance is positive

- Ablation (melt) period
  - Energy balance becomes positive
  - Mass balance becomes negative
  - Simplified 3-phase process:
    (1) Warming, (2) Ripening, increase of LWC, (3) Output of mass

- Once the snow is at 0^{o}C, further energy input results in melt and mass loss

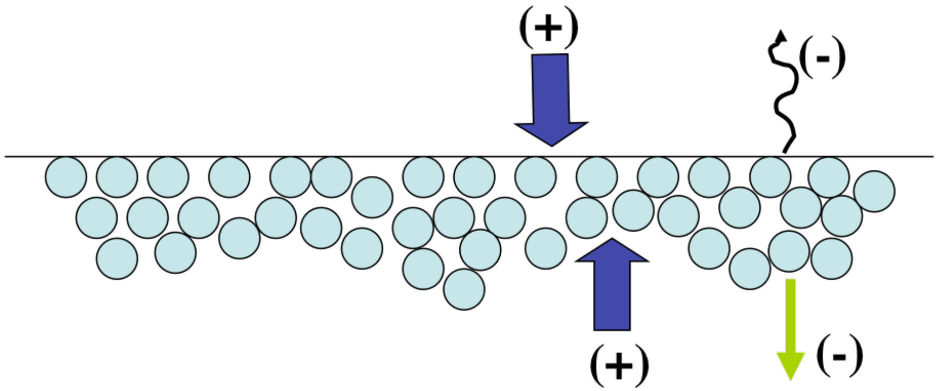
Difficult to directly measure evaporation, sublimation and melt →
Need to compute these quantities by measuring energy inputs and outputs

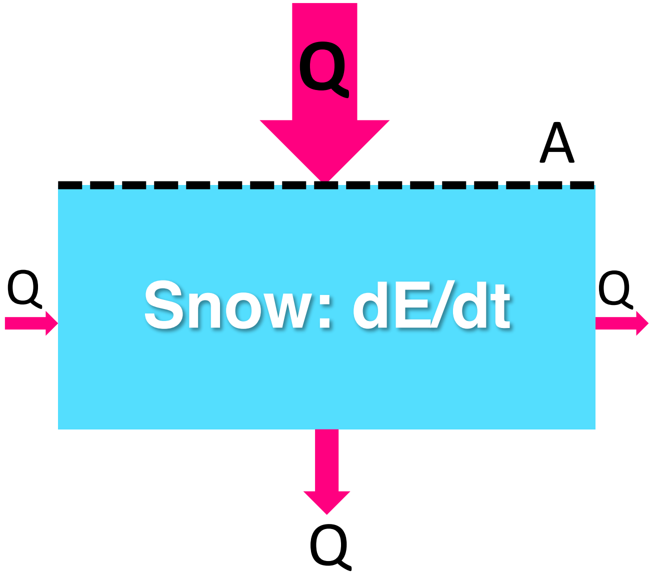
- For a snowpack slab or layer of a given thickness *dz*:
  Energy in – Energy out = Change in heat content, internal energy (U), (temperature or phase)

$$\frac{dE}{dt} = \sum(Q_i * A_i)$$

Q: energy flux (J $s^{-1}m^{-2}$ = Wm^{-2}), perpendicular to
A: surface area (m^2) → [J s^{-1}]

- For the snow surface, which has zero thickness (no volume):
  Energy in – Energy out = 0





- Fluxes into the snowpack are positive
- Fluxes out of the snowpack are negative

Mechanisms of energy transfer

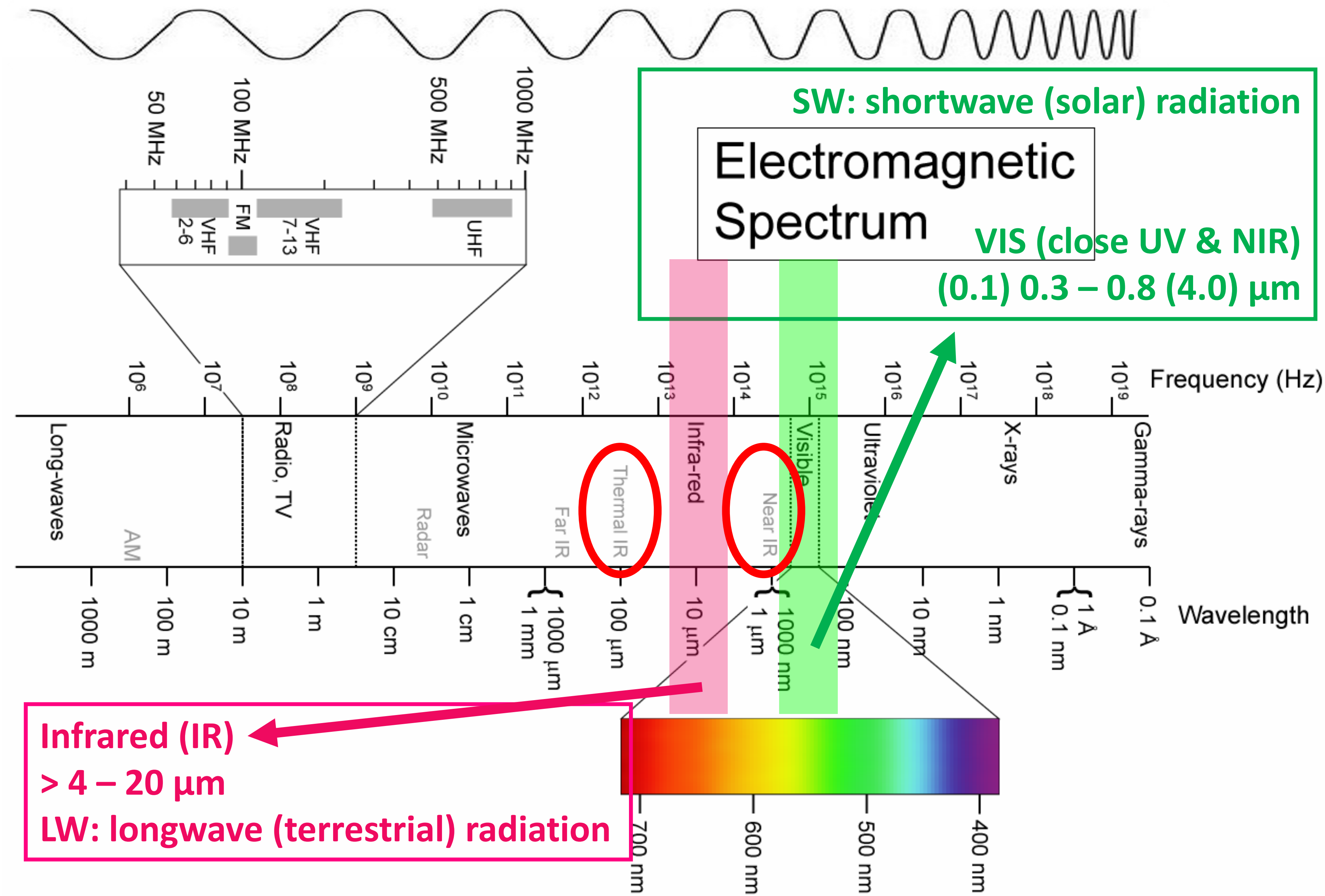
- Radiation
  - Transfer of energy by electromagnetic waves:

  a) absorption, b) reflection, c) transmission

  d) emission, e) refraction, f) scattering
- Conduction
  - transfer of energy by direct contact in a solid (molecular scale)
- Convection
  - involves (turbulent) mixing and motion of a fluid
- Diffusion
  - involves thermal or chemical gradient (of scalar variable)
- Advection
  - externally forced mass transport

Short review of radiation

Electromagnetic spectrum



Quick review of radiation

- Planck equation: monochromatic intensity of emission of a black body (proportional to surface temperature!)

$$B_\lambda = \frac{2hc^2}{\lambda^5}\frac{1}{\exp\left(\frac{hc}{\lambda kT}\right) - 1} \quad , \quad \left[\frac{\mathrm{J}}{\mathrm{sm}^3} = \frac{\mathrm{W}}{\mathrm{m}^3} = \frac{\mathrm{W}}{\mathrm{m}^2}\frac{1}{\mathrm{m}}\right]$$

Subscript λ indicates spectral quantities (wavelength)

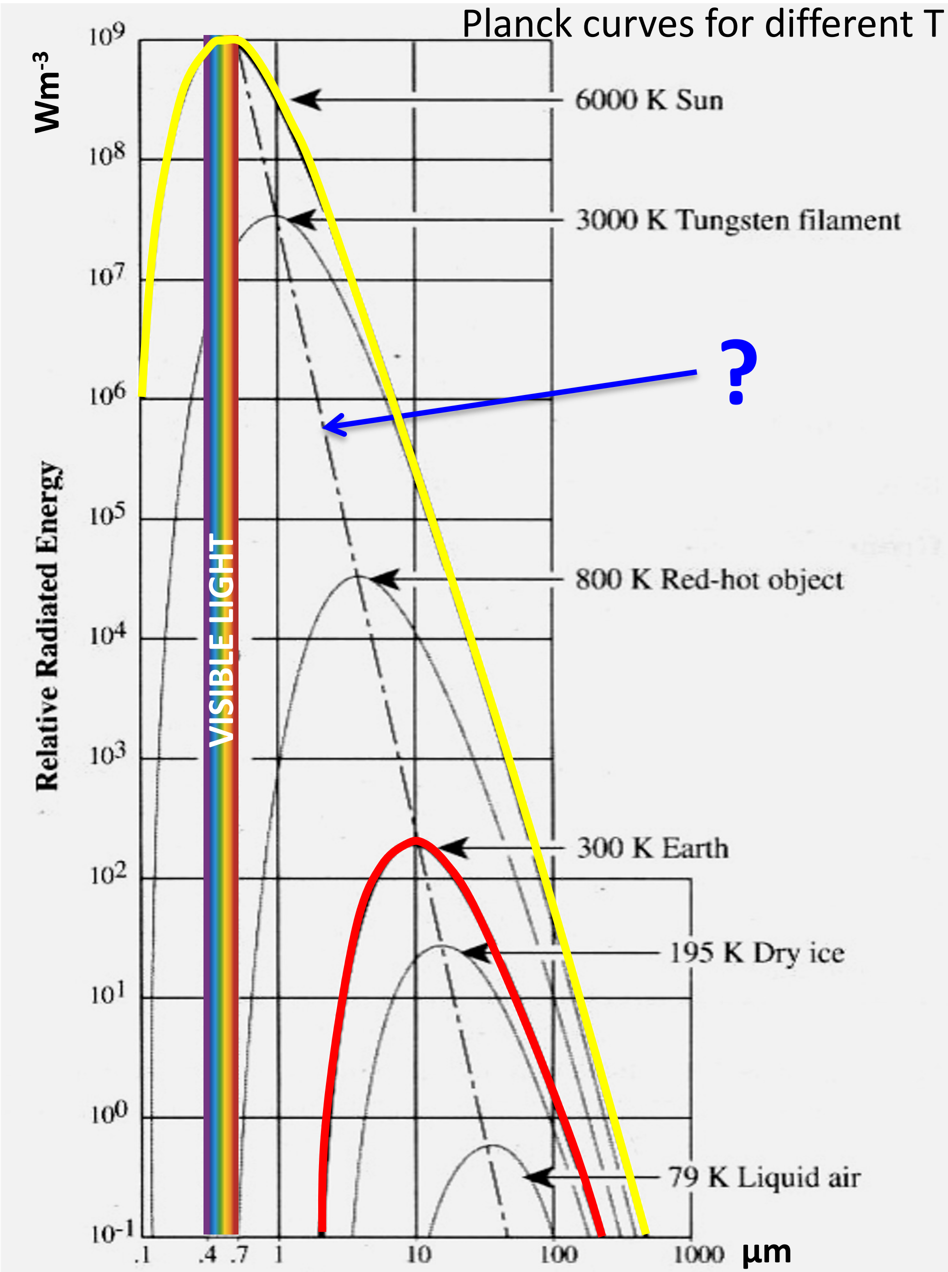
h = Planck constant
c = speed of light in vacuum
k = Boltzmann constant

- Wien's Displacement Law

$$\frac{dB_\lambda(T)}{d\lambda} = 0 \rightarrow \lambda_{\mathrm{max}} T = \mathrm{const.} = 2.897 \cdot 10^{-3} \quad [\mathrm{m} \cdot \mathrm{K}]$$

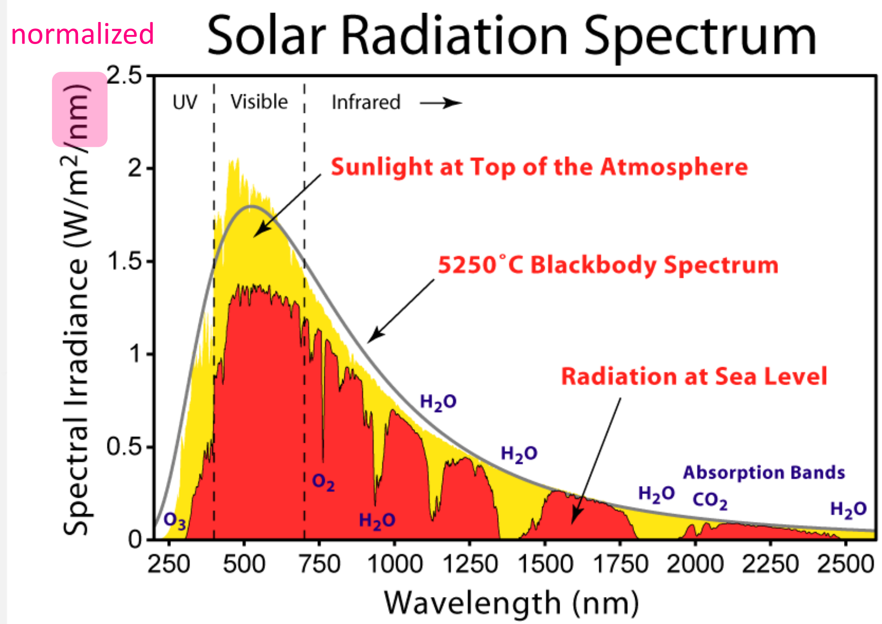
- Example: snow at -10°C: $\lambda_{max} \approx 11\mu m$ (→ IR)

Thermal radiation



Maximum solar energy is in the visible portion of the electromagn. spectrum (0.4 – 0.7 µm) but is still high in the near-infrared portion of the spectrum (0.7 – 2.5 µm)

Longwave portion of the electromagnetic spectrum is approximately (2.5 – 100 µm)



Black and gray body radiation

Stefan-Boltzmann equation:
Radiative emission (B) is proportional to physical temperature (in [K])

$$B = \sigma T^4$$

is the Stefan-Boltzmann constant, equal to 5.67*10^{-8} [W m^{-2} K^{-4}]

This assumes a black body being a perfect emitter and perfect absorber for all wavelengths λ, for each T

A gray body is an object which is not a perfect absorber/emitter;
it reflects part of the incident radiation, depending on its reflectivity (emissivity)

$$B = \varepsilon \sigma T^4$$

Emissivity, **ε** [-], is the ratio of gray body emission to black body emission

$$\varepsilon = \frac{B_{\text{gray body}}}{B_{\text{black body}}}$$

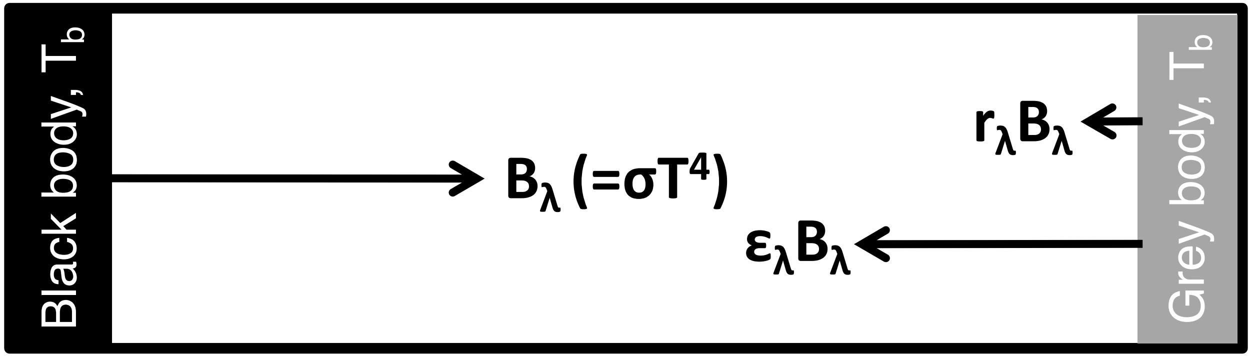
Kirchhoff's law of thermal radiation

"Absorptivity equals emissivity for all wavelengths." (Kirchoff's Law).

