
Snow
physical
properties

Outline

- Physical properties of snow
  - Volume
  - Depth
  - Snow covered area
  - Porosity
  - Liquid water content
  - Density
  - Snow water equivalent
  - Permeability
  - Specific surface area
- Thermal properties of snow
- Electromagnetic (radiative) properties of snow
- Mechanical properties of snow
- (Chemical properties: not discussed)

Snowpack volume

$$V_s = V_i + V_w + V_a = h_s A$$

V_s = volume of snow [m^3]

V_i = volume of ice [m^3]

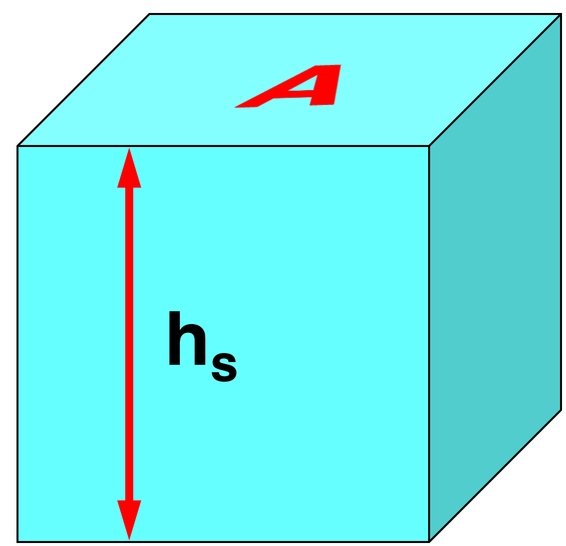
V_w = volume of water [m^3]

V_a = volume of air [m^3]

h_s = snow depth [m]

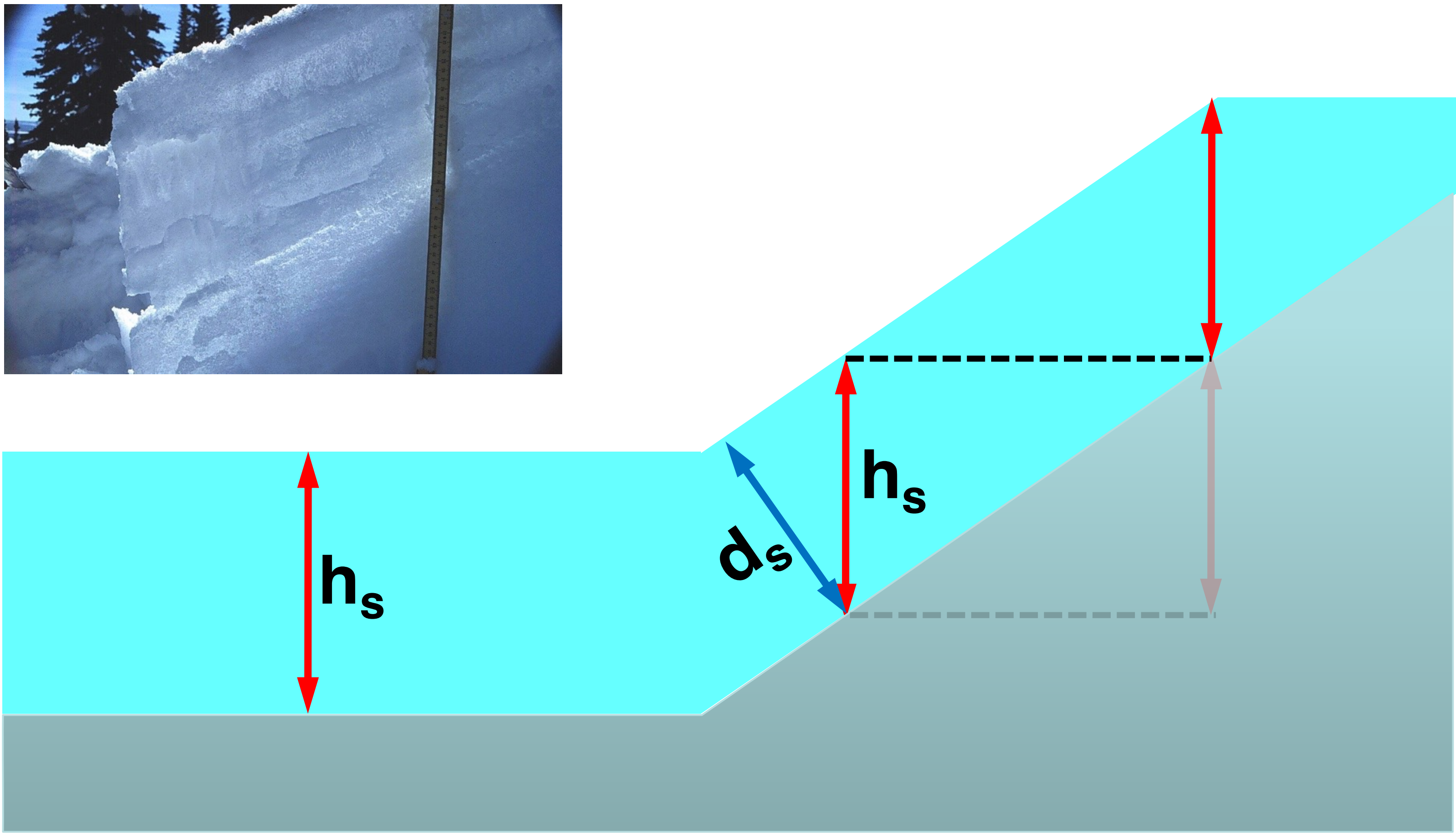
A = area [m^2]

Subscripts: s = snow, i = ice, w = water, a = air

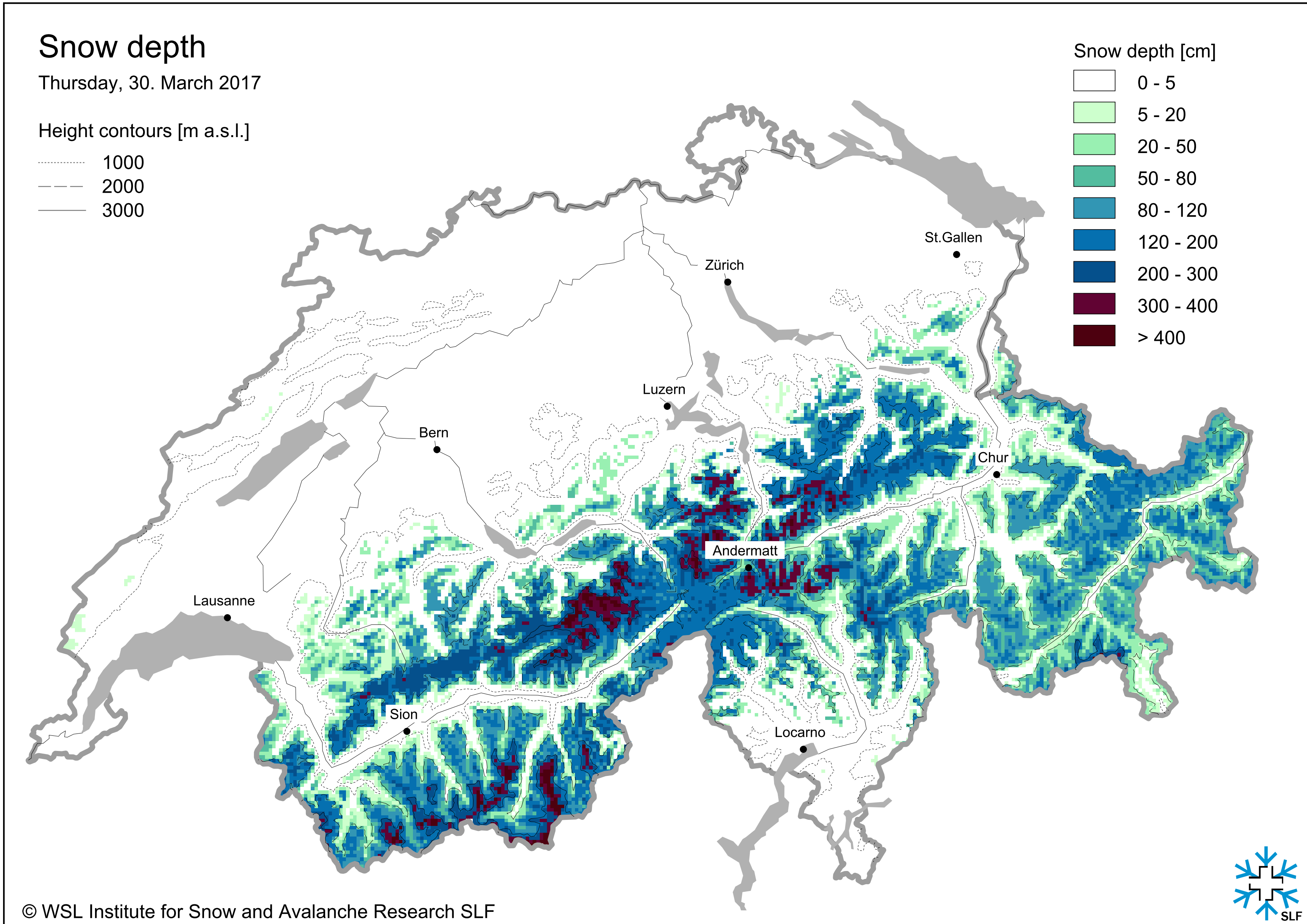


Snowpack depth (h_s)

Vertical distance [m] or [cm] from the top surface of the snowpack to the bottom of the snowpack (snow-soil or snow-ice interface).



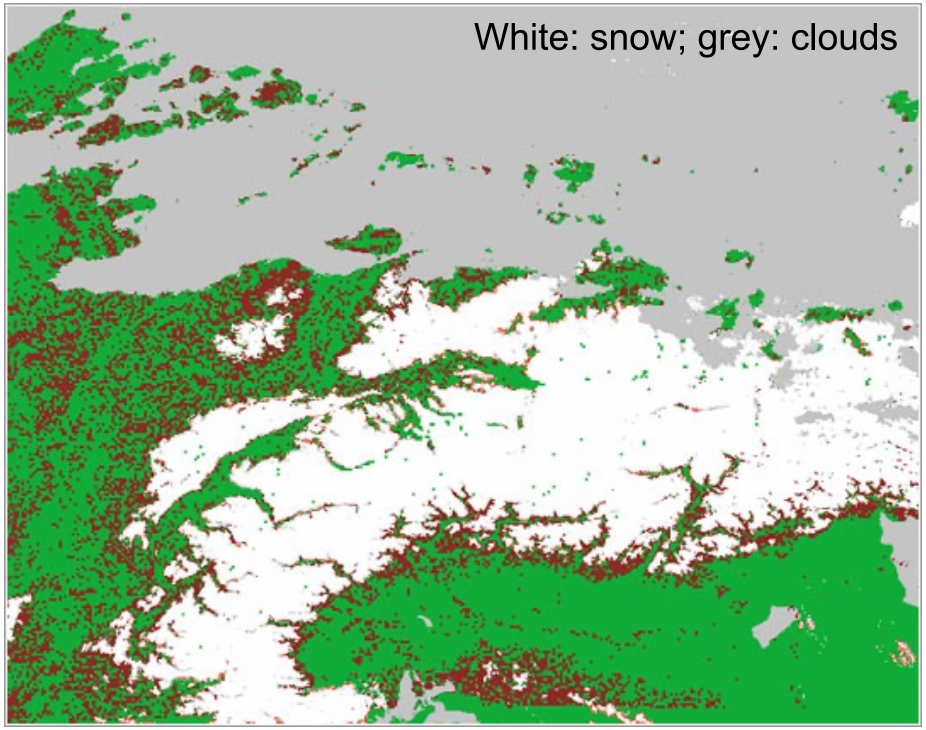
Snow depth map Switzerland



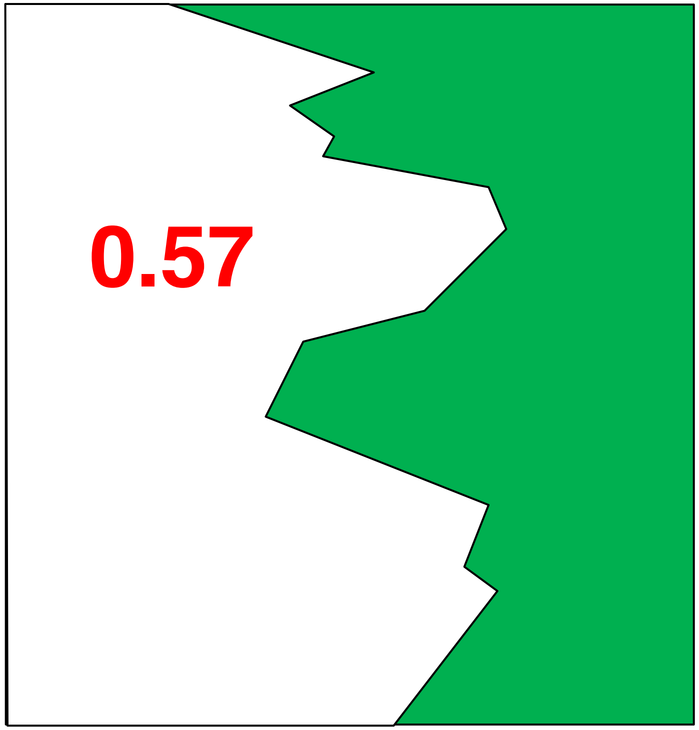
Snow Covered Area (SCA)

The areal extent of the snow cover. Typically, this is the projected area but sometimes topography is considered [km^2] or [m^2]

Binary (snow / no snow per pixel)



Fractional (percentage per pixel)



Snowpack porosity

Porosity ϕ is defined as the ratio of pore volume to total volume:

$$\phi \equiv \frac{V_a + V_w}{V_s} \;, \; [\,\text{--}\,]$$

If liquid water content = 0, then there is a direct relationship between density and porosity:

$$\rho_s = \rho_i (1 - \phi) \; [\text{kg m}^{-3}]$$

where ρ_i = 917 kg m^{-3}
is the density of pure ice at 0°C

Liquid Water Content (LWC)

LWC θ is defined as the amount of liquid water in wet snow:

Specific (mass): $$\theta_m = \frac{m_{water}}{m_{wet\,snow}} * 100, \ [\%]$$

or

Volumetric: $$\theta_V = \frac{V_w}{V_s}, \quad [--]$$

While $$\theta_V = \theta_m \frac{\rho_s}{\rho_w}, \quad [m^3/m^3]$$

LWC_{max} = maximum water holding capacity (amount of water) = f (texture).

Snow type	LWC max (vol %)	LWC max (mass %)
Fresh snow (dry)	0 - 3	
Fine grain snow (wet)	3 - 15	
Coarse grain snow (soaked)	> 15	

Evolution of LWC of melting snow

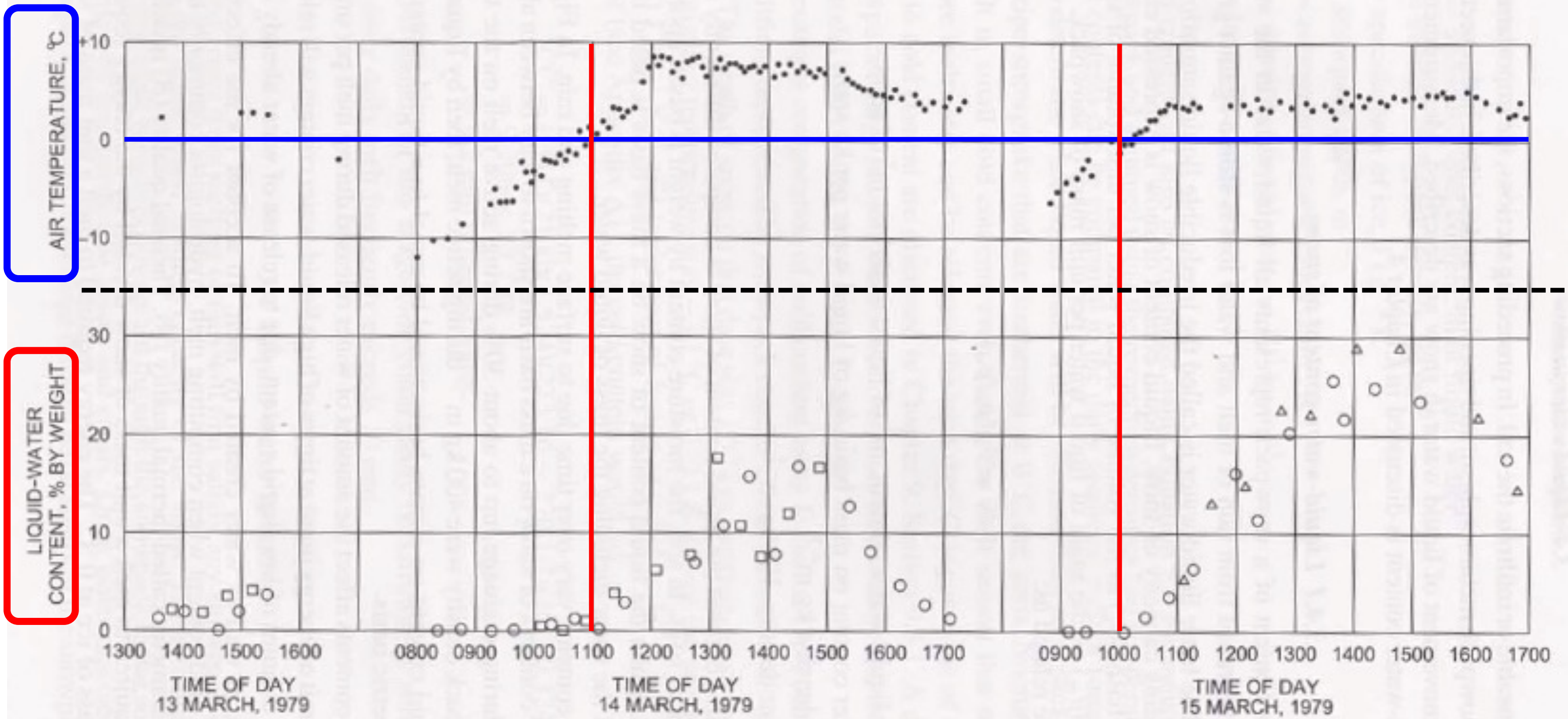


Figure 3.9 Daily variation of snowpack liquid-water content (mass of liquid water per unit mass of snow) determined by three different operators using the freezing calorimetry method at Fraser, Colorado (Jones *et al.*, 1983, copyright 1983 IWA Publishing with permission).

Snowpack density

Snow density ρ_s (m/V, [kg m^{-3}] or [g cm^{-3}]) is defined as the mass per unit volume of snow:

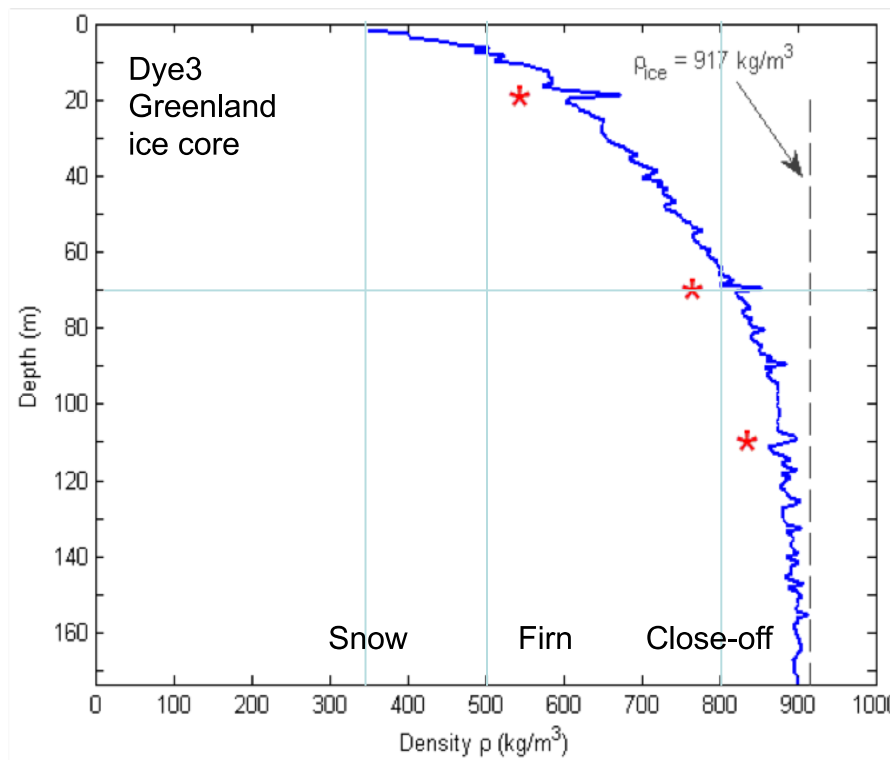
$$\rho_s = \frac{m_i + m_w + m_a}{V_s} = \frac{\rho_i V_i + \rho_w V_w + \rho_a V_a}{V_s}$$

Air can be neglected.