Black body: $\varepsilon_\lambda = a_\lambda = 1$, for all λ and in thermodyn. equil. (i.e., same T)

Grey body: $\varepsilon_\lambda = a_\lambda$; < 1, with $a_\lambda = (1 - r_\lambda)$; → a_λ (or ε_λ) $+ r_\lambda = 1$.

ε_λ = emissivity, r_λ = reflectivity, a_λ = absorptivity,



Radiation terminology

- SW: Solar radiation
  Shortwave radiation
  Visible light (VIS)
  Spectral range: visible (0.4-0.7 μm)
- LW: Terrestrial radiation
  Longwave radiation
  Infrared radiation (thermal)
  Spectral range: IR/TIR (4-20 μm)

These are the two principal wavelengths (radiative heat fluxes) relevant in the earth's energy balance controlling the global climate. (UV is relevant for photo-chemistry).

SW radiation gets reflected, absorbed and emitted, and contributes to the change of the surface temperature. Given the surface temperature on earth, the surface emits in the IR range and almost nothing in the visible range.

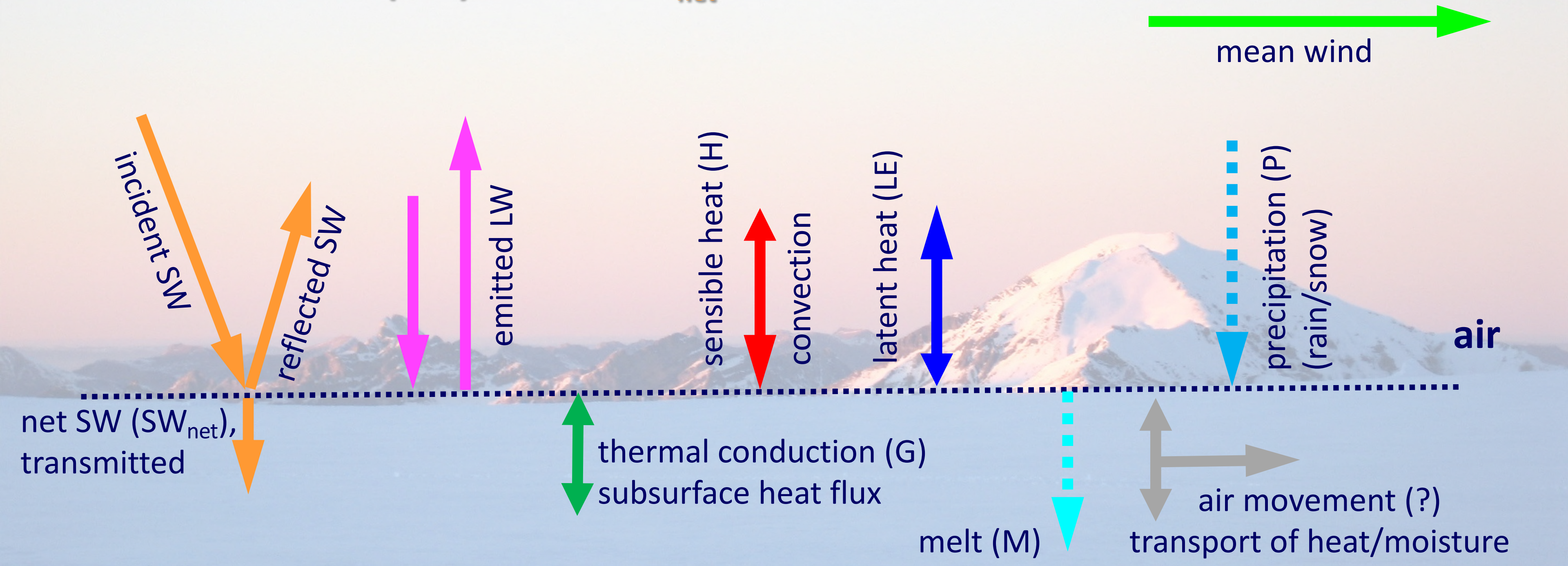
LW radiation gets absorbed, emitted, and reflected, and contributes to the change of the surface temperature. Given absorptivity/emissivity of the earth for typical temperatures, most energy is absorbed and re-emitted in the IR range.

Snow Energy Balance

Surface energy balance: $R + G + H + LE = 0$ [W m^{-2}]

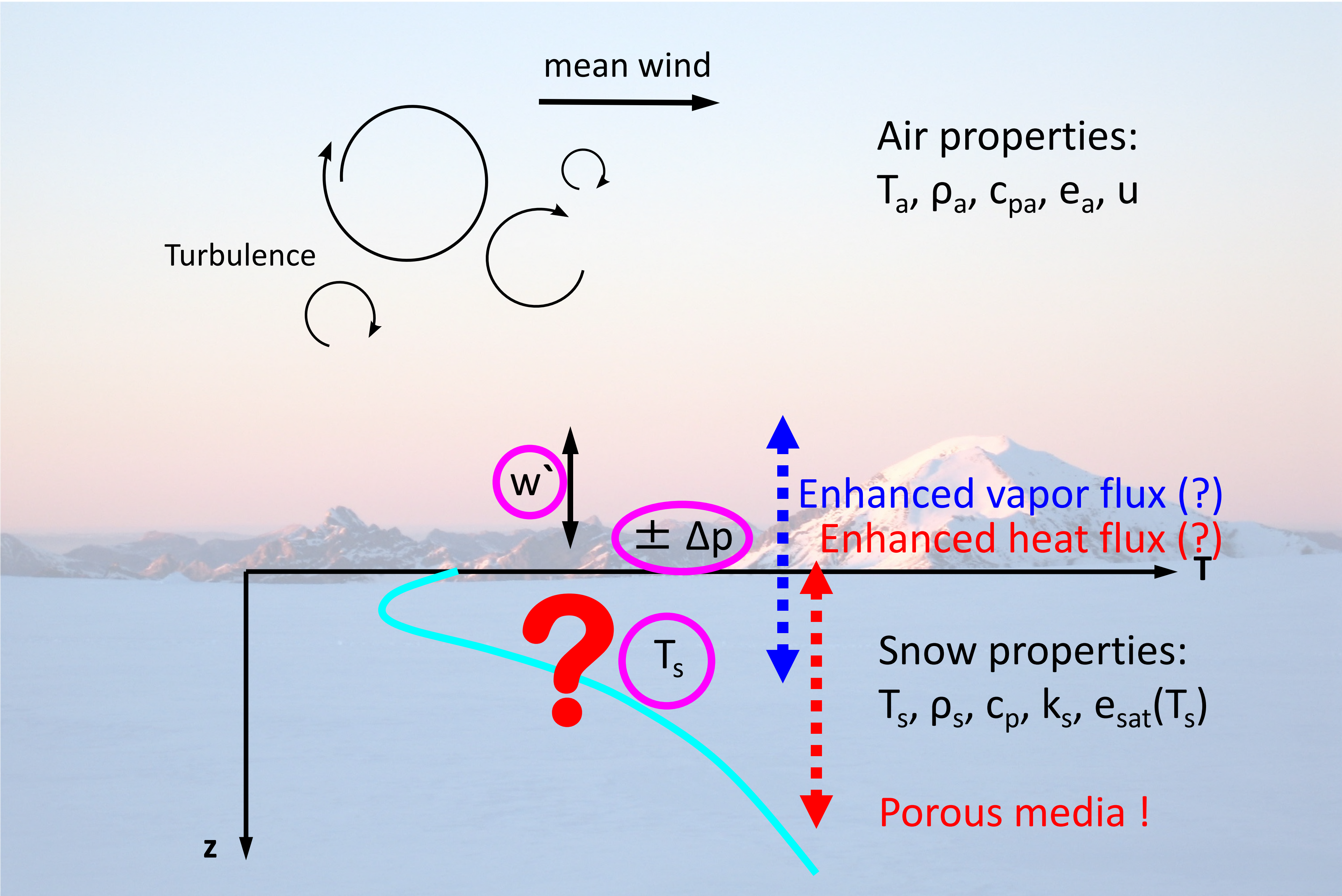
Net radiation $R = SW\Downarrow - SW\Uparrow + LW\Downarrow - LW\Uparrow$

Transmitted SW : $(1-\alpha)\, SW\Downarrow = SW_{net}$



Snow **slab** or **layer:** $dE/dt = (R + G + H + LE + M + P) * A$ [J s^{-1}]

Snow Energy Balance



Main factors in terrestrial energy exchange

- Air temperature
  - temperature gradient between snow and overlying air
  - sensible heat exchange
- Water vapor
  - vapor pressure gradient between snow and overlying air
  - latent heat exchange
- Wind
  - turbulent energy exchange
  - aerodynamic surface roughness
- Radiation (net)
  - shortwave and longwave
- Advected heat
  - rain, snowpack ventilation (wind pumping)?
- Soil temperature (gradient)
  - heat flux at snow–soil interface
- Terrain and vegetation
  - reflected solar radiation, emitted longwave radiation, modification of air flow

Energy Balance Equation

Energy available for temperature or phase change:

$$dE/dt = (\, SW_{net} + LW_{net} + H + \boxed{LE} + \boxed{P} + G + \boxed{M}\,) * A$$

dE/dt = rate of change in snowpack internal sensible & latent heat storage

SW_{net} = net shortwave flux [Wm^{-2}]

LW_{net} = net longwave flux [Wm^{-2}]

H = sensible heat flux [Wm^{-2}]

LE = latent heat flux [Wm^{-2}]

P = heat flux due to input of precipitation(sensible + latent) [Wm^{-2}]

G = ground heat flux (conduction) [Wm^{-2}]

M = loss of L_f due to melt water exiting the snowpack [Wm^{-2}]

A = surface area [m^2]

Note: individual terms can have positive or negative sign.

$\boxed{\ldots}$: terms imply a mass change of the snowpack

Net shortwave radiation (SW_{net})

Net shortwave radiation is the difference of energy received from incident solar radiation and reflected solar radiation. --- Global = direct + diffuse SW radiation.

$$SW_{net} = SW_{down} - SW_{up} \quad ; \quad SW_{up} = \alpha\, SW_{down}$$

$$SW_{down} = SW_{dir} + SW_{diff} = SW_{global}$$

$$SW_{down} = I_{norm} \cos(Z) + SW_{diff} \qquad \text{(slopes)}$$

α : albedo/reflectivity (broadband, integrated over 0.4-2.0µm)

I_{norm} : normal irradiance [Wm^{-2}]

Z : zenith angle of the sun [deg]

S_0 : solar constant, 1366 Wm^{-2}

Clear day: ≈ 80% SW_{dir}, Cloudy day: ≈ 80% SW_{diff}

Empirical estimations of daily total SW↓: (DeWalle & Rango, 2008, p.151)

$$SW_{down} = I_{pot} (0.25 + 0.5\, n/N)$$

$$SW_{down} = I_{pot} (0.85 - 0.47\, C)$$

$$SW_{down} = I_{pot} (0.7 (1 - \exp(-0.01\, \Delta T_a^{2.4})))$$

n : # of sunshine hours per day

N : max. possible hrs of sunshine

C : cloud cover fraction

ΔT_a : daily range of air temperature

I_{pot} : potential SW at clear sky cond.