Combining ρ_s, ϕ and θ yields:

$$\rho_s = (1 - \phi)\rho_i + \theta \rho_w$$

Substance	Density [kg m^{-3}]
Water @ 4°C	1000
Air (at ca. 2000 m a.s.l.)	1
Air (at 0°C, 1013 hPa)	1.3
Ice (at 0°C)	917
Fresh snow	20 – 200
Settled snow	200 – 300
Wet or wind-packed snow	350 – 450
Firn	500 – 800



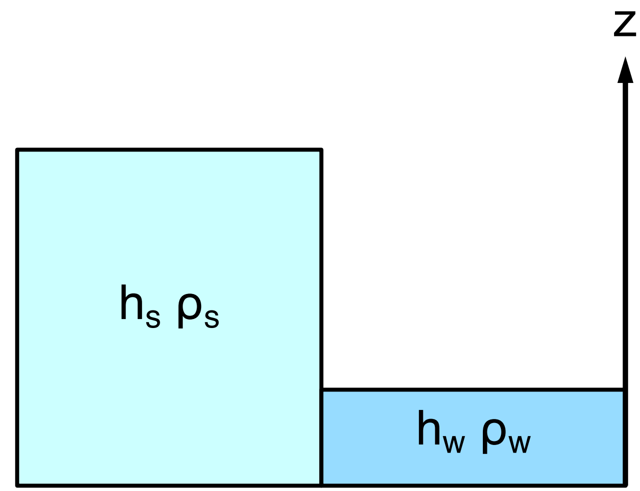
Snow Water Equivalent (SWE)

SWE: The amount of liquid water represented by a snowpack:
Mass of water per unit area [kg/m²];

With 1kg H_2O = 1liter = 1dm³ → 1mm/m²
1mm = 1kg/m² * 9.81 m/s² = 9.81 N/m² = 9.81 Pa

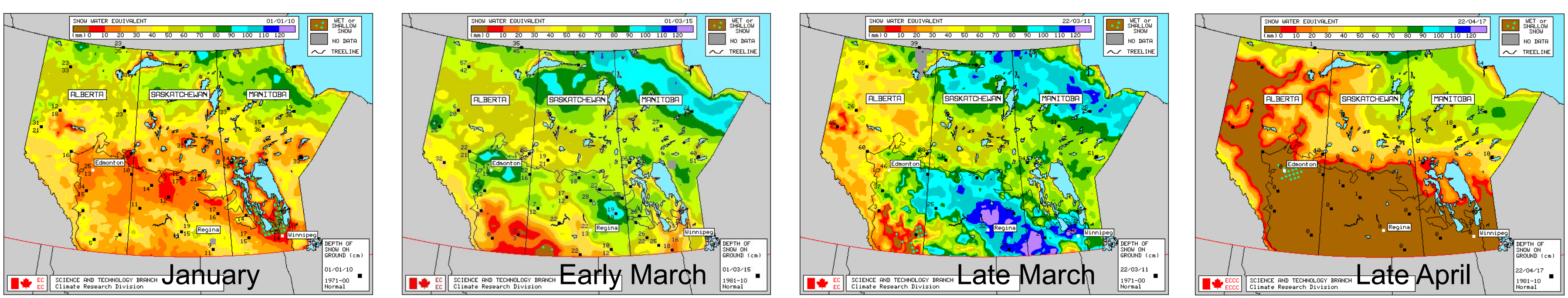
$$SWE = (m_i + m_w)/A = (\rho_i V_i + \rho_w V_w)/A$$

$$h_w \rho_w = h_s \rho_s \rightarrow h_w = \frac{\rho_s}{\rho_w} h_s \equiv SWE$$



SWE is the most important parameter for hydrologists and stakeholders in water resources management (agriculture, hydropower, flood management, etc.)

Example: SWE distribution in Canadian provinces during different seasons.



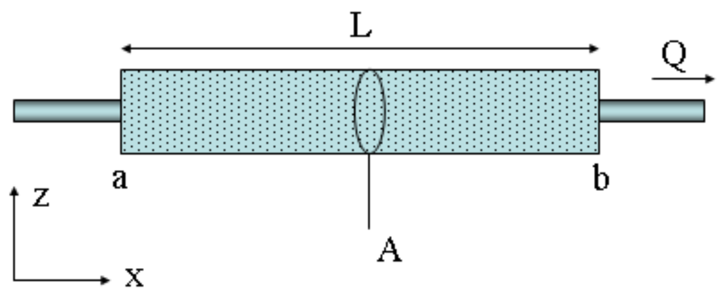
Snowpack permeability

Permeability [m²] describes the ability of a porous material to transmit fluids (liquid or gaseous).

Relevant for

- Air/gas movement in the snowpack
- Energy balance (why?)
- Snow metamorphism
- Soil/vegetation – atmosphere exchange of air

- Snow: mostly continuous pore system
- Ice: only isolated pores (not connected)
- Firn: very few connected interstitial cavities
- Transition firn - ice → (close-off density) 800-850 kg/m³



Darcy's law: flow in porous media:

- κ is the permeability [m²]
- μ is the dynamic viscosity of the fluid [Pa s]
- v is the mean flow velocity (hydraulic conductivity) [m s⁻¹]
- Q is the horizontal flow, driven by pressure gradient, over the cross-section A (including the effect of immobile matrix)

$$Q = \frac{-\kappa A}{\mu} \frac{P_b - P_a}{L} \rightarrow v = \frac{-\kappa}{\mu} \nabla P$$

Q = v*A

Snowpack permeability

Permeability κ is part of proportionality constant in Darcy's law which relates discharge (flow rate) and fluid physical properties (e.g. viscosity), to a pressure gradient applied to the porous media:

From $v = \frac{-\kappa}{\mu}\frac{dP}{dx}$ we derive the permeability $\kappa = v\frac{-\mu dx}{dP}$

In coarse grain snow, capillary forces can be ignored.

Gravity-driven flow (percolation of melt water in snow): $v_z = \rho_w g \frac{\kappa}{\mu} \rightarrow \kappa = \frac{v_z \mu}{\rho_w g}$

In natural materials, permeability values range over orders of magnitude

- *v* is the fluid flow velocity through the medium (i.e., average velocity in the porous medium) [m/s]
- κ is the permeability [m^2]
- μ is the dynamic viscosity of the fluid [Pa s]; μ = ρ η, with η = kinematic viscosity [$m^2\ s^{-1}$]
- Δ*P* is the acting pressure difference [Pa]
- Δ*x* is the distance in the porous medium [m]

Dividing by the porosity ϕ we obtain the mean flow of the fluid through the pore space.

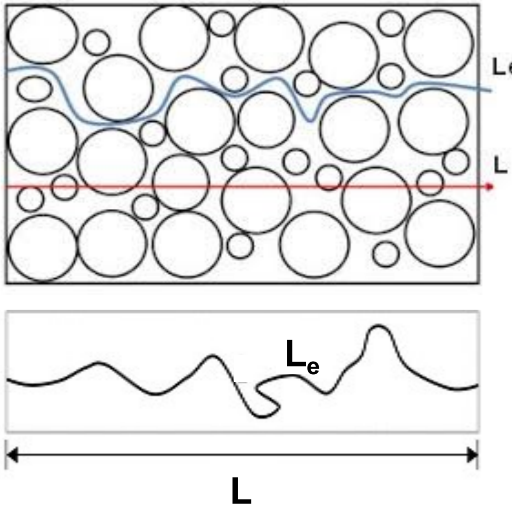
Effective gas diffusivity in snow

Gas diffusion (Fick's law) in snow:

$$q_c = -D_{eff}\nabla c$$

The effective diffusion coefficient, D_{eff}, describes diffusion through the pore space of porous media. It is macroscopic in nature, not individual pores but the entire pore space needs to be considered. D_{eff} is estimated as follows:

$$D_{eff} = D\delta\phi/\tau$$



- D : diffusion coeff. (diffusivity) in gas or liquid, filling the pores [m^2s^{-1}]
- ϕ : porosity available for the transport [dimensionless]
- δ : constrictivity* [dimensionless], ($0 < \delta < 1$)
- τ : tortuosity (L_e/L), [dimensionless], ($\tau > 1$)

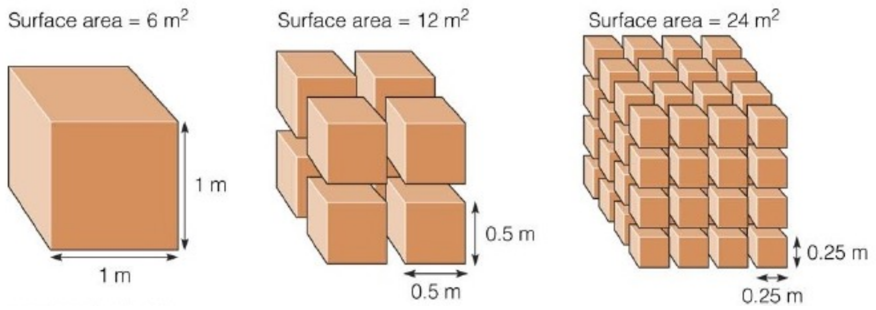
* Constrictivity describes the slowing down of diffusion by increasing viscosity in narrow pores as a result of greater proximity to the average pore wall. It is a function of pore diameter and the size of the diffusing particles (ratio of the diameter of the diffusing particle to the pore diameter).

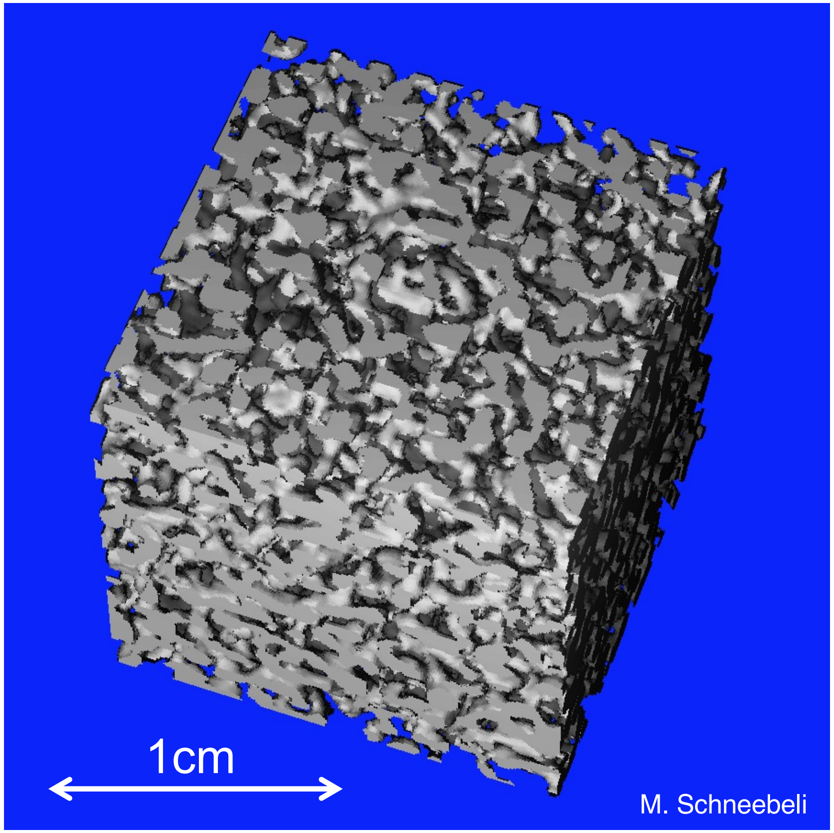
Resistance to transport in porous media increases because the viscosity of the fluid (which fills the pores) increases in the vicinity of the pore walls.

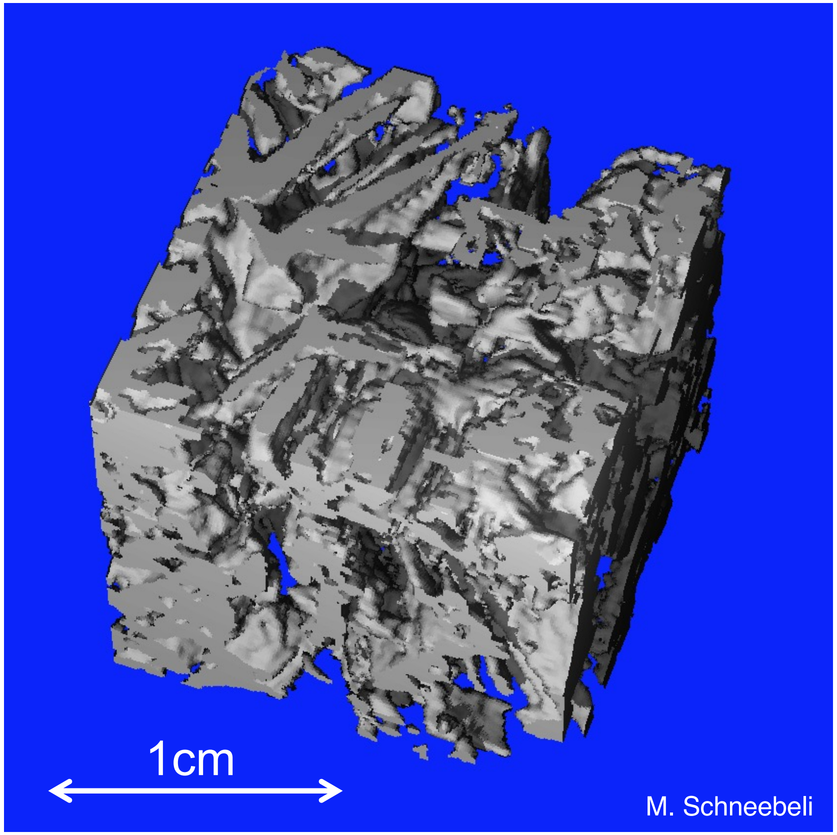
Specific surface area (SSA)

Surface area of ice / volume of ice:

$$SSA = A_{ice} / V_{ice}$$

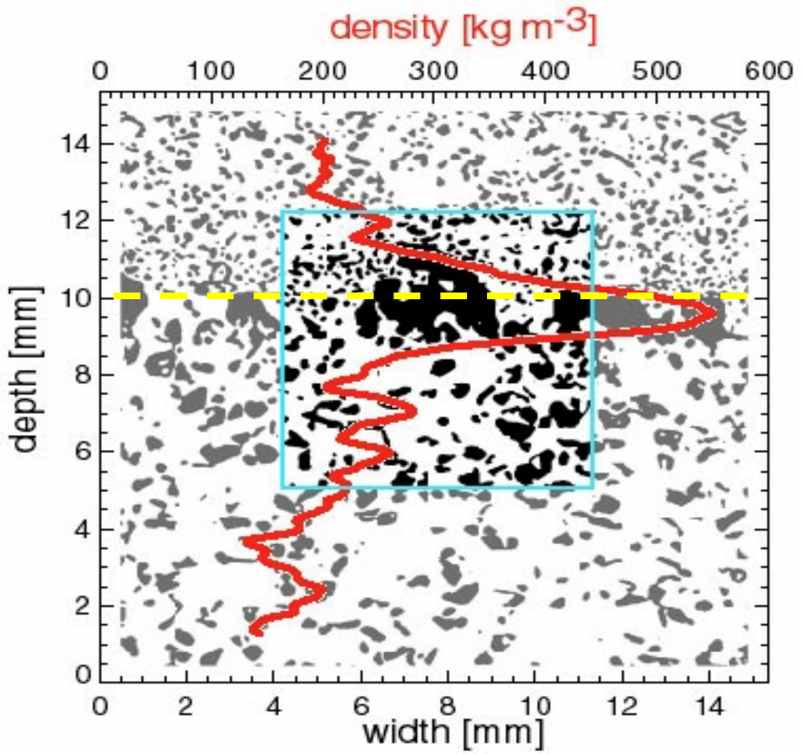






Significance of density and SSA

- Density and SSA are the most important parameters influencing mechanical and electromagnetic behavior.
- A fundamental problem is the layering and spatial variability of density and SSA in a natural snowpack (as well as the precise measurement of both).
- Uncertainty in these two variables explains some of the discrepancies often observed between model results and measurements



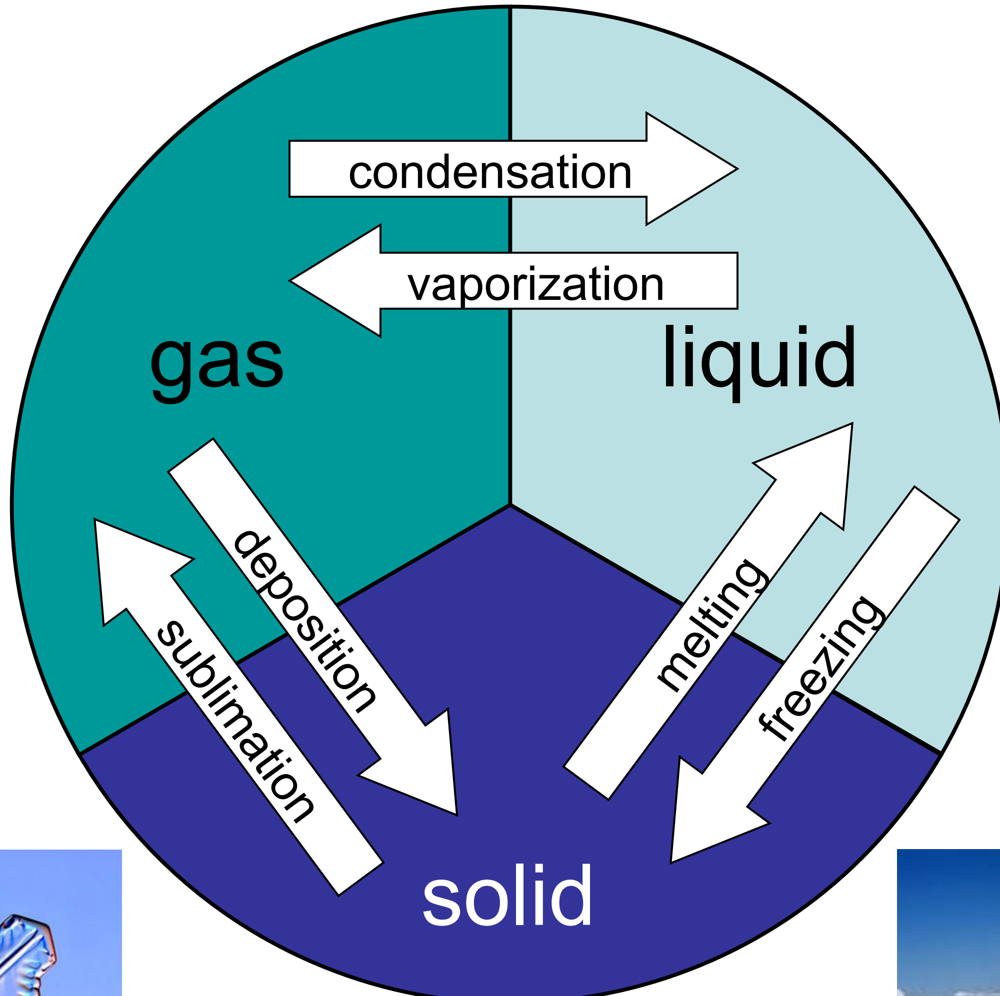
Geometric-physical properties jump

Outline

- Physical properties of snow
- Thermal properties of snow
- Electromagnetic (radiative) properties of snow
- Mechanical properties of snow
- (Chemical properties: Not discussed)

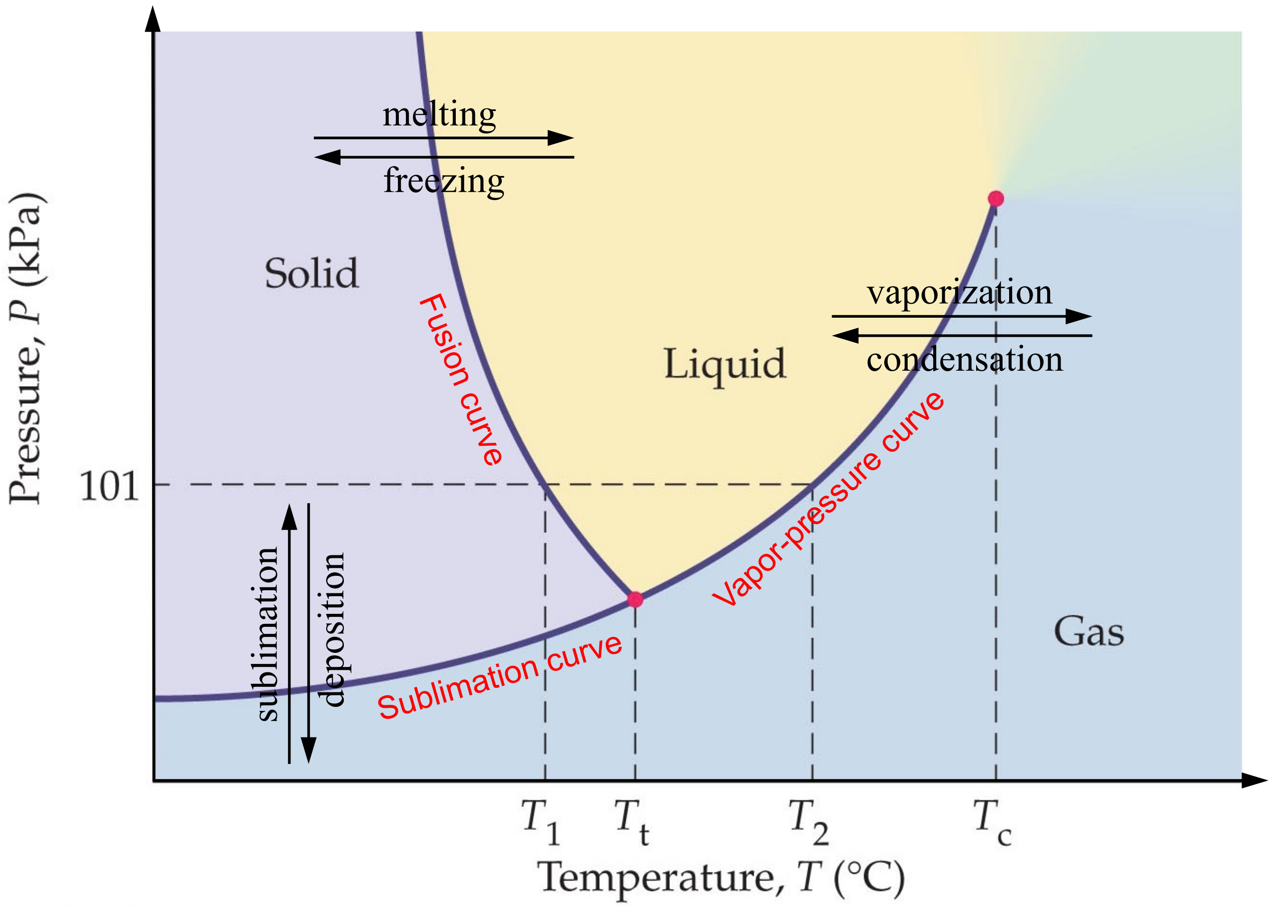
- Phase change
- Specific and latent heat
- Heat transfer in snow: thermal conductivity
- Cold Content of snow

Phases of water



Phase diagram of water

- The fusion curve indicates the equilibrium of the solid and liquid phases.
- The sublimation curve indicates the equilibrium of the solid and gas phases.
- At triple point (T_t), all three phases, solid, liquid, and gas, are in equilibrium.
- The vapor-pressure curve ends at the critical point (T_c); beyond, no distinction between liquid and gas.
- In water, the triple point is at T = 273.16 K (0.01° C) and P = 611.2 Pa.