Albedo

- is an intrinsic property of a material that describes the total amount of energy reflected from a surface relative to the amount of incident energy

$$\alpha_{vis} = \frac{SW\uparrow}{SW\downarrow} \; ; \; [-]$$

- vis = visible light (shortwave / solar radiation)
- α = f (λ), but also f (zenith angle, surface type, grain size, type, density…)
- in VIS ($0.3 < \lambda < 0.8$ µm) α is a weak function of λ
- α is proportional $1/D^2$, where D is the grain diameter
- α decreases with LWC and pollutants in snow

Albedo of various surfaces

Water (ocean), zenith angle: 45, 60, 70, 80°	0.05, 0.08, 0.12, 0.22
Fresh/worn asphalt	0.04/0.12
Conifer forest (summer)	0.08–0.15
Deciduous trees	0.15–0.20
Savanna	0.20–0.25
Green grass	0.25
Desert sand	0.30–0.40
New concrete	0.55
Ocean Ice	0.50–0.70
Old snow	0.45–0.80
Clouds	0.60–0.90
Fresh snow	0.80–0.90

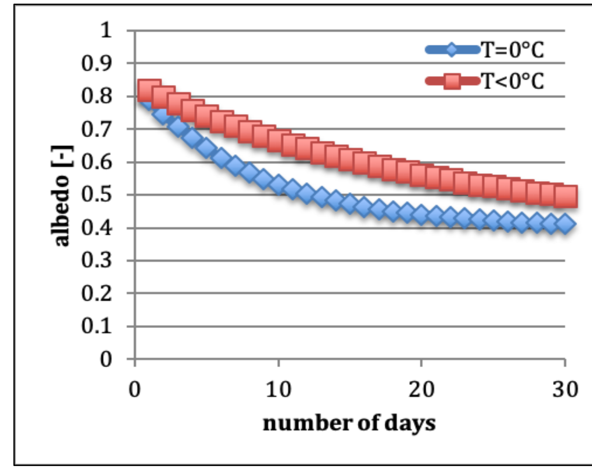
Planetary albedo Incl. atmosphere & clouds	≈ 0.30

Albedo parametrizations

Empirical models for computing albedo (DeWalle & Rango, 2008, p.154):
(a) is based on the age of snow and (b) on snow density

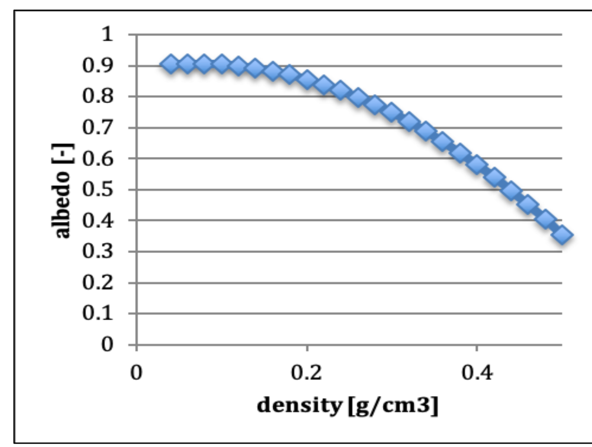
(a) $\alpha \approx \alpha_0 + Ce^{-nr}$

α_0 = minimal α for snow (≈ 0.4),
C = const. (≈ 0.44),
n = # of days since last major snowfall
r = recession coefficient, with
r = 0.05 for $T_{air} < 0°C$ and
r = 0.12 otherwise

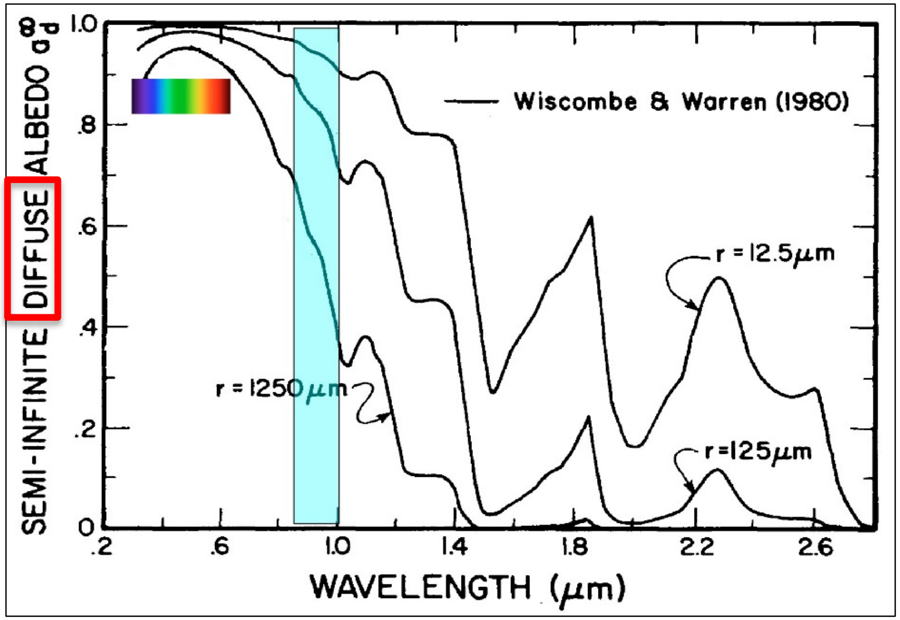


(b) $\alpha \approx 0.892 + 0.417\rho_s - 2.99\rho_s^2$

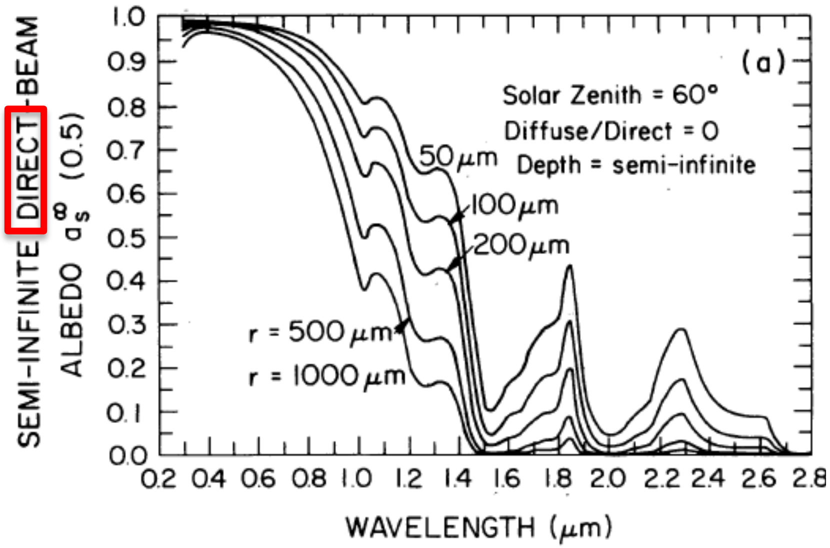
ρ_s in [g/cm³], 2nd order polynomial for density

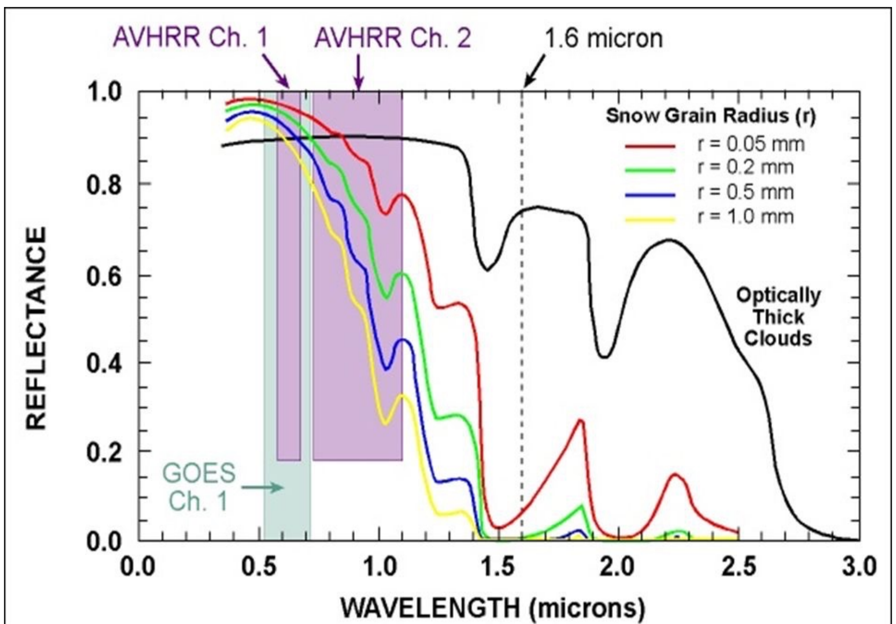


Spectral albedo, grain size, angle

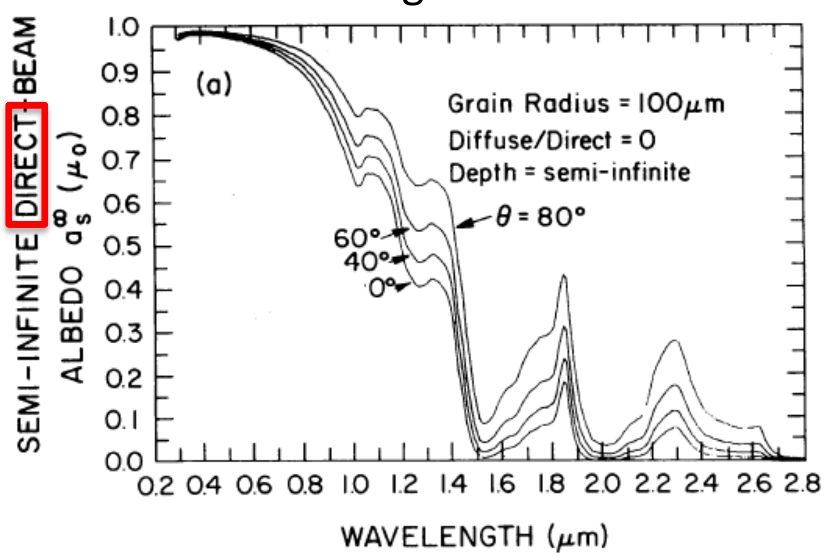


r = grain size

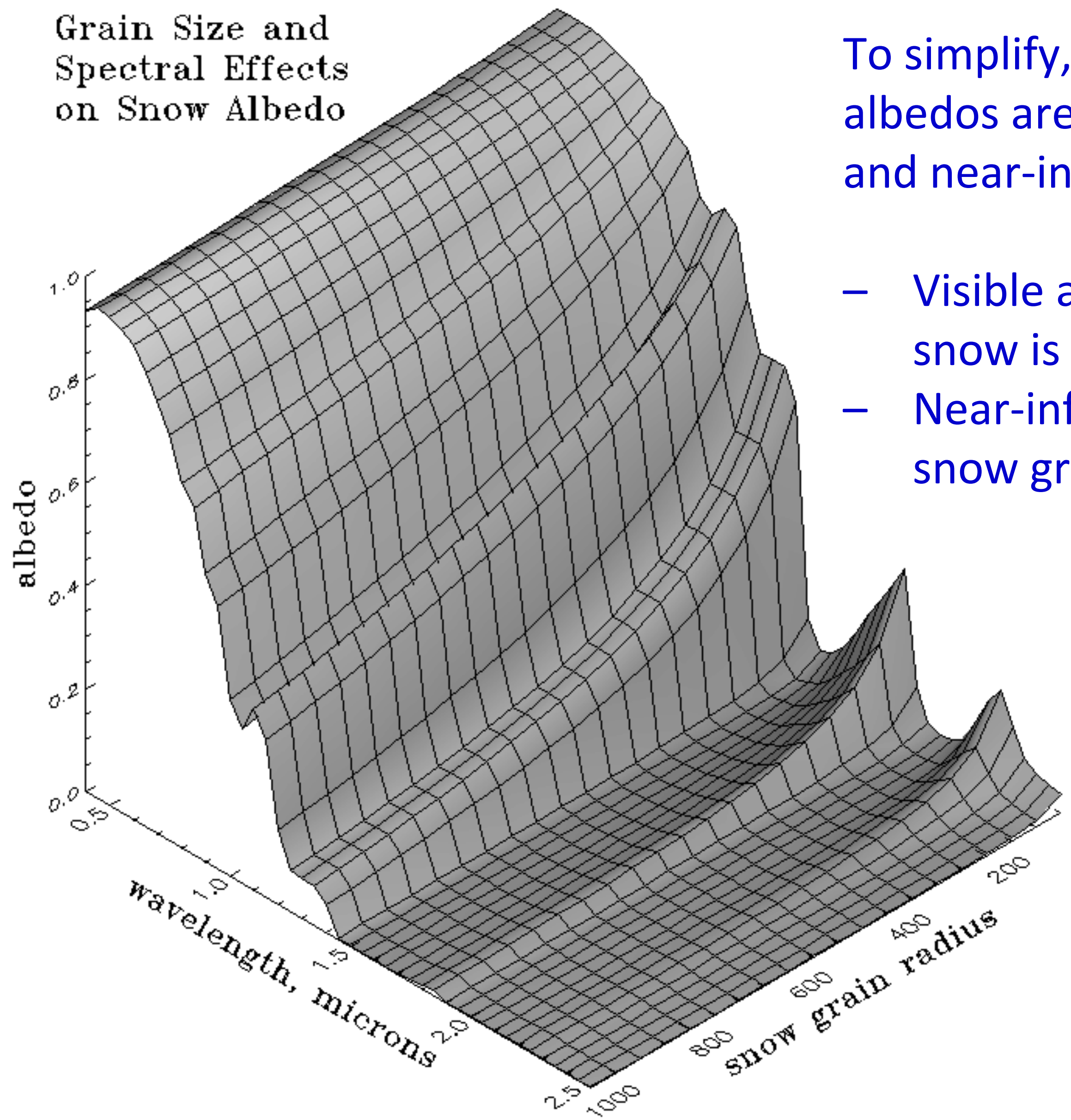




θ = zenith angle



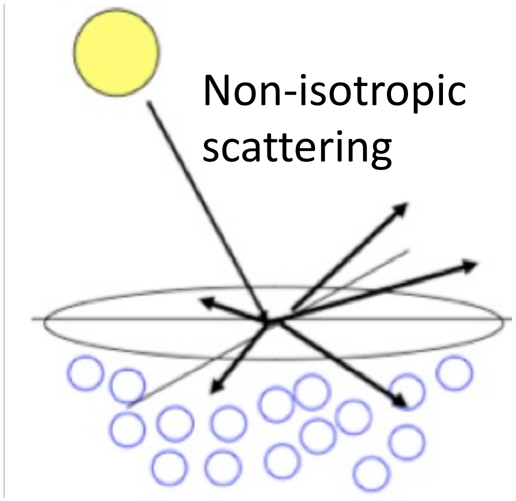
Snow albedo

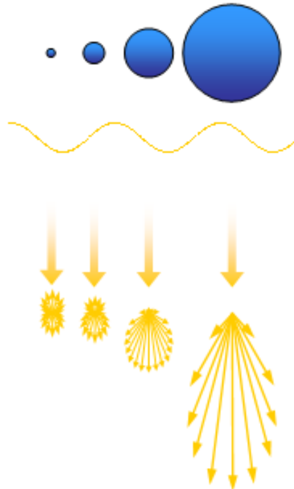


To simplify, sometimes separate mean albedos are computed for visible (VIS) and near-infrared (NIR)

- Visible albedo is very high unless the snow is dirty
- Near-infrared albedo is influenced by snow grain size