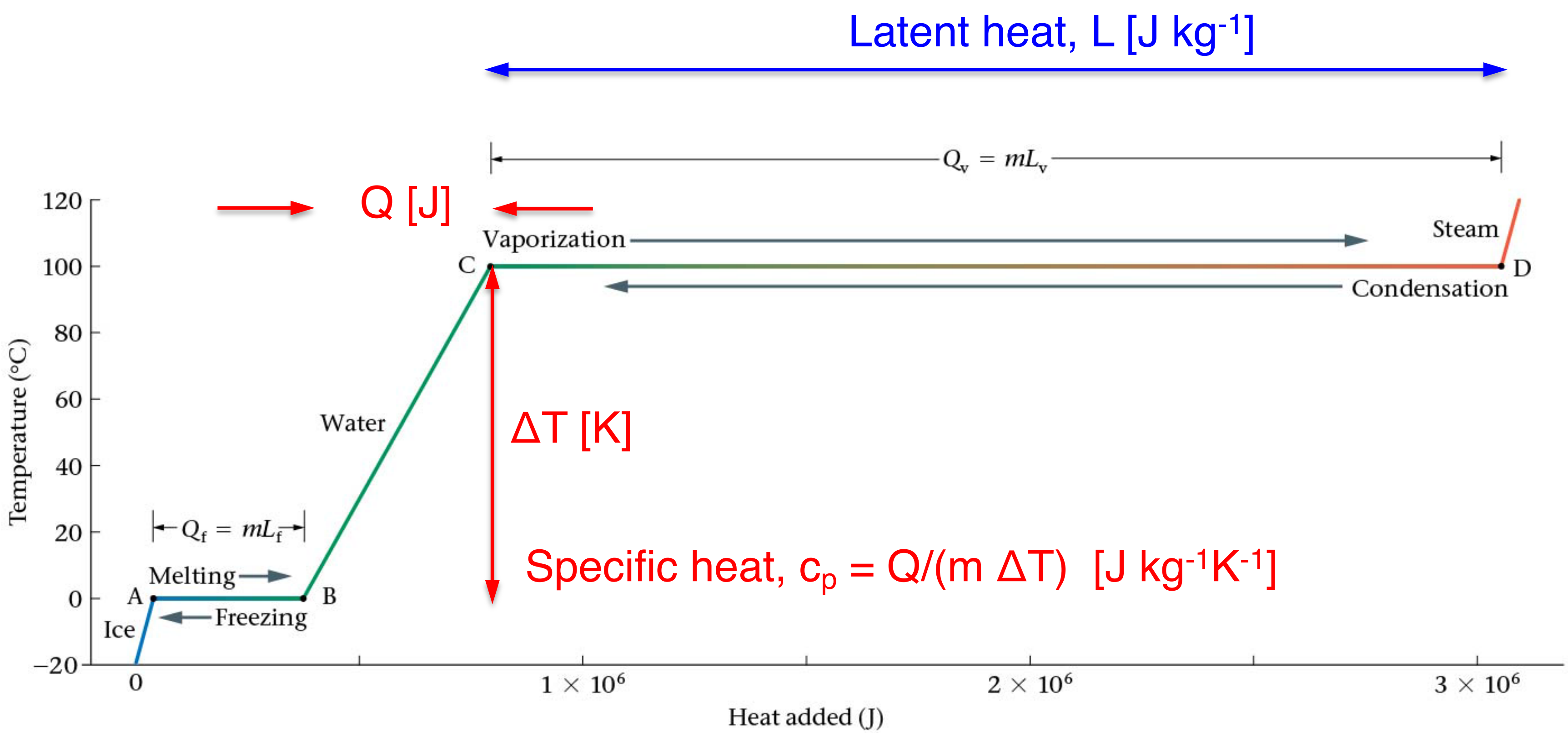


At 1013.25 hPa:

T_1 = 0° C
T_t = 0.01° C
T_2 = 100° C
T_c = 374° C (ca.)
218 atm

Latent and specific heat

Example of water



Phase changes of water

Phase Change	Process	Energy exchanged	MJ kg^{-1} at 0^oC *
Solid ←→ Liquid	Melting / Freezing	Latent heat of fusion (L_f)	±0.334
Liquid ←→ Gas	Evaporation / Condensation	Latent heat of vaporization (L_v)	±2.501
Solid ←→ Gas	Sublimation	Latent heat of sublimation (L_s)	±2.835

* Values are function of temperature

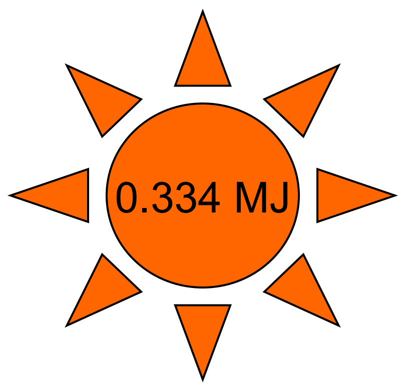
Latent heat, L

The amount of energy required for a phase change (isotherm) per unit mass (kg).
Here: Latent heat of fusion, L_f

Note: L_f(ice) ≈ corresponds to an amount of energy equivalent to heat water from 20 → 100°C

1 kg ice @ 273K

+



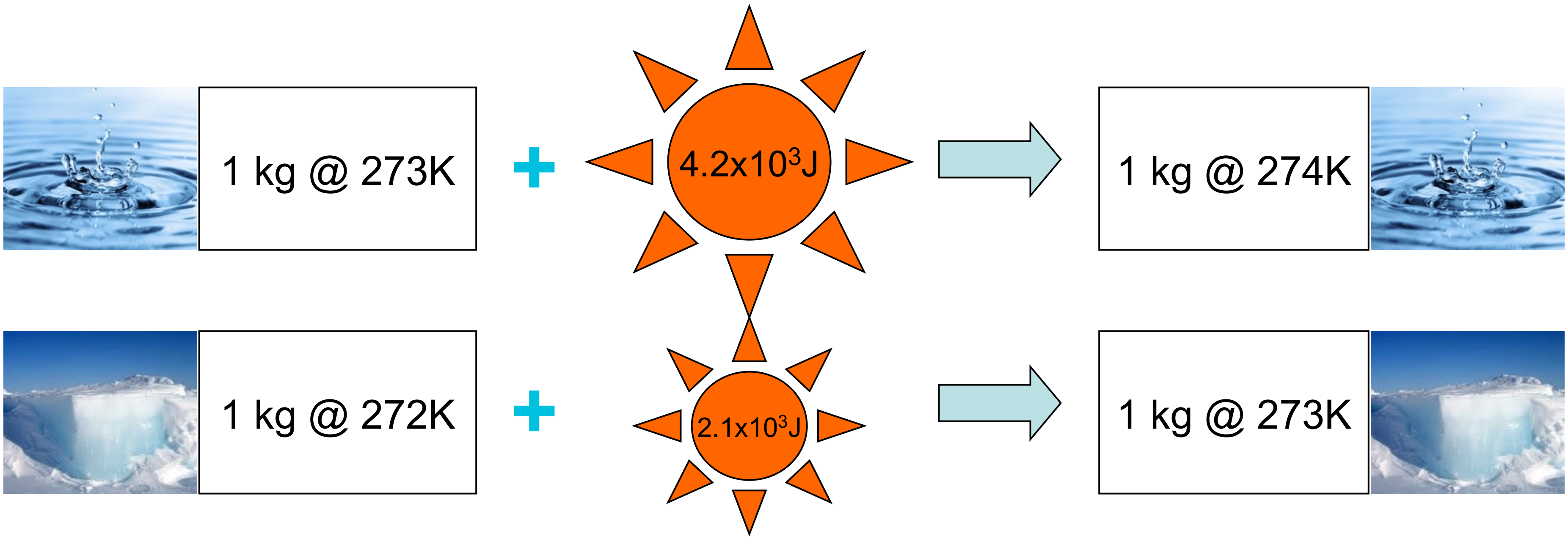
→

1 kg water @ 273K

Specific heat capacity (c_p)

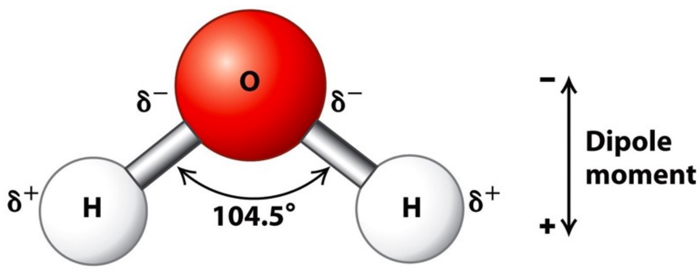
The amount of energy required to raise the temperature of a unit mass (kg) by 1 K:

Water ($c_{p,w}$) = 4220 J kg^{-1} K^{-1} (at 0^{o}C)
Ice ($c_{p,i}$) = 2110 J kg^{-1} K^{-1} (at 0^{o}C)

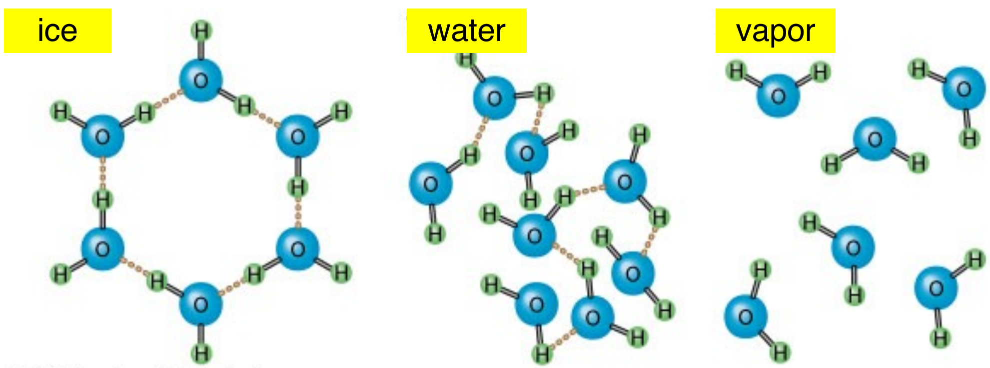


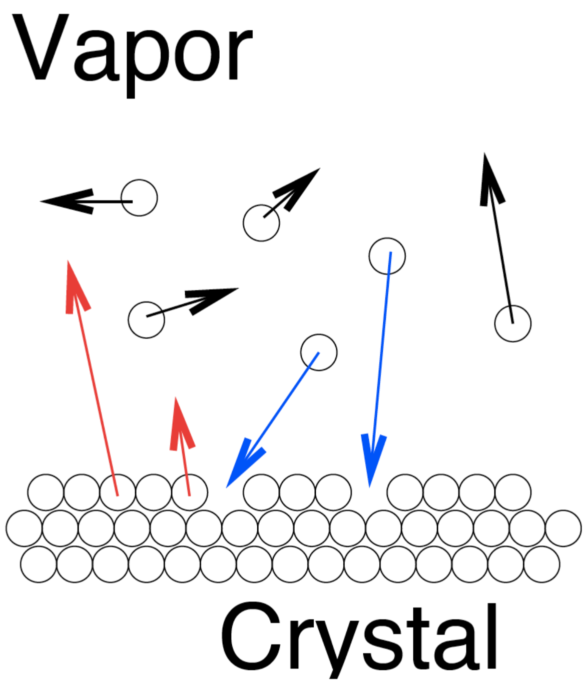
Analog for “volumetric heat capacity”: unit volume (m^3) by 1 K