Forward scattering of snow

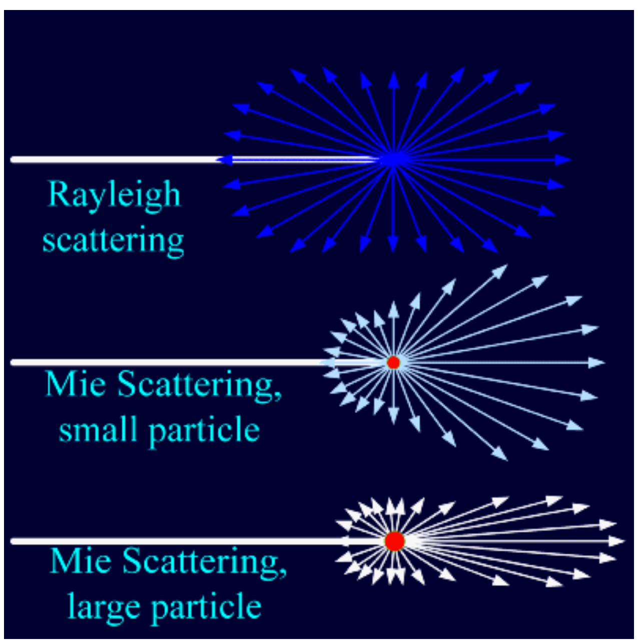




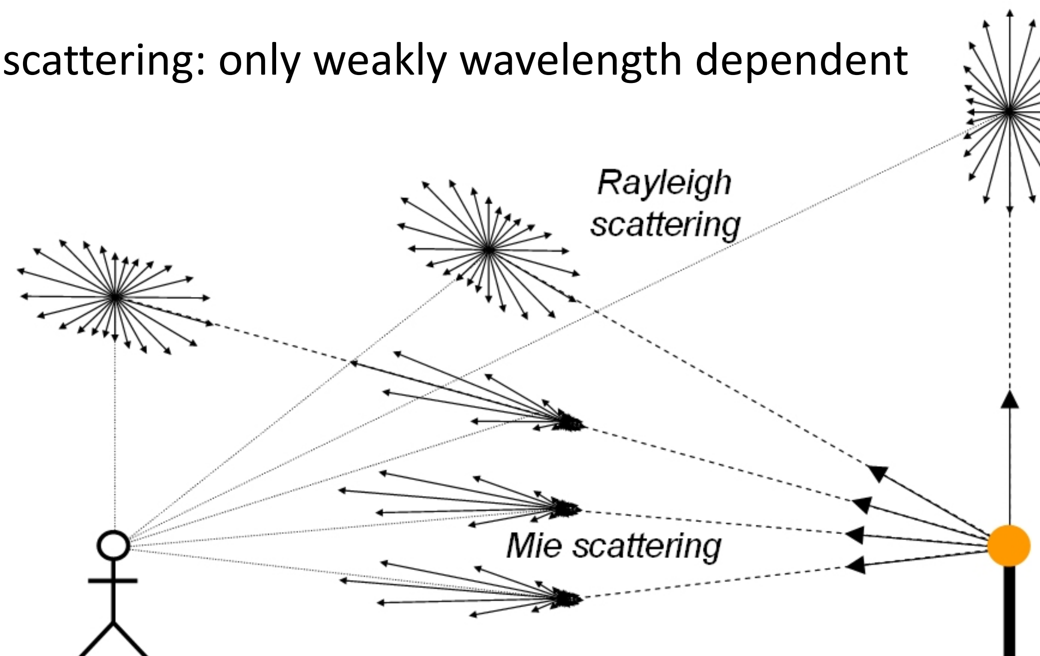
Consequences:

- Directional albedo
- Implications for
  - terrestrial
  - air-borne
  - satellite

  remote sensing



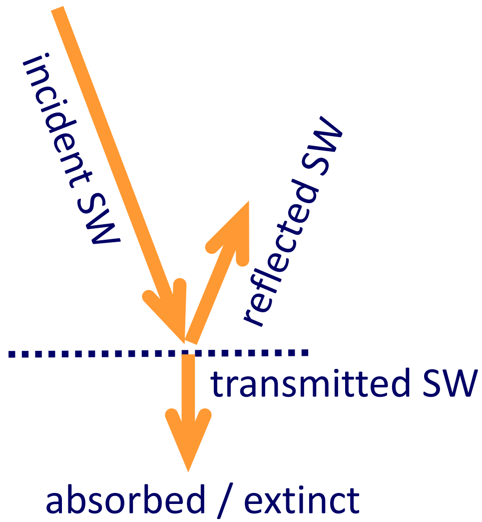
Mie scattering: only weakly wavelength dependent



Snow albedo and particles

Shortwave radiation transmission

- Transmission: Beer's Law
- Extinction (absorption)
- Leads to local heating

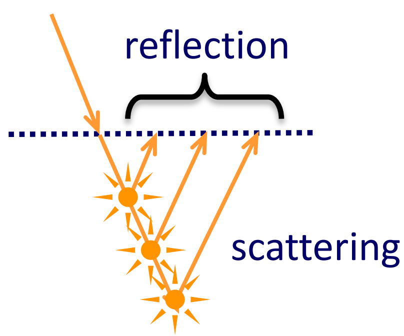


$$SW_{\text{trans}}(z) = SW_{net}\,e^{-\kappa z}\,, \qquad \kappa[\mathrm{m}^{-1}]$$

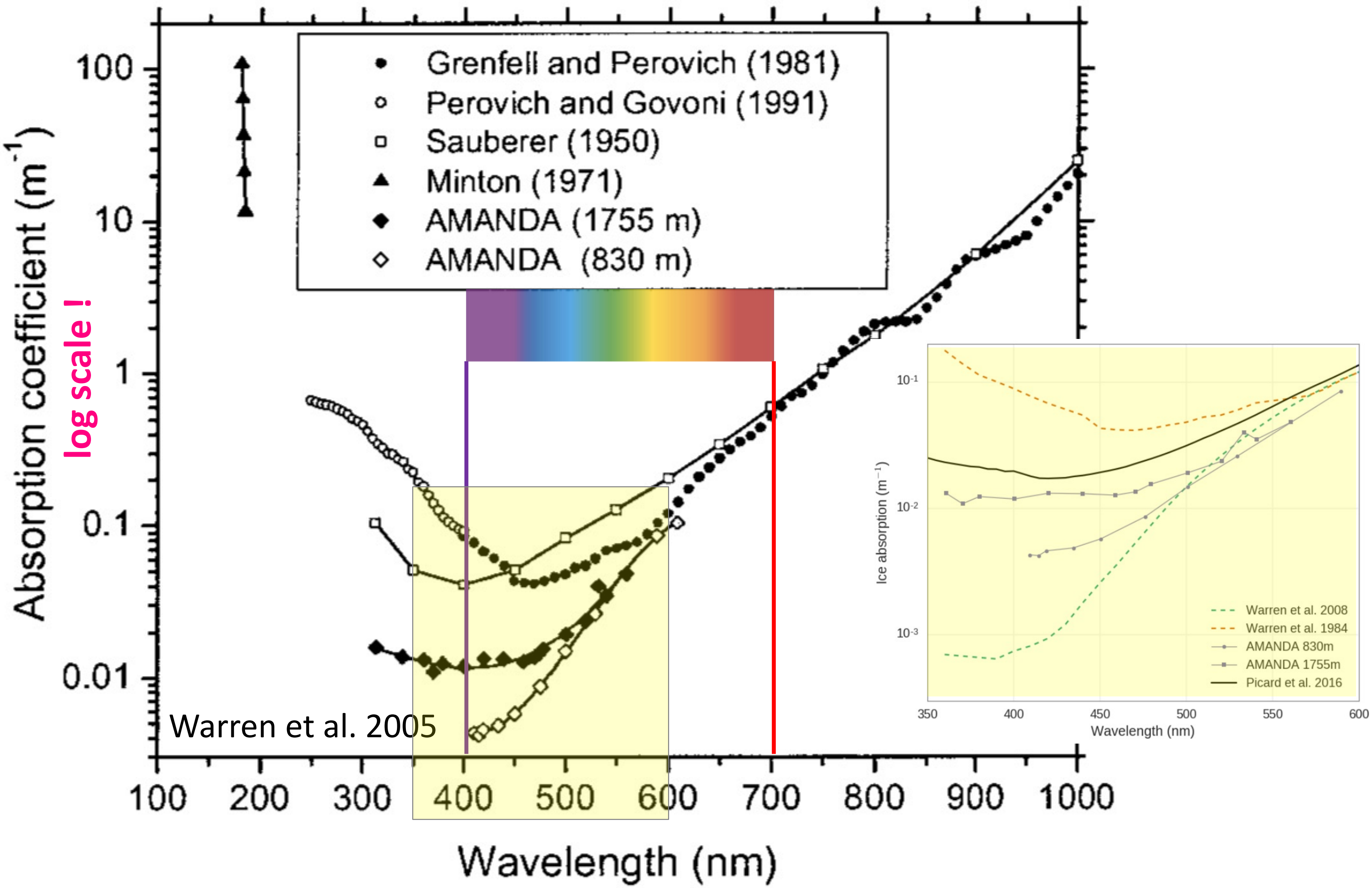
$$SW_{net} = (SW_{\text{inci.}} - SW_{\text{refl.}})$$

κ is the extinction coefficient
κ = f(λ), here use avg(κ) over VIS
SW_{net} = net SW radiation

low density snow:	$\kappa \approx 20\ \mathrm{m}^{-1}$
high density snow:	$\kappa \approx 10\ \mathrm{m}^{-1}$
pure ice:	$\kappa \approx 1\ \mathrm{m}^{-1}$



Absorption / extinction coefficient of ice



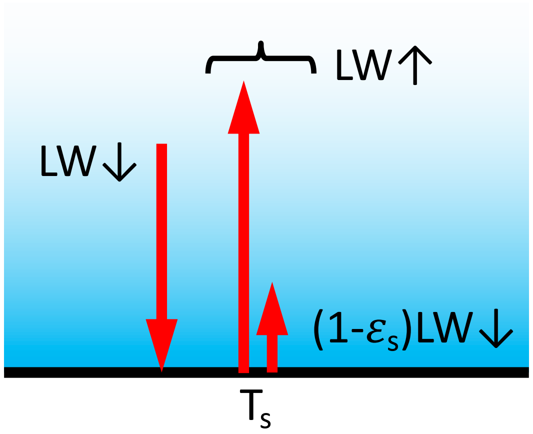
Net longwave radiation (LW_{net})

Net longwave radiation: difference between incoming and outgoing LW radiation

$$LW_{net} = LW\downarrow - LW\uparrow = \varepsilon_a \sigma T_a^4 - \varepsilon_s \sigma T_s^4$$

And more precisely for LW_{up} (considering the reflected part):

$$LW\uparrow = \varepsilon_s \sigma T_s^4 + (1-\varepsilon_s)\, LW\downarrow$$



LW↓: incoming longwave radiation (from atmosphere) [Wm^{-2}]

ε : emissivity of medium [-]

σ : Stefan-Boltzmann constant = $5.67*10^{-8}$ [$Wm^{-2}K^{-4}$]

T : temperature of medium [K]

subscripts *a* and *s* indicate air (or atmosphere) and snow (or surface), respectively.

Radiant vs. kinetic temperature

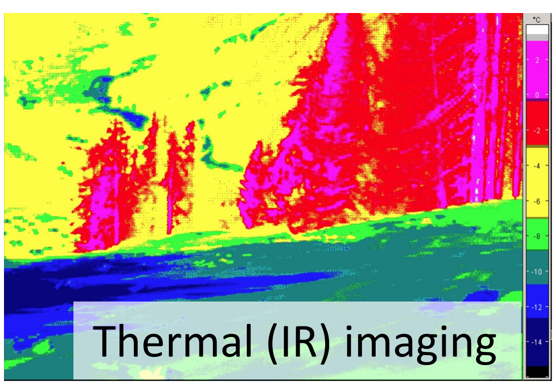
Two objects can have the same kinetic temperature, but different radiant temperatures due to different emissivity.

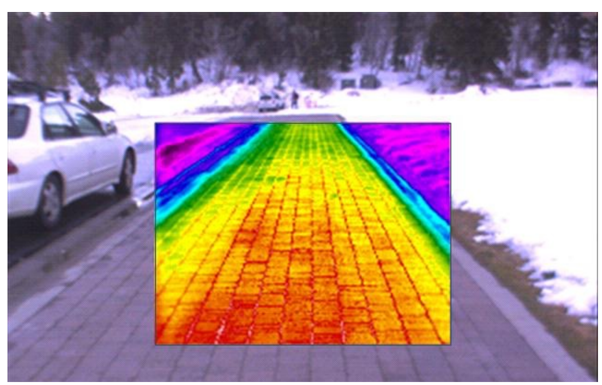
$$B_{IR} = \varepsilon \sigma T_{kin}^4 = \sigma T_{rad}^4$$

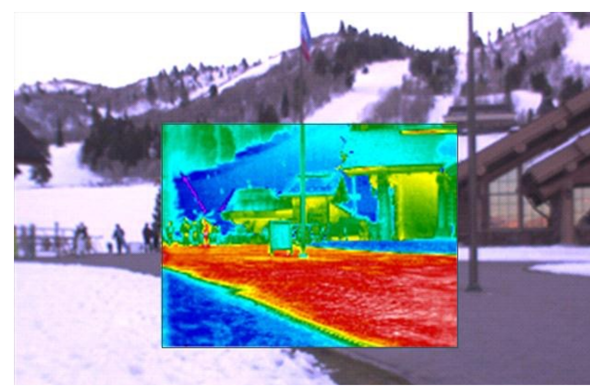
The emissivity **ε** is defined as the ratio of emitted radiant flux at a given kinetic temperature and the kinetic flux of a black body at the same T.

$$T_{rad} = \varepsilon^{0.25} * T_{kin}$$

Object	Emissivity	Kinetic Temp. [K] ; (°C)	Radiant Temp. [K] ; (°C)
Blackbody	1.0	273.15 ; 0.00	273.15 ; 0.00
Snow	0.97	273.15 ; 0.00	271.08 ; -2.07
Mirror	0.02	273.15 ; 0.00	102.72 ; -170.43