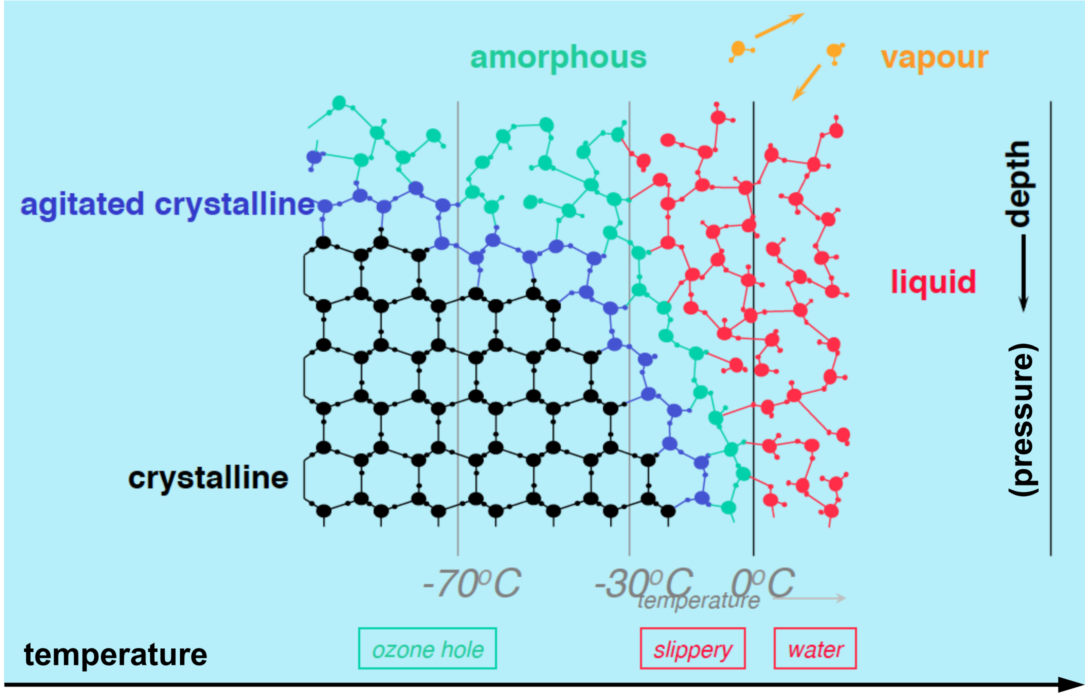
Molecular water dynamics



The physical states of water







Saturation vapor pressure

First Law of Thermodynamics: conservation of energy

This is the basis for the Clausius Clapeyron Equation, which gives the saturation vapor pressure over ice (snow) or water as a function of temperature under equilibrium conditions

$$\frac{\partial p}{\partial T} = \frac{\Delta H}{T \Delta V}$$

In meteorology, the CCE is written for the saturation vapor pressure using state variables of the ideal gas law:

$$\frac{1}{e_s}\frac{\partial e_s}{\partial T} = \frac{L_v}{R_v T^2}$$

differential form

$$e_s(T) = e_{s0} exp\left[\frac{L_v}{R_v}\left(\frac{1}{T_0} - \frac{1}{T}\right)\right]$$

integrated form

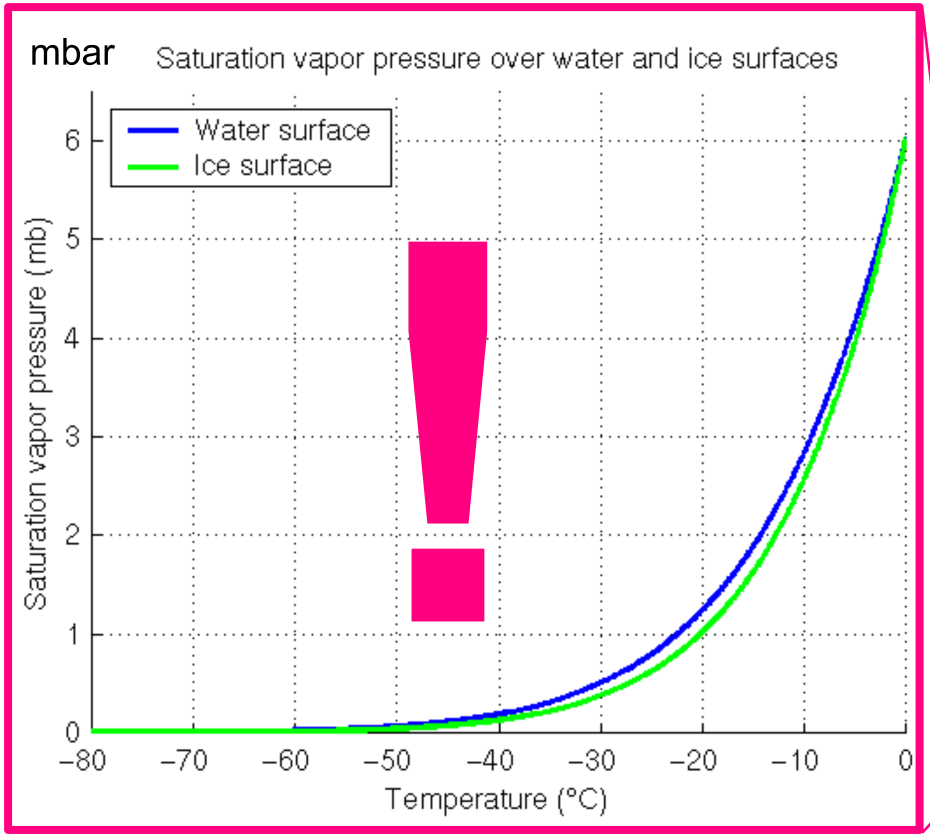
- p (vapor) pressure [Pa]
- T temperature [K]
- H enthalpy [J]
- V volume [m^3]

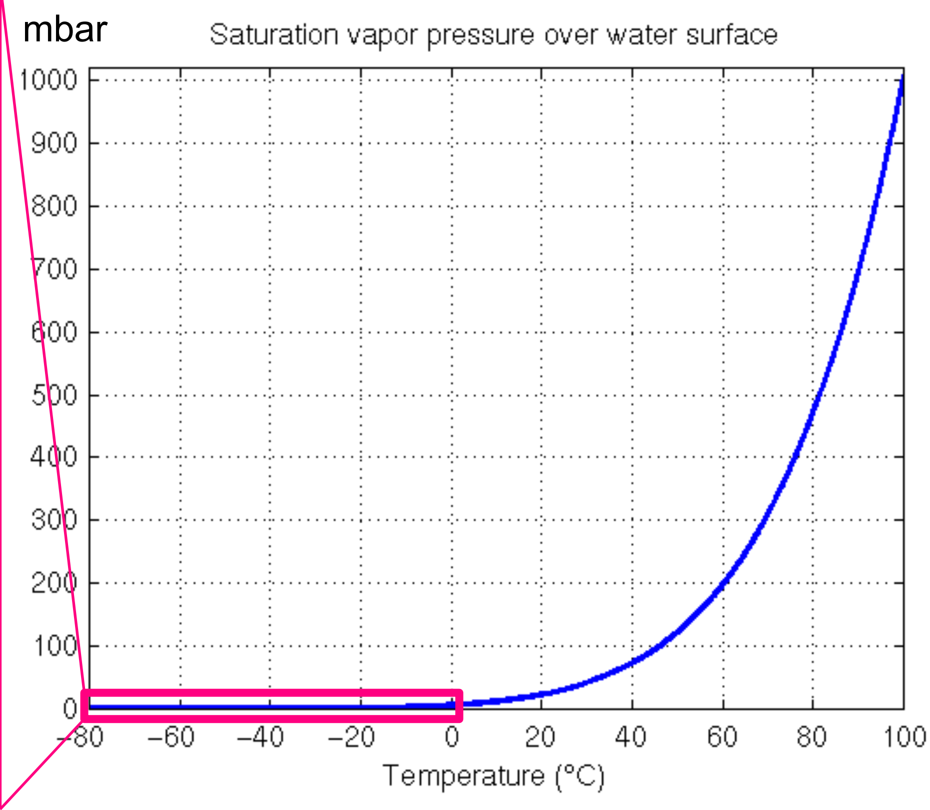
- e_s saturation vapor pressure
- e_{s0} saturation vapor pressure at T(0^{o}C)
- L_v spec. latent heat of vaporization, 2.501 MJ kg^{-1}
- R_v gas constant of water vapor, 461.5 [J kg^{-1} K^{-1}]

Saturation vapor pressure

- $e_s(T)$: strong f(T)
- different over water and ice surfaces (for T < 0°C)
- various approximations (equations) available (both for water and ice)

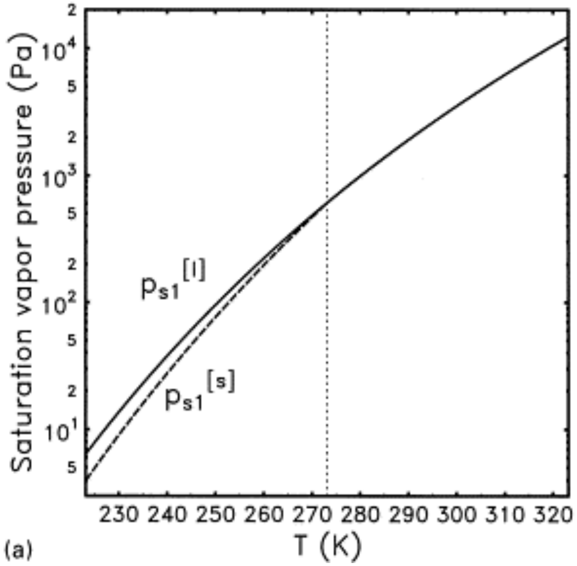
Relative humidity:
$RH = e/e_s(T) * 100$
e = vapor pressure
e_s = saturation vapor pressure

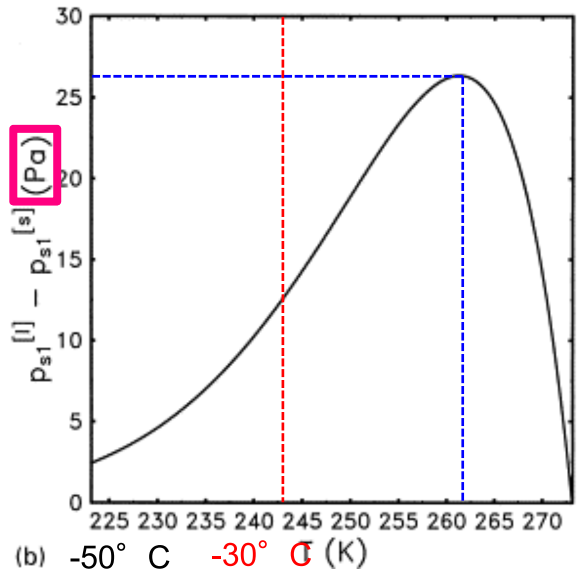


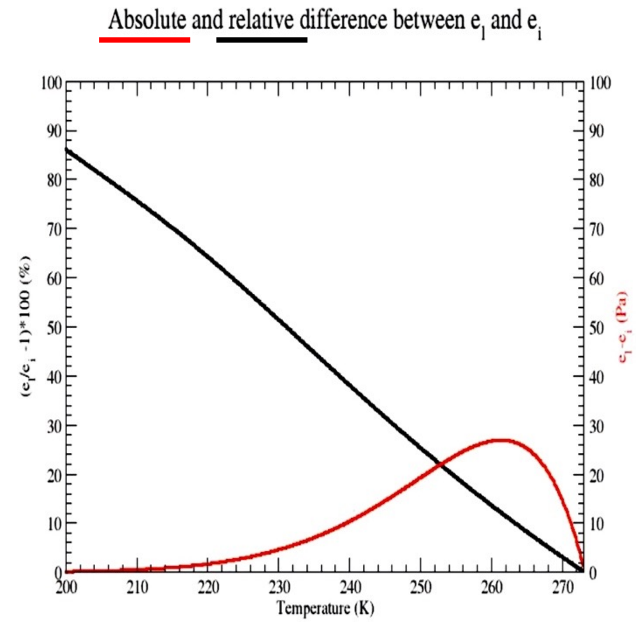


Saturation vapor pressure

Log-scale!







Vapor pressure over liquid water < 0°C

Magnus Tetens
(Murray, 1967)

$e_w = 6.1078\, e^{17.269388 * (T-273.16) / (T - 35.86)}$
with T in [K] and e_w in [hPa]

Vapor pressure over solid water (ice)

Magnus Teten
(Murray, 1967)

$e_i = 6.1078\, e^{21.8745584 * (T-273.16) / (T - 7.66)}$
with T in [K] and e_w in [hPa]

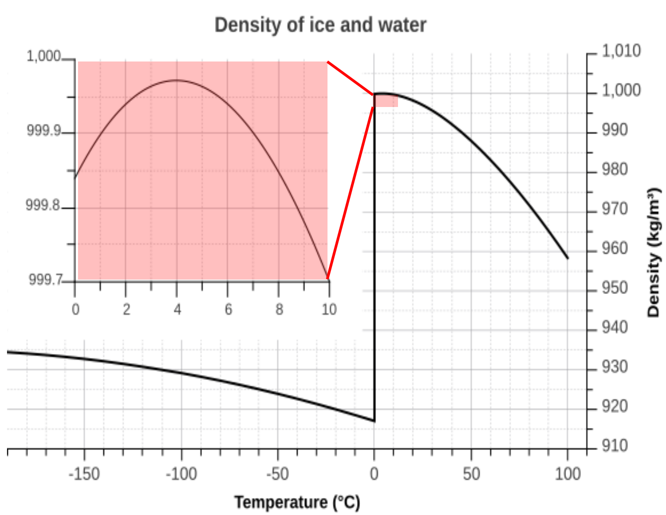
(note: “e” at RHS of eqns. is exp{…})

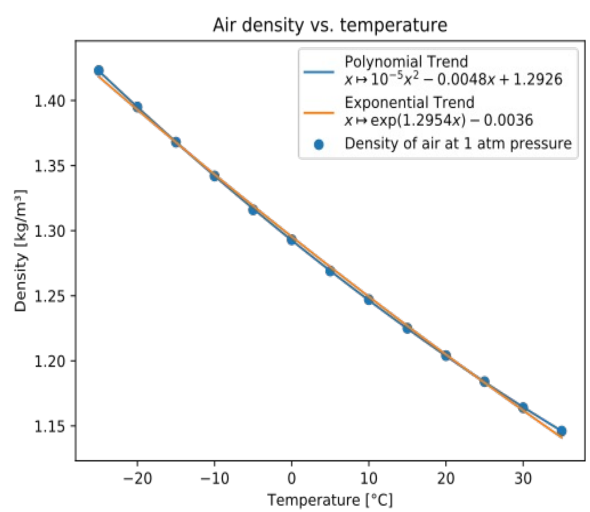
Review of vapor pressure formulations (equations):
http://cires.colorado.edu/~voemel/vp.html

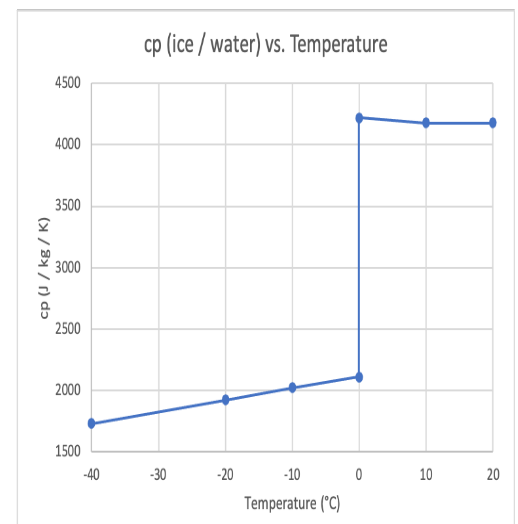
Physical properties as f(T)

Ice/Water and Air *, * Dry and at sea level pressure (1013 hPa);
Density of air f(p,T,q), pressure, temperature, spec. humidity

Temperature (°C)	Density (kg m^{-3})		Specific heat (J kg^{-1} K^{-1})		Thermal conductivity (W m^{-1} K^{-1})	
-40	924	1.51	1730	1005	2.65	0.023
-20	922	1.39	1920	1005	2.44	0.024
-10	921	1.34	2020	1005	2.33	0.025
0	917 / 1000	1.29	2110 / 4220	1005	2.23 / 0.567	0.025
10	1000	1.25	4180	1005	0.582	0.026
20	999	1.20	4180	1005	0.597	0.027







Thermal conductivity

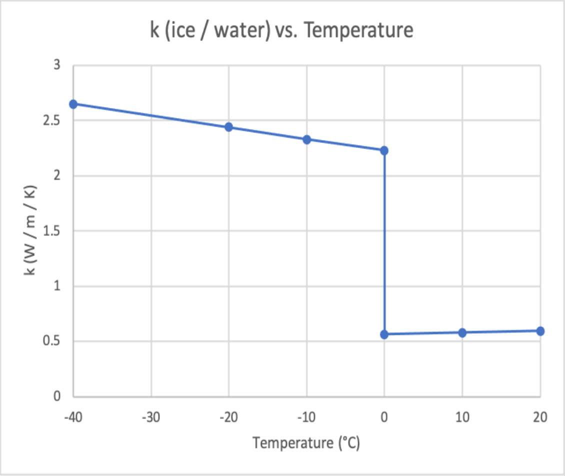
- Conductive heat transfer via ice matrix
- Typically, 70-80% of heat flux in snow. Rest:
  - Latent heat transport through vapor advection
  - Heat diffusion through air in cavities
  - Convection in pore system

Current and future research may change this statement

$$Q_c = -k\frac{\partial T}{\partial z}; \quad k\left[\frac{W}{mK}\right]$$

$$Q_c = -k_{eff}\nabla T$$

Temperature	k (ice/water)
-40	2.65
-20	2.44
-10	2.33
0	2.23/0.567
10	0.582
20	0.597



Substance	k (W/m/K)
Air	0.025
Water (0°C)	0.567
Wood	0.04 – 0.4
Soil	1.5
Concrete, Stone	1.7 – 2.4
Copper	400

Subsurface heat transfer

- Heat transfer by conduction depends on magnitude of temperature gradient and thermal conductivity (k) of material
- Rate at which snow temperature will change depends on the ratio of thermal conductivity to specific heat capacity, as a function of snowpack density
- Combined these quantities constitute the thermal diffusivity:

$$\kappa = \frac{k_s}{\rho_s c_{p,i}}; \quad [m^2 / s]$$

media	density [kg m^{-3}]	thermal conductivity [Wm^{-1} K^{-1}]	thermal diffusivity [m^2 s^{-1}]
Water	1000	0.567 (at 0°C)	1.4 x 10^{-7}
Ice	917	2.230 (at 0°C)	1.2 x 10^{-6}
Air	1.29	0.025 (at 0°C)	2.0 x 10^{-5}
Snow	**200**	**0.14**	**3.3 x 10^{-7}**
Snow	**500**	**0.72**	**6.8 x 10^{-7}**

Conductivity – Density relation

$$q = -k_{eff} \frac{dT}{dz}$$

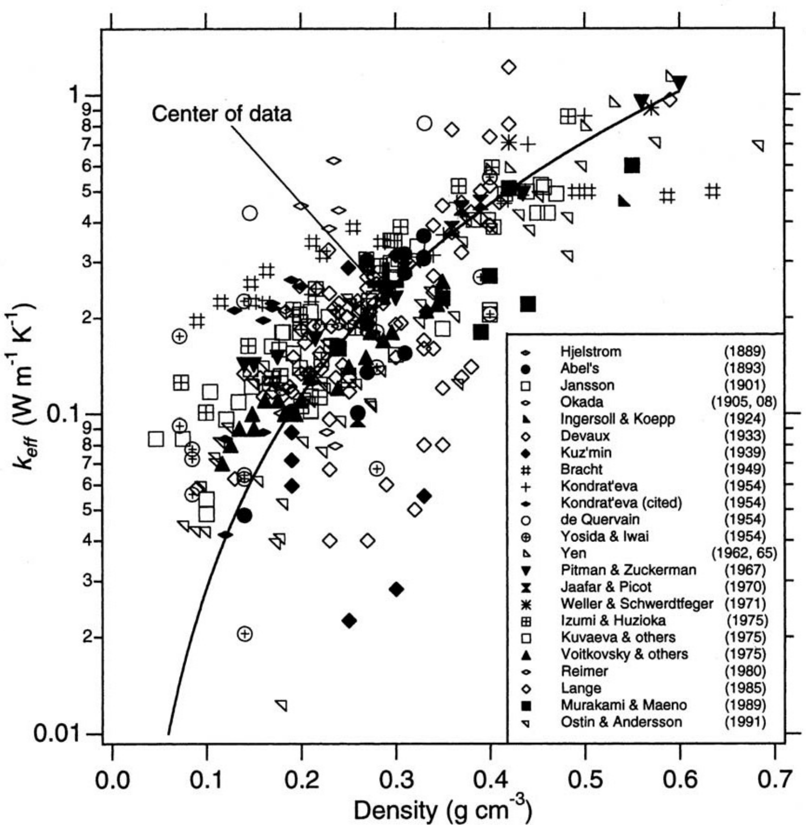
$$k_{eff} = 0.138 - 1.01\rho_s + 3.233\,\rho_s^2 \qquad [0.156 \le \rho_s \le 0.6]$$

$$k_{eff} = 0.023 + 0.234\rho_s \qquad [\rho_s < 0.156]$$

k_{eff} = effective thermal conductivity

What else than pure conduction through ice matrix?