Longwave: Snow emissivity ε_λ

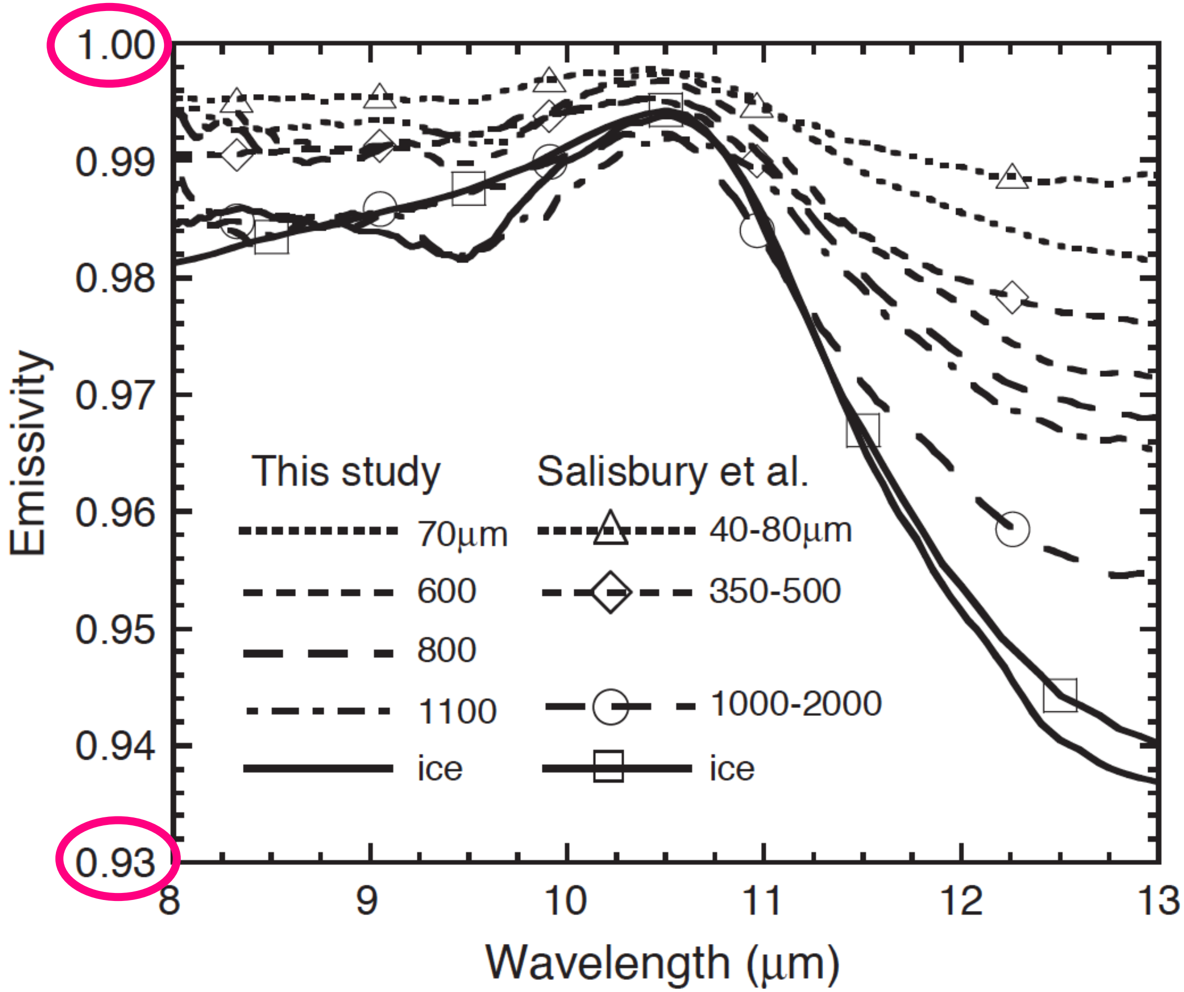
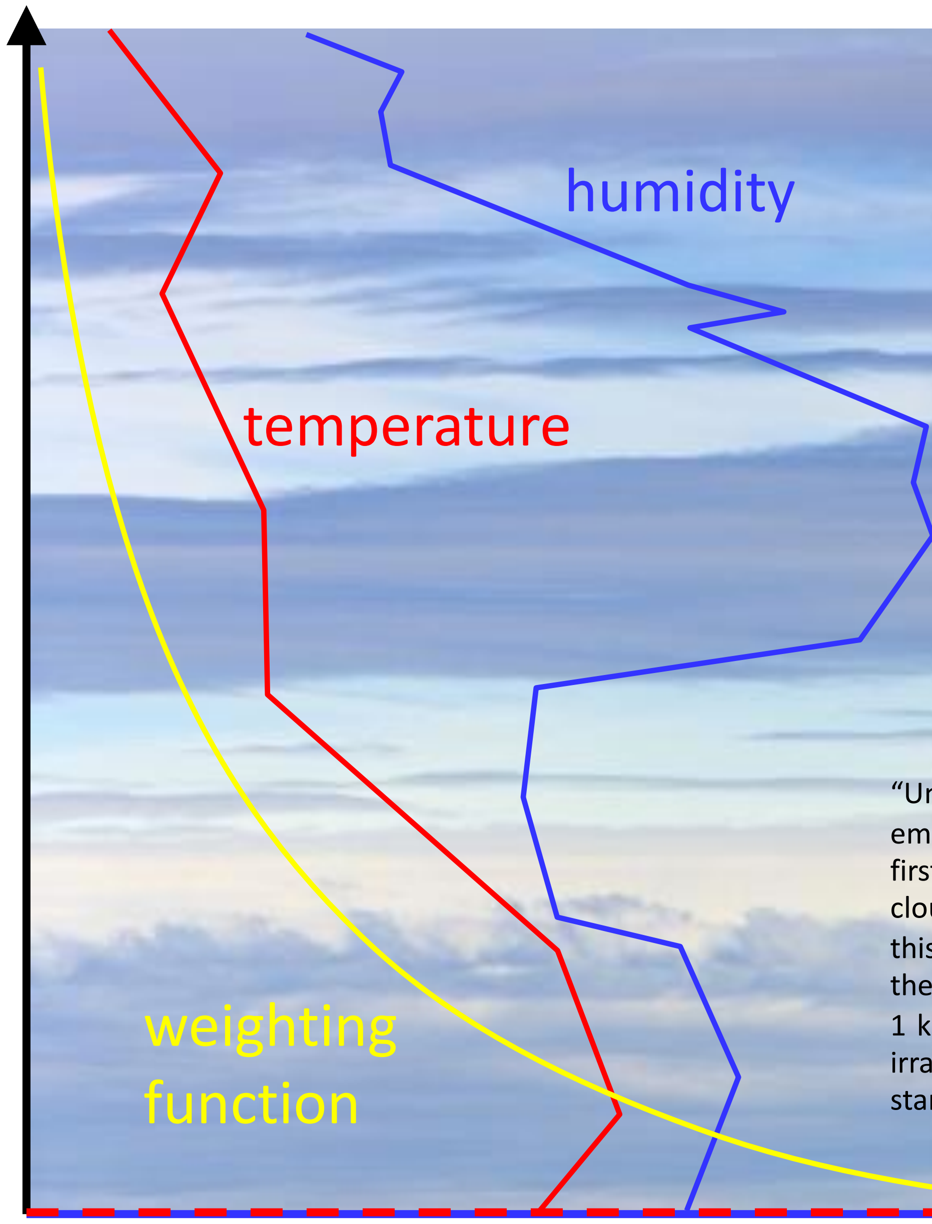


Fig. 9. Emissivity spectra at the exitance angle $\theta_{ext} = 10°$ measured for various snow types by this study and Salisbury et al. (1994). The spectra of this study are interpolated with those at 0° and 15°. Snow grain sizes observed for the individual spectra are shown as median diameter for this study and as diameter range for Salisbury et al. (1994) both in µm. The spectra by Salisbury et al. (1994) are reproduced from the ASTER Spectral Library (1999).

from:
Hori et al., 2006

Incoming LW radiation – non-trivial quantity



Temperature varies with z
Humidity varies with z
→ Emissivity of air varies with z
→ Air density varies with z

Where does the LW radiation received at the surface originate?

Lower layers contribute more than air masses far away. Presence of humidity largely increases emission ($0.7 < \varepsilon < 0.95$)
Rule of thumb:

"Under cloudless-sky conditions, as much as 60% of the atmospheric emission is derived from within the first 100 m and 90% from the first 1 km of the atmosphere. When the sky is overcast with the cloud bottom within the first 1 km, more than 90% originates within this layer between the surface and the bottom of the cloud. When the sky is overcast with the cloud bottom higher than 1 km, the first 1 km of the atmosphere still makes up about 70% of the longwave irradiance at the surface, for which the air temperature measured at standard screen level is the single most influential factor."

Ohmura, A., (2001): J. Appl. Meteorol., 40, 753-761

Incoming longwave radiation

Depends on atmospheric humidity, temperature, and cloud cover.
Parametrization by Brutsaert 1975:

$$LW \downarrow_{\text{clear sky}} = \left(1.24(e_a/T_a)^{1/7}\right) \sigma T_a^4$$

e_a = atmos. water vapor pressure [hPa] or [mbar]
T_a = air temperature at the surface [K]

$$LW \downarrow_{\text{cloudy sky}} = LW \downarrow_{\text{clear sky}} (1 + a_c C^2)$$

a_c = empirical coeff., f(cloud type)
C = cloud cover fraction [-]

Typically, $\varepsilon_{air} < \varepsilon_{sfc}$; why?

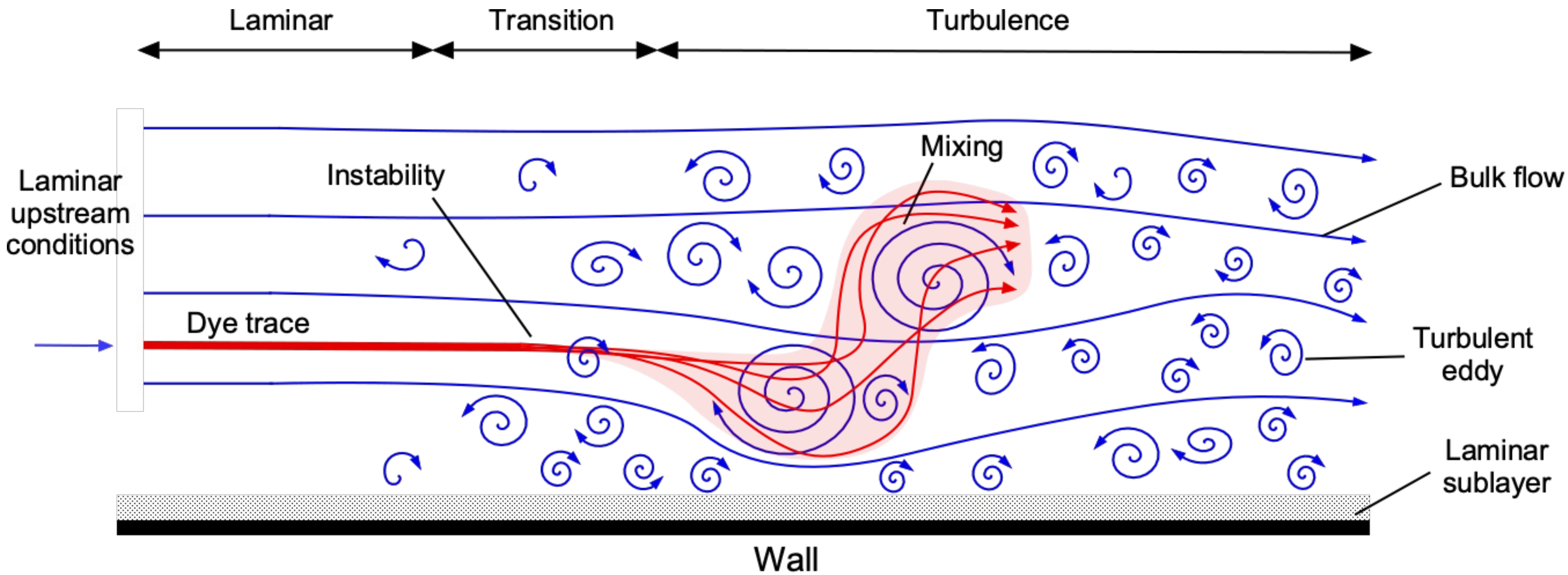
Relative humidity:
RH = $e/e_s(T)$ * 100
e = vapor pressure
e_s = saturation vapor pressure

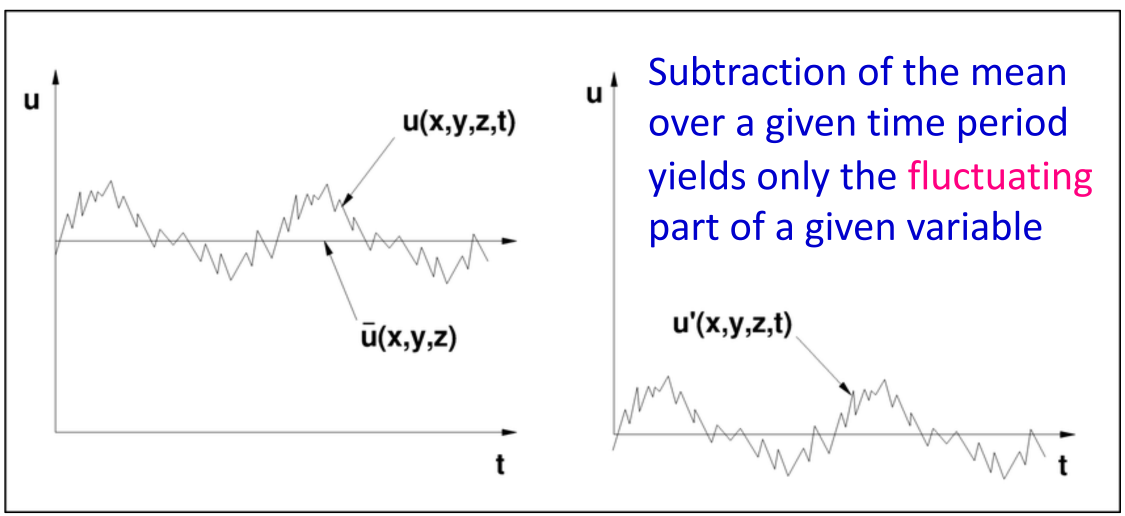
Cloud Type	a_c
cirrus	0.04
cirrostratus	0.08
altocumulus	0.17
altostratus	0.20
cumulus	0.20
stratocumulus	0.22
stratus	0.24
fog	0.25

atmos. emission occurs in spectral bands rather than uniform over a range of λ
Recall: $a_\lambda = \varepsilon_\lambda$ and thus bulk ε of atm. lower than a non-selective material.

Turbulent fluxes

Turbulent flow





Consider the variable **u**, with averaging period T

$$\bar{\boldsymbol{u}} \equiv \frac{1}{T}\int_0^T \boldsymbol{u}(t)dt$$

and $u' = u - \bar{u}$ is the fluctuating part, with $\bar{u} = 0$

$\bar{\boldsymbol{u}}$ is the average value,

u’ is the fluctuating part.

Reynolds decomposition (RD) and averaging

NS

$$\frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} = f_i - \frac{1}{\rho}\frac{\partial p}{\partial x_i} + \nu \frac{\partial^2 u_i}{\partial x_j \partial x_j}$$

Computationally expensive!
Need modelling

\+

RD

$$u_i(x, y, z, t) = \bar{u}_i(x, y, z) + u_i'(x, y, z, t)$$

=

RANS

$$\rho\frac{\partial \bar{u}_i}{\partial t} + \rho \bar{u}_j \frac{\partial \bar{u}_i}{\partial x_j} = \rho \bar{f}_i + \frac{\partial}{\partial x_j}\left[-\bar{p}\delta_{ij} + 2\mu \bar{S}_{ij} - \rho \overline{u_i' u_j'}\right]$$

Non-linear terms

$$\overline{uw} = \overline{(\overline{u} + u')(\overline{w} + w')} = \overline{u}\,\overline{w} + \overline{u'w'}$$

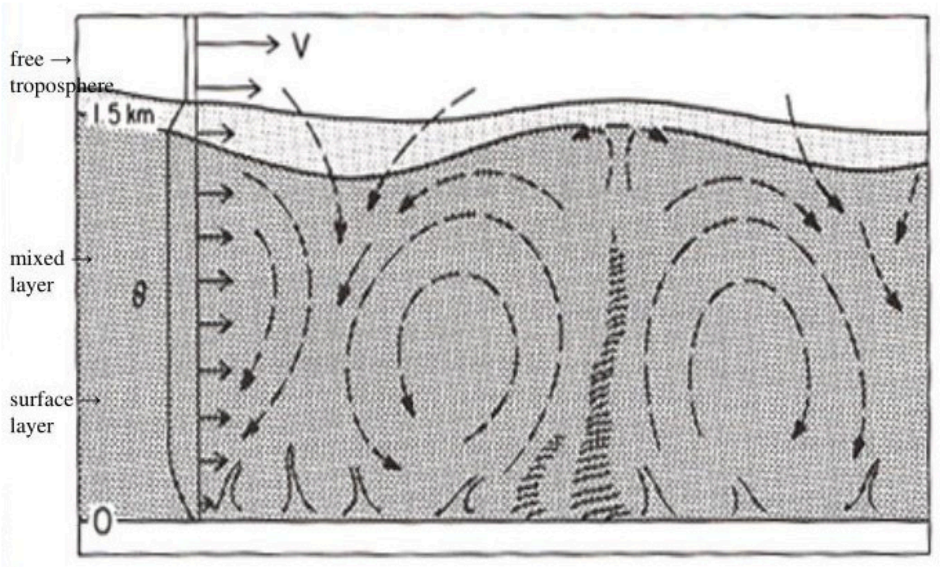
NS: Navier-Stokes Equation
RD: Reynolds Decomposition
RANS: Reynolds Averaged Navier-Stokes Equation

From Reynolds averaging we get covariance terms (turbulent fluxes)
from the non-linear terms of the Navier-Stokes Eq. and the non-linear terms from the energy Eq.)
involving the fluctuations of the vertical wind speed component w'
defining turbulent transport of momentum (u'w'), heat (T'w') and humidity (q'w')

u, v, w, are the three wind speed components in x, y, z direction, respectively (cartesian coordinates).
T is the air temperature
q is the specific humidity

Turbulent fluxes

- Momentum
- Sensible heat
- Latent heat



Rate of energy transfer depends on:
1) Temperature gradient in the air,
2) wind speed,
3) surface roughness,
4) stability of the near-surface atmosphere

Turbulent heat fluxes are estimated using

- Eddy covariance method
- Bulk aerodynamic method, assuming
  - A logarithmic profile of mean wind speed above the (snow) surface
  - Neutral atmospheric stability conditions (or requires stability corrections)
  - Fluxes of momentum and heat are constant with height (within surface layer)

Difficult to measure and calculate the mixing of air even at a well-instrumented site

Some assumptions are necessary to compute turbulent fluxes, e.g., homogeneity and stationarity, as well as dimension analysis (Monin-Obukhov Similarity Theory, MOST)

Turbulent fluxes (eddy covariance)

- Vertical component of horizontal momentum: $F_{mz} = \rho_a \overline{w'u'} \equiv -\tau_{xz} = -\tau_0$ [N m^-2]
- Vertical component of turbulent sensible heat: $F_{hz} = \rho_a c_{pa} \overline{w'T'} \equiv H$ [W m^-2]
- Vertical component of turbulent latent heat: $F_{vz} = \rho_a \overline{w'q'} L \equiv LE$ [W m^-2]

In case of momentum, there is a sink at the surface in form of a shear stress, $\boldsymbol{\tau}_0$ [N m^-2]

With τ_0 we can define the friction velocity u* : $u^* = (\tau_0/\rho_a)^{1/2}$ [m s^-1]

Assumptions / conditions: homogeneity (space) and stationarity (time)

ρ_a density of dry air [kg m^-3]
c_{pa} specific heat capacity of dry air [J kg^-1 K^-1]
L latent heat (vaporization or sublimation) [J kg^-1]
u' fluctuating part of horizontal wind component [m s^-1]
w' fluctuating part of vertical wind component [m s^-1]
q' fluctuating part of the specific humidity [kg kg^-1]
Subscript "0" denotes "at the surface", "E" denotes evaporation, and "H" stands for heat.

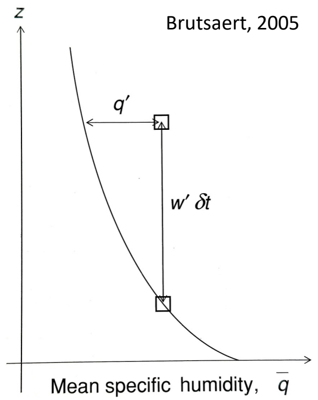
Parameterizing turbulent transport (bulk approach)

- Vertical turbulent momentum flux
- Vertical turbulent sensible heat flux
- Vertical turbulent spec. humidity flux

$$\overline{w'u'} = -C_D(\bar{u}_2 - \bar{u}_1)^2$$
$$\overline{w'T'} = -C_H(\bar{u}_2 - \bar{u}_1)(\bar{T}_2 - \bar{T}_1)$$
$$\overline{w'q'} = -C_E(\bar{u}_2 - \bar{u}_1)(\bar{q}_2 - \bar{q}_1)$$

where

- C_D is the transfer coefficient for momentum, called Drag coefficient, [-]
- C_H is the sensible heat transfer coefficient, called Stanton number, [-]
- C_E is the water vapor transfer coefficient, called Dalton number, [-]



If u_1 is the surface level where u = 0, the turbulent fluxes can be written as:

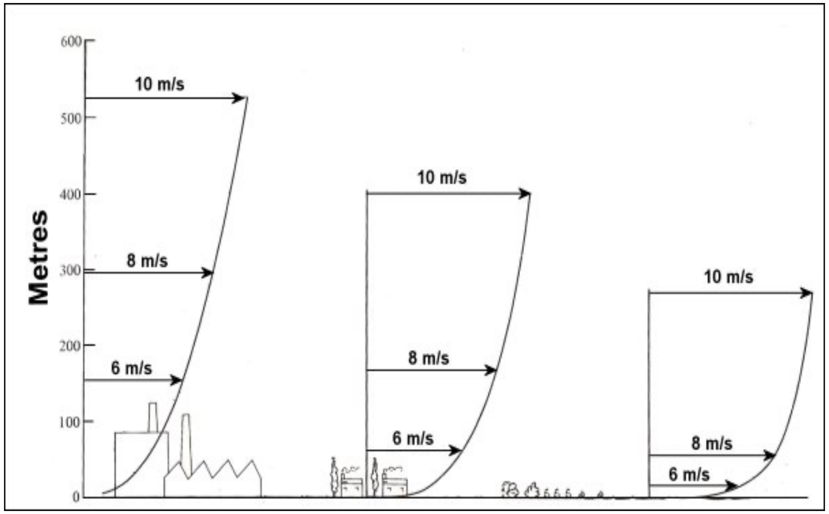
→ Values for C_D, C_H, C_E ?

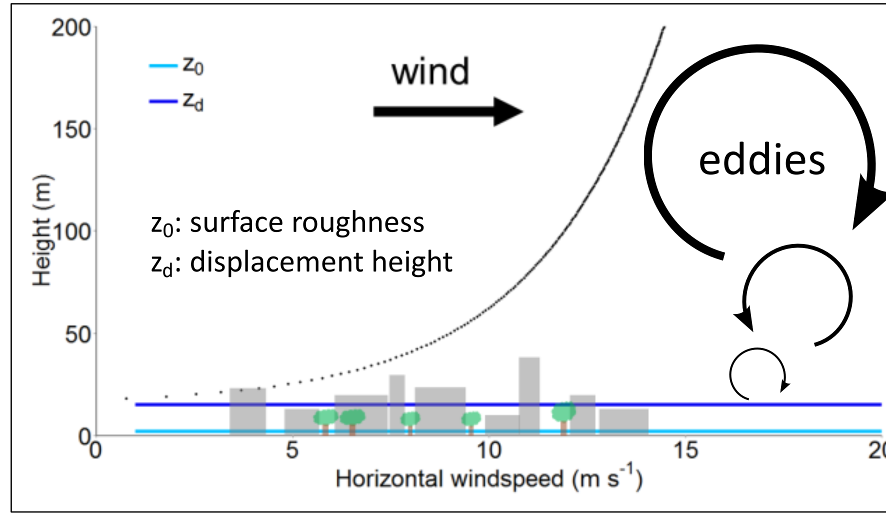
They are z-dependent !

(Measurement height).

$$\tau_0 = -C_D \rho_a \bar{u}^2$$
$$H = -C_H \rho_a c_{pa} \bar{u}_a (\bar{T}_a - \bar{T}_s)$$
$$LE = -C_E \rho_a\, L\, \bar{u}_a (\bar{q}_a - \bar{q}_s)$$

Turbulence similarity, dimensional analysis





- <u>Objective 1</u>: find a scaled vertical velocity profile valid for neutral atmos. surface layer
- <u>Objective 2</u>: calculate turbulent fluxes using standard measured meteorological variables
- Logarithmic profile of mean wind speed in general form: u = a · ln(b·z); a, b = const. (also of potential temperature, specific humidity, etc.)
- In plane-parallel flow, an increase in velocity in z-direction (du/dz) means downward momentum flux with a sink at the surface (friction).
- The mean velocity gradient in a fluid of density, ρ, is determined by the shear stress at the surface, τ_0, and the distance from the surface, z.

Logarithmic wind profile and roughness length

The previous variables can be combined into a single dimensionless quantity, k:

$$\frac{u_*}{z(d\bar{u}/dz)} = k$$

reformulated:

$$\frac{\partial \bar{u}}{\partial z} = \frac{u_*}{k}\frac{1}{z}$$

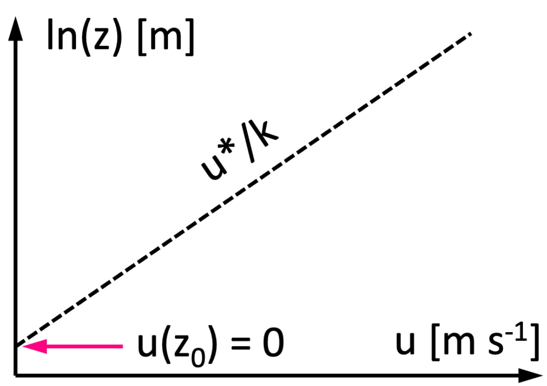
k ≈ 0.4 and is nearly invariant, and is known as the 'von Karman's constant'.

Integration of the above relation directly results in the logarithmic wind profile:

$$u(z) - \underbrace{u(z_0)}_{= 0} = \frac{u_*}{k} ln\left(\frac{z}{z_0}\right)$$

where z_0 is the aerodynamic roughness length for momentum, the theoretical height where the mean wind speed approaches zero for a logarithmic wind profile.

u_*/k = slope of log. profile under neutral stability conditions



z_0 for snow ranges from 0.0002 – 0.02 m;
Avg. values are about 0.001 – 0.005 m; otherwise:

$$z_0 = 0.136 h_0$$ (Brutsaert, 1982)

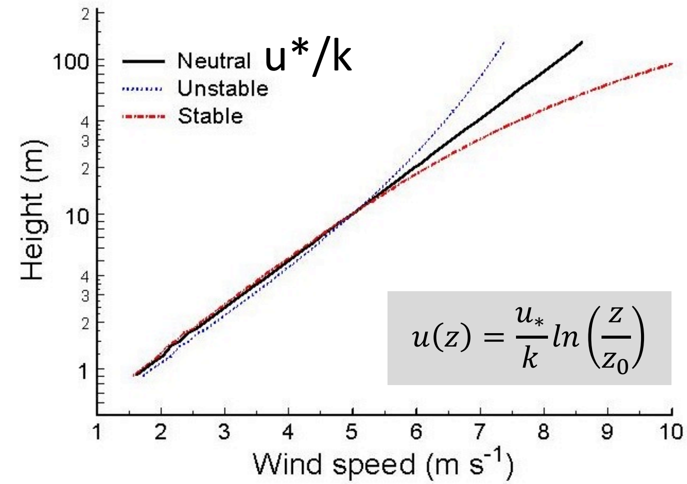
h_0 is the average height of roughness obstacles

Transfer coefficient in surface layer

For neutral atmospheric conditions (subscript "n"):

$$C_{Dn} \approx C_{Hn} \approx C_{En} = k^2[ln(z/z_0)]^{-2}$$

C_{Xn} = bulk transfer coeff. under neutral conditions (-)
k = von Karman's constant (0.4)
z = height of wind measurement (m)
z_0 = aerodynamic roughness length of surface (m)
u_* = friction velocity (m s^{-1}), related to shear stress



- Constant flux layer assumption
- Turbulent exchange coefficients
  for non-neutral conditions: $C_x(z_0, z/L)$ where L is the Obukhov length

For turbulent convective sensible heat (under neutral conditions), we can write:

$$H_n = \rho_a c_{pa} u_a C_{Hn}(T_a - T_s)$$

→

$$H_n = \frac{\rho_a c_{pa} u_a k^2}{[ln(z/z_0)]^2}(T_a - T_s)$$

For stable and unstable conditions, stability correction functions need to be applied.

Latent heat flux (LE)

Convective water vapor exchange between snow and overlying atmosphere.
Note that water vapor is invisible (fog is already condensed water, i.e., droplets).

Rate of energy transfer depends on:

- Vapor pressure gradient (defines magnitude & direction of energy transfer)
- Wind speed (enhances turbulent transport)
- Surface roughness (favors turb. transport)
- Atmospheric stability (stratification)

LE can be converted into a mass flux (rate) [mm s^{-1} = kg m^{-2} s^{-1}] of liquid water equivalent (evaporation or sublimation):

$$E = LE/(L_v \rho_w)$$

$$S = LE/(L_s \rho_i)$$

Latent heat flux (LE)

$$LE = \rho_a L\, C_e u_a (q_a - q_s)$$

$$q = 0.622 e_a \,/\, (P_a - 0.378 e_a)\text{ , with } e_a \ll P_a \text{ (*)}$$

$$LE = \rho_a L\, (0.622/P_a)\, C_e u_a (e_a - e_s)$$

m_{vapor} = 18 g mol^{-1}
$m_{dry\text{-}air}$ = 29 g mol^{-1}

m_{vap}/m_{air} = 0.622
1 – 0.622 = 0.378

(*) : see Brutsaert (2005) “Hydrology”

Subscripts “a”, “s” refer to air, surface, resp.

q = specific humidity of air (kg kg^{-1})

ρ_a = density of air (kg m^{-3})

L = latent heat of vaporiz. or sublim. (J kg^{-1})

P_a = total atmospheric pressure (Pa)

C_e = bulk transfer coeff. for vapor exchange (-)

u_a = wind speed at height z, (m s^{-1})

e_a = vapor pressure (moist air) at height z (Pa)

e_s = vapor pressure at the (snow) surface (Pa)
assumed to be saturated for snow!

Rain on snow

Three main effects:

1. Advection of warmer rain (heat and mass) into the snowpack
2. Release of latent heat (L_f) if rainfall freezes in sub-zero temperature snowpack
3. Condensation on snowpack due to saturated air above snow

$$Q_{rH} = P_r \rho_w c_{p,w} (T_r - T_s), \quad \text{for } T_s = 0^oC \text{ (and } T_r > T_s\text{)}$$

$$Q_{rLE} = P_r \rho_w L_f, \quad \text{for } T_s < 0^oC$$

(DeWalle & Rango, 2008, p.170)

Q_{rH} = sensible heat transfer due to rain on snow (Wm^{-2})
Q_{rLE} = latent heat release due to rainfall freezing (Wm^{-2})
P_r = precipitation rate, intensity ($m\ s^{-1}$)
ρ_w = density of water ($kg\ m^{-3}$)
c_{pw} = specific heat of water at T_r ($J\ kg^{-1}\ K^{-1}$)
T_r = temperature of the rain (K), assume $T_r \approx T_{air}$
T_s = temperature of the snow (K)
L_f = latent heat of fusion ($J\ kg^{-1}$)

Conductive heat flux (Fourier equation):

1D: $$\frac{\partial T}{\partial t} = \kappa \frac{\partial^2 T}{\partial z^2}$$

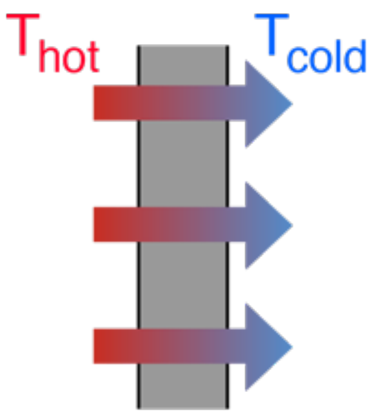
3D: $$\frac{\partial T}{\partial t} = -\kappa \nabla^2 T = 0$$

where $$\kappa = \frac{k}{\rho c_p}$$

$$\rho c_p \frac{\partial T}{\partial t} = k \frac{\partial^2 T}{\partial z^2} = \frac{\partial}{\partial z}\left(k \frac{\partial T}{\partial z}\right)$$

Heat flux: $$Q_c = -k_s \frac{\partial T}{\partial z} \approx -k_s \frac{T_s - T_b}{z_s - z_b}$$

- κ = thermal diffusivity [$m^2\ s^{-1}$]
- Q_c = conductive heat flux in the snow ($W\ m^{-2}$)
- k_s = thermal conductivity of snow ($W\ m^{-1}\ K^{-1}$)
- z = depth / vertical axis (m)
- T_s = temperature at surface (depth z_s) (K)
- T_b = temperature at bottom (depth z_b) (K)



Note that $k_s = k_{eff}$ (effective thermal conductivity of snow)

Conductivity – Density relation

$$q = -k_{eff}\frac{dT}{dz}$$

$$k_{eff} = 0.138 - 1.01\rho_s + 3.233\,\rho_s^2 \qquad [0.156 \le \rho_s \le 0.6]$$

$$k_{eff} = 0.023 + 0.234\rho_s \qquad [\rho_s < 0.156] \quad \text{(g/cm}^3\text{)}$$

k_{eff} = effective thermal conductivity

What else than pure conduction through ice matrix?

- vapor transport
- forced convection (ventilation)
- heat conduction in the air

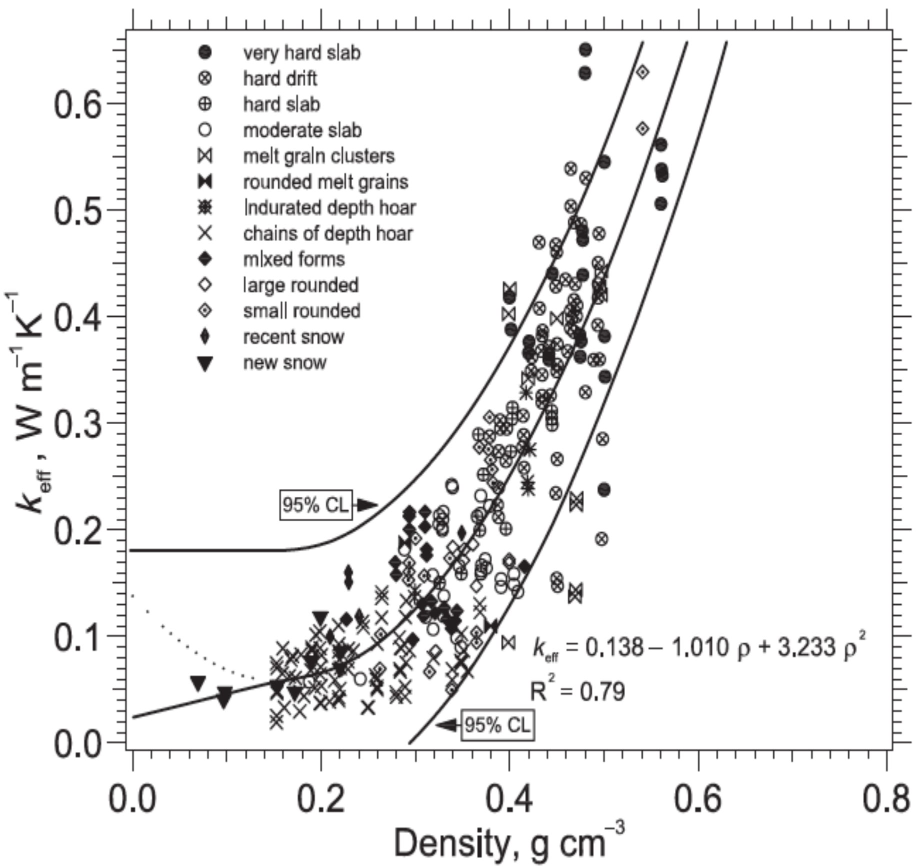
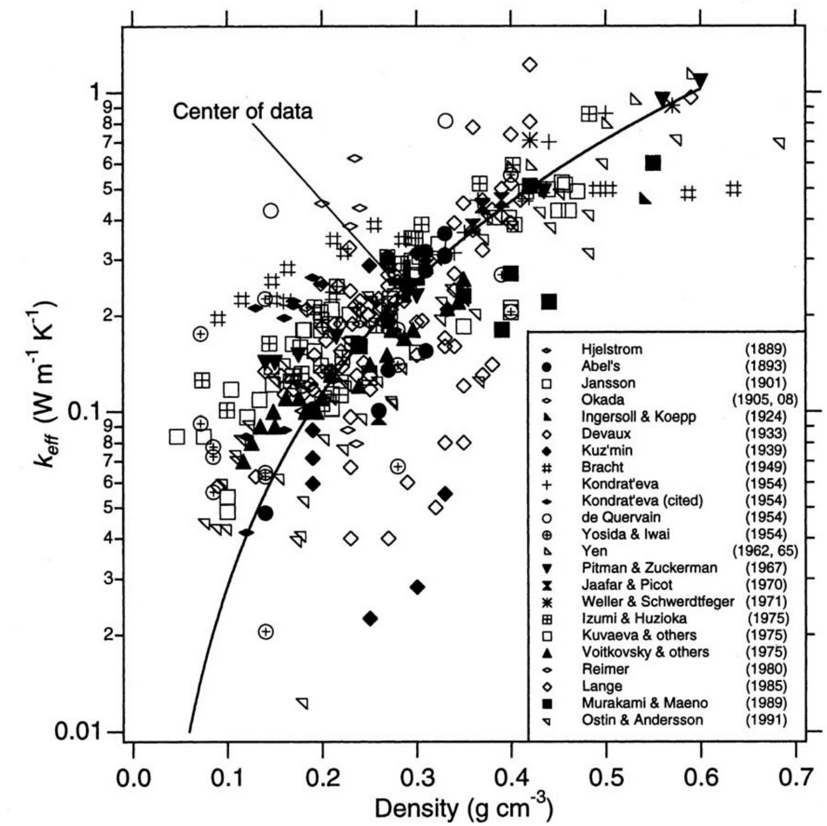


Figure 3.6 Effective thermal conductivity of snow as a function of density (modified from Sturm *et al.* 1997 from *Journal of Glaciology* with permission of the International Glaciological Society).



Snowpack internal energy (“cold content”)

Integral form:

$$\Delta Q = \int_{z=0}^{z=d} \left(\rho_s c_{pi} \delta T_s\right) dz$$

Discrete form:

$$\Delta Q = \sum_j \left(\rho_{s,j} c_{pi}\right) \Delta T_{s,j} \Delta z_j$$

Q_i = snowpack internal energy per unit area ($J\ m^{-2}$)
ρ_s = snow density ($kg\ m^{-3}$)
c_{pi} = specific heat of ice ($J\ kg^{-1}\ K^{-1}$)
T_s = snow temperature (K)
z = height above soil surface (m); z=0 at soil surface
d = snowpack depth (m)

Corresponding heat flux F over time interval Δt: $F = \Delta Q/\Delta t$ ($J\ m^{-2}\ s^{-1} = W\ m^{-2}$)

Melt energy

Energy required for phase change (latent heat of fusion)

$$M_{\mathrm{SWE}} = \frac{\rho_s}{\rho_w}\frac{\partial h_s}{\partial t} = \frac{Q_m}{\rho_w L_f}$$

$$M_{\mathrm{snow}} = \frac{\partial h_s}{\partial t} = \frac{Q_m}{\rho_s L_f}$$

M = melt rate (m s^{-1})

h_s = snow depth (m)

Q_m = heat flux available for melting (W m^{-2}), i.e., net energy balance excess

ρ_w = density of liquid water at 0°C (kg m^{-3})

ρ_s = density of snow (kg m^{-3})

L_f = latent heat of fusion, 334 (kJ kg^{-1})

Effects of Topography and Canopy

Topographic influences

Topography controls distribution of snow in mountainous terrain

- Elevation
- Slope (inclination)
- Aspect (exposure)

} influence distribution of net SW and net LW rad.

Consequences:

- Differential melt
- Corrections for effect of top. necessary
- Shading !
- Precipitation patterns (amount and type)
- Wind speed
- Temperature
- Humidity

} patterns

Topographic influences: Aspect

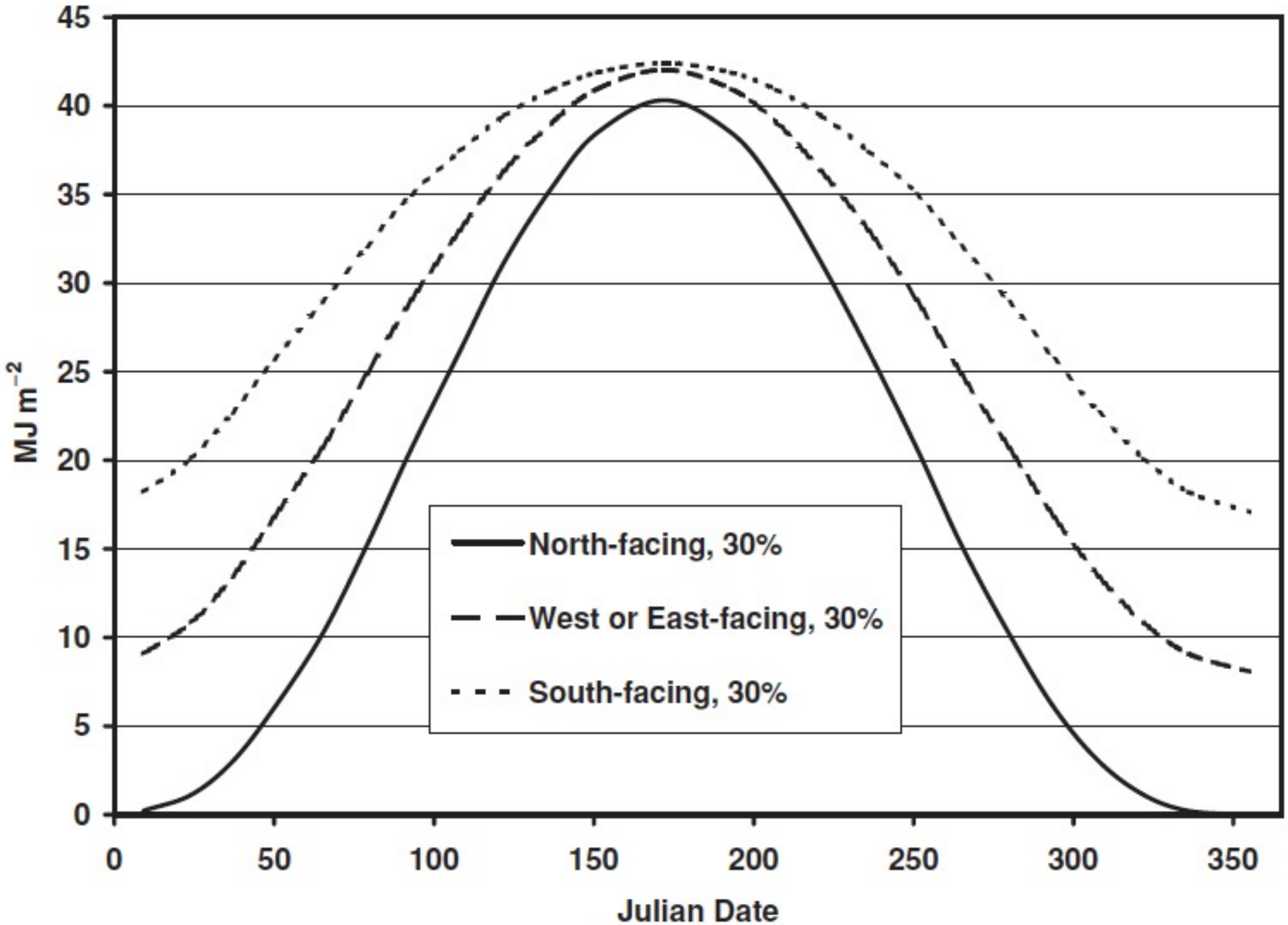
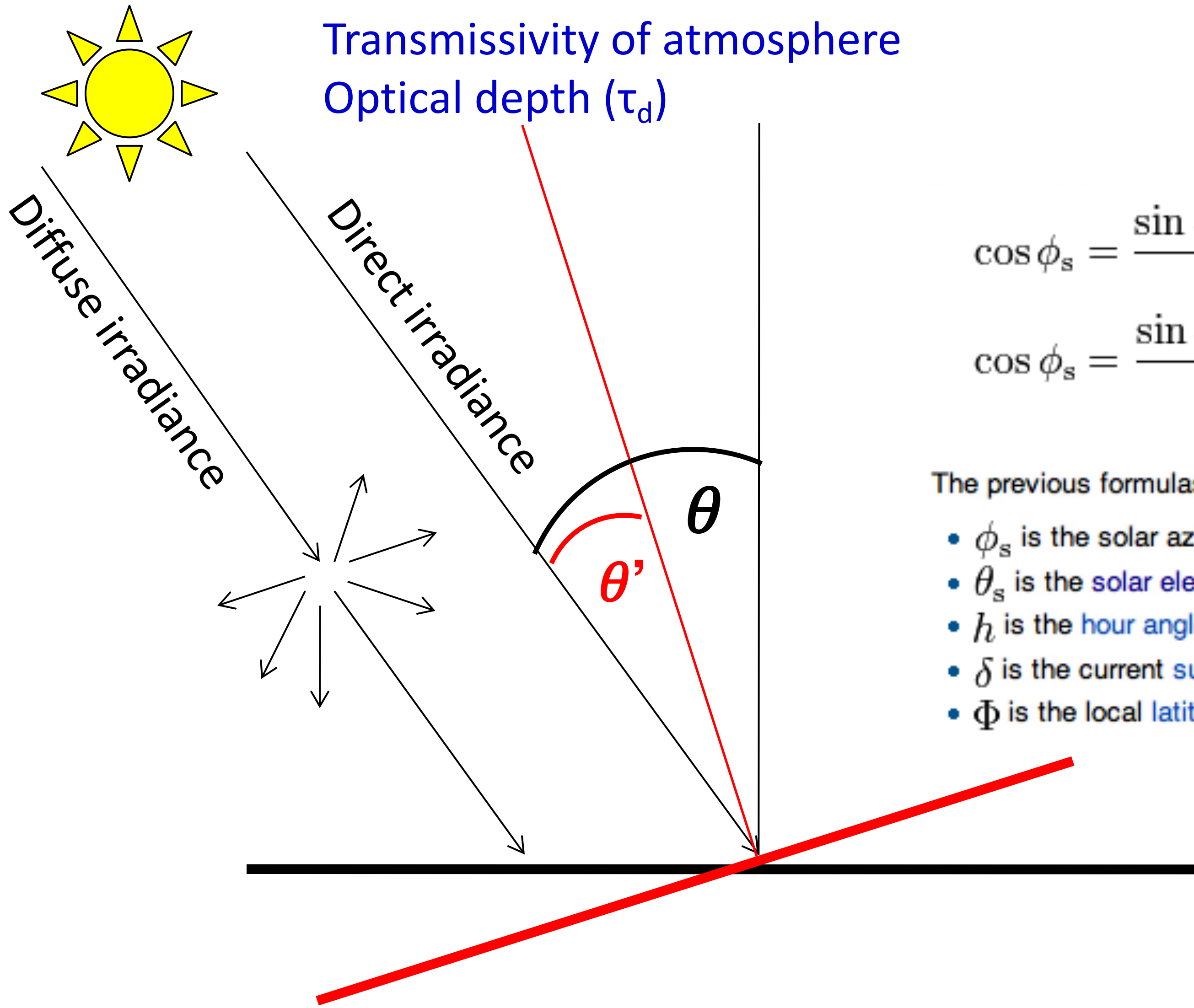


Figure 7.2 Daily potential solar irradiation for 30% slopes facing N, S and E–W at various times of the year and a latitude of 50 °N (Frank and Lee, 1966; see Appendix B).

Irradiance on inclined surfaces



$$\cos\phi_s = \frac{\sin\delta\cos\Phi - \cos h\cos\delta\sin\Phi}{\cos\theta_s}$$

$$\cos\phi_s = \frac{\sin\delta - \sin\theta_s\sin\Phi}{cos\theta_s\cos\Phi}$$

The previous formulas use the following terminology:

- ϕ_s is the solar azimuth angle
- θ_s is the solar elevation angle
- h is the hour angle of the present time
- δ is the current sun declination
- Φ is the local latitude[1]

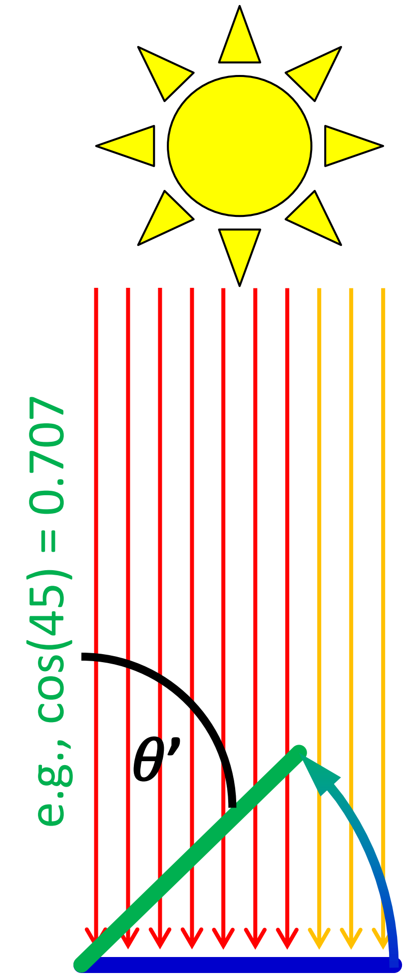
Both slope and aspect are important

Inclined terrain: cosine effect

Net shortwave (solar) radiation

$$SW_{\perp}\downarrow = SW_{\text{dir}}\cos(\theta') + SW_{\text{diff}}$$

$$SW'_{\text{net}} = (1-\alpha)\, SW_{\perp}\downarrow$$



$SW_{\perp}\downarrow$: incoming shortwave radiation normal to the slope
SW_{dir} : incoming slope-normal direct beam solar radiation
SW_{diff} : incoming diffuse solar radiation
θ' : solar zenith angle normal to slope
SW'_{net} : slope-normal net shortwave radiation

Consider direct and diffuse reflectance from adjacent slopes!
Various expressions for SW_{dir} and SW_{diff} on slopes exist.
→ Sky view factor

Other topographic effects, T, H, u

- Temperature (elevation): use lapse rate from local climate data:
  dry adiabatic: 1.0K/100m
  moist adiabatic: 0.6K/100m

$$\Delta T = \gamma(h_{\text{ref}} - h_{\text{est}})/100$$

γ = temperature lapse rate (K/100m)
h_{ref} = elevation of reference T station
h_{est} = elevation of station to estimate T

- Wind speed:
  influences sublimation & accumulation;
  convective heat exchange in the snowpack

- Humidity: assumed invariant
  (was found to vary less at basin scale)

Forest influences

- Forest canopy shadows the surface, reducing incoming shortwave radiation
- Needles, bark, and other forest litter reduce snow albedo
- Longwave emission from trees leads to increased heating/melt around trunks
- Longwave emission reduces radiative losses (during night)
- Forests reduce wind speeds, thus reducing sensible & latent heat exchange
- Trees intercept snow and increase the surface area
- Open forest traps snow (reduced wind speed & wind erosion
- Overall, forest cover reduces the rate of snowmelt
- 15-60% reduction in daily snowmelt rates
  (reported for a deciduous study area in Japan; Ohta et al., 1993)

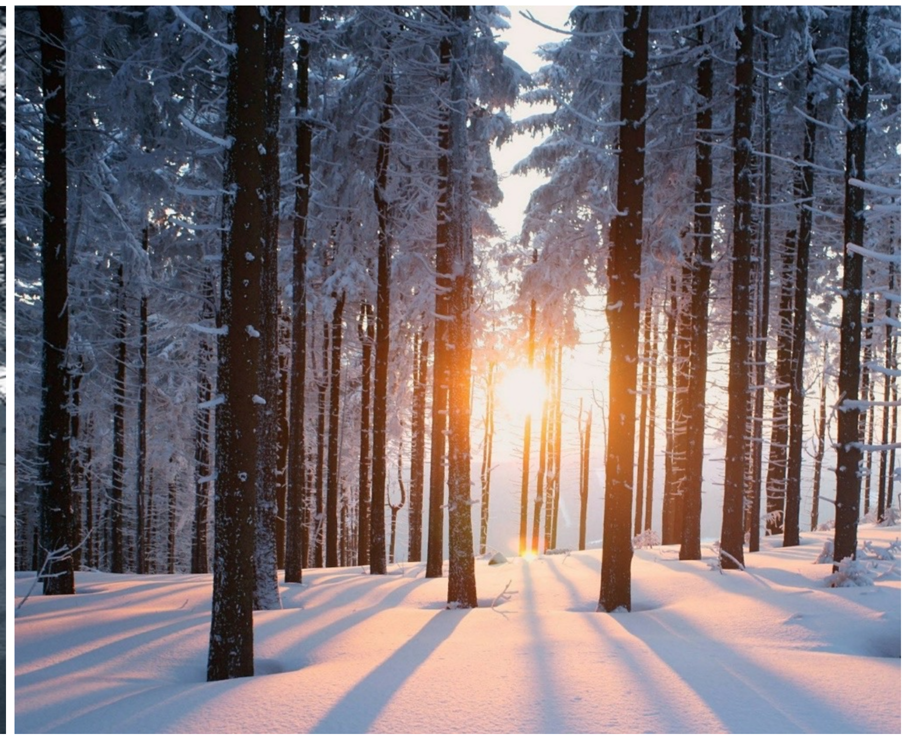
Forest influences

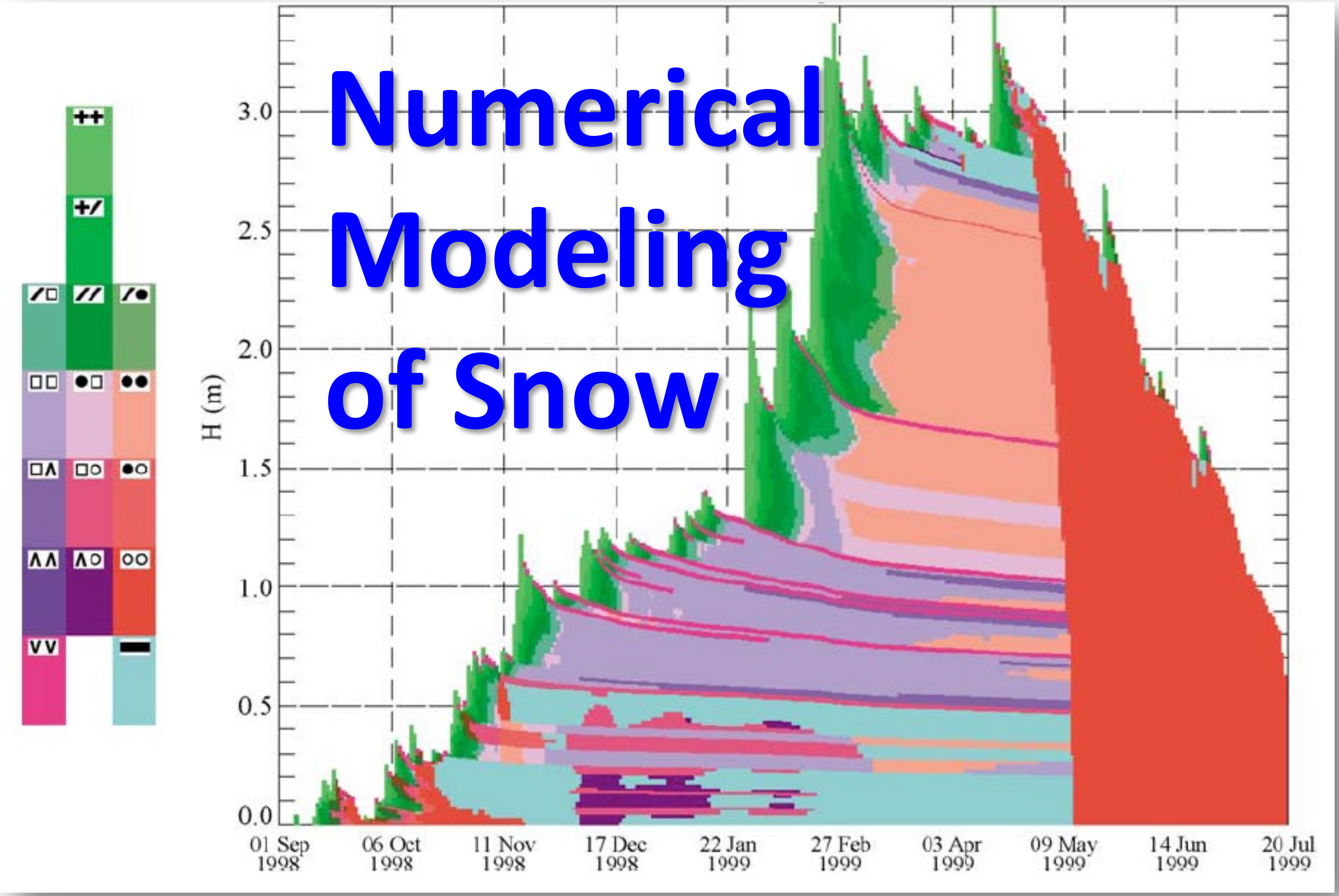
SW radiation in forests $$SW_f \downarrow / SW \downarrow = \exp(-\kappa \cdot LAI)$$

$SW_f\downarrow$: incoming shortwave radiation flux in forest [W m^{-2}]

κ : extinction coefficient for SW radiation in forest [-], specific for forests

LAI : leaf area index [-]





Numerical Modeling of Snow
H (m)
3.0
2.5
2.0
1.5
1.0
0.5
0.0
01 Sep 1998
06 Oct 1998
11 Nov 1998
17 Dec 1998
22 Jan 1999
27 Feb 1999
03 Apr 1999
09 May 1999
14 Jun 1999
20 Jul 1999

Grain Type Classification

Class	*Symbol*	*Code*	*Colour*[1]	Web colour name	*RGB*[2] (0–255)
Precipitation Particles	+	PP		Lime	0 / 255 / 0
Machine Made snow	◎	MM		Gold	255 / 215 / 0
Decomposing and Fragmented precipitation particles	/	DF		ForestGreen	34 / 139 / 34
Rounded Grains	●	RG		LightPink	255 / 182 / 193
Faceted Crystals	□	FC		LightBlue	173 / 216 / 230
Depth Hoar	Λ	DH		Blue	0 / 0 / 255
Surface Hoar	V	SH		Fuchsia	255 / 0 / 255
Melt Forms	○	MF		Red	255 / 0 / 0
	◎◎	MFcr			
Ice Formations	▬	IF		Cyan/Aqua	0 / 255 / 255