- vapor transport
- forced convection (ventilation)
- heat conduction in the air



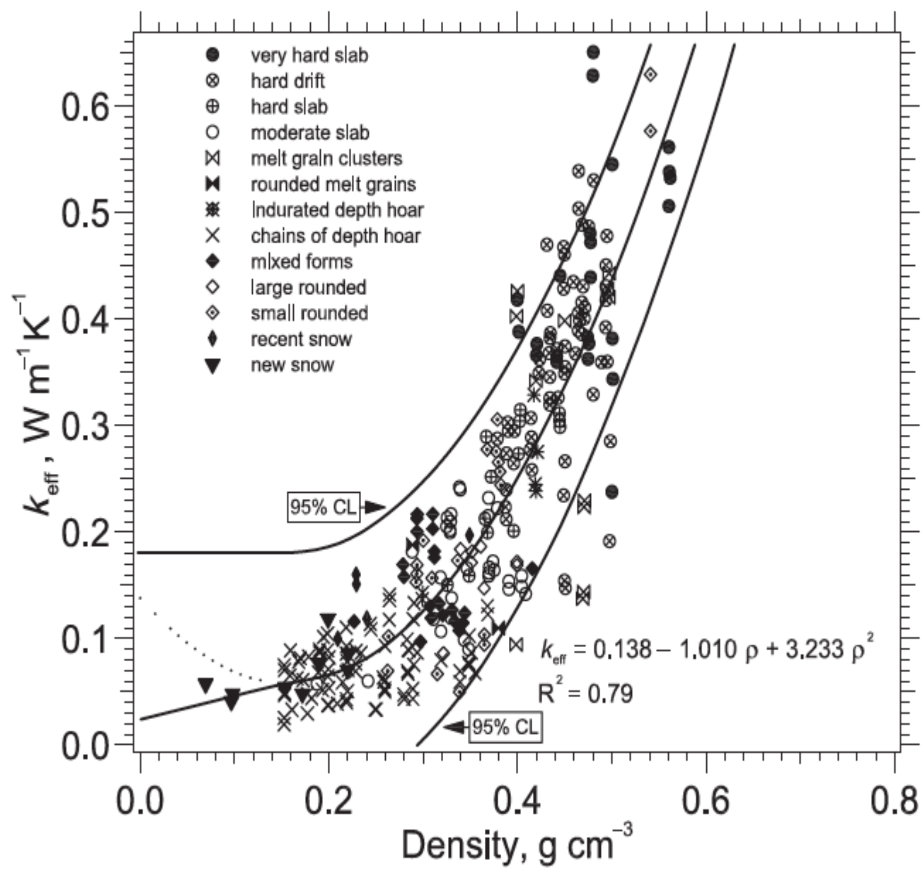


Figure 3.6 Effective thermal conductivity of snow as a function of density (modified from Sturm *et al.* 1997 from *Journal of Glaciology* with permission of the International Glaciological Society).

Cold Content of the snowpack

Cold Content (CC) = negative heat storage = energy deficit, w.r.t. a reference state

Definition: amount of energy required to bring a snowpack to 0°C isothermal.
Note: mass remains constant!

CC = 0: defines a “ripe snowpack” of 0°C.
Snow must reach 0°C and LWC_{max} , before liquid water can be released from snowpack

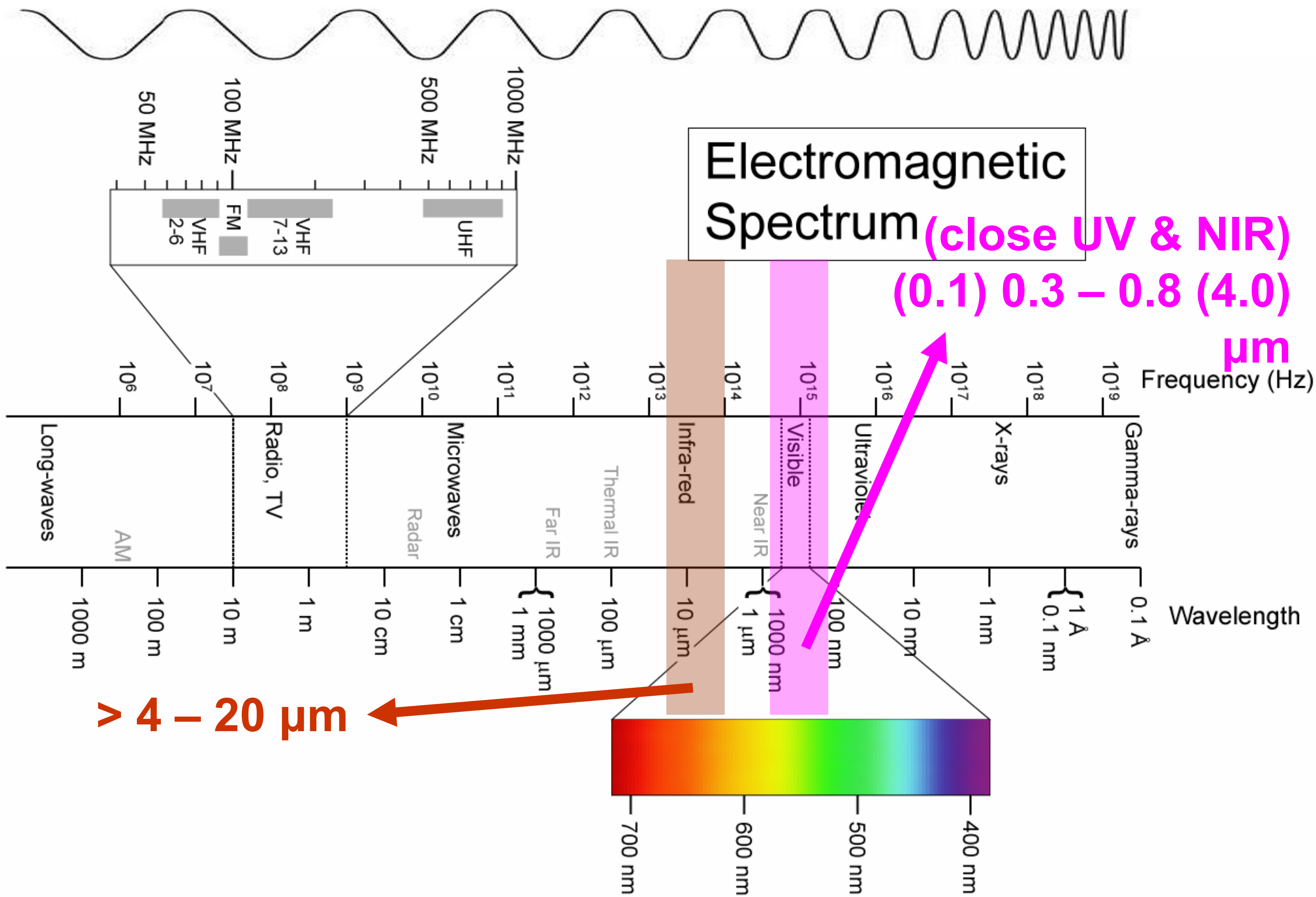
$$CC = \Delta E = \rho_s c_{p,i} \Delta z \Delta T \quad [\mathrm{J/m^2}]$$

Outline

- Physical properties of snow
- Thermal properties of snow
- Electromagnetic (radiative) properties of snow
- Mechanical properties of snow
- (Chemical properties: not discussed)

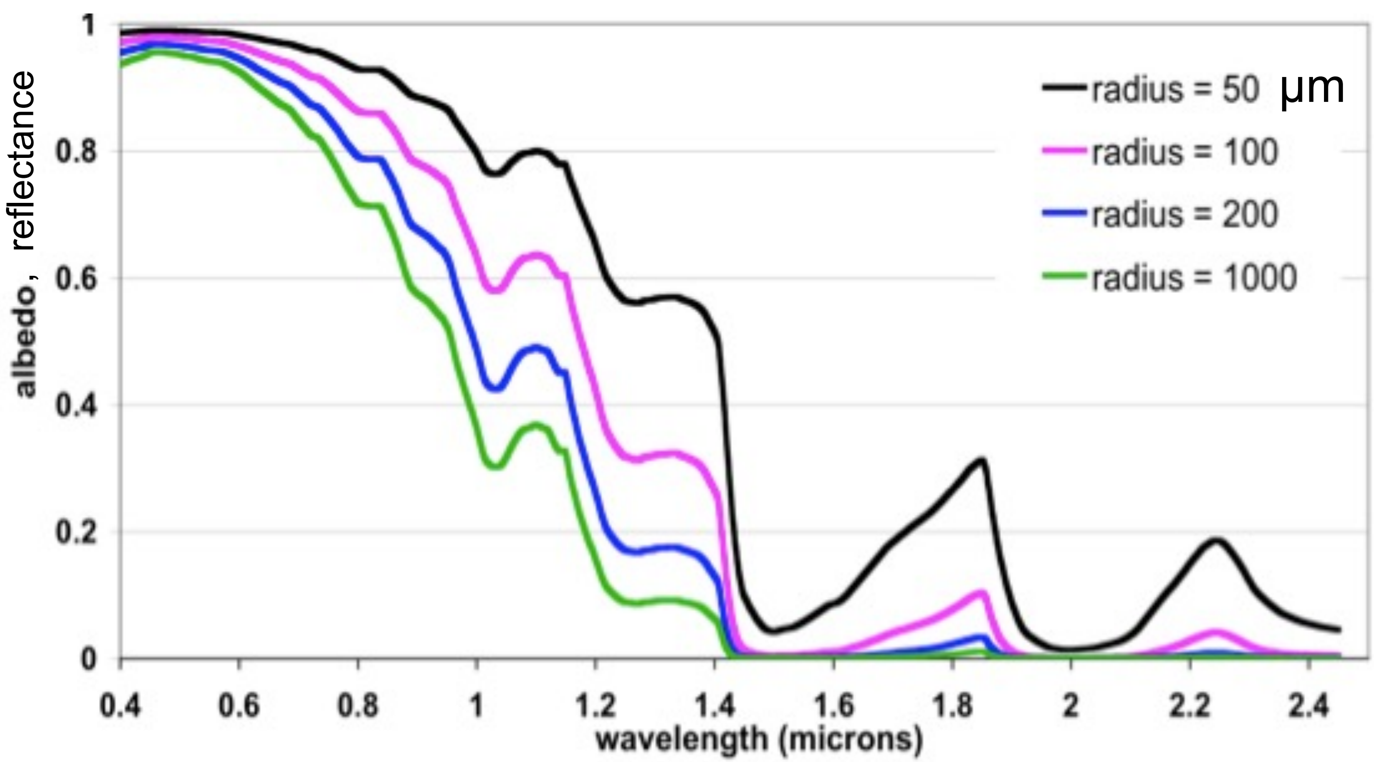
- EM Spectrum
- Scattering
- Reflection
- Absorption
- Emission

Electromagnetic spectrum

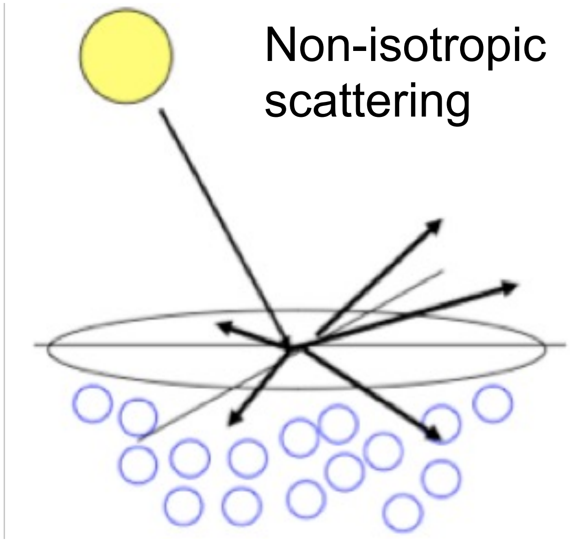


Optical properties of snow

- Highly reflective in visible light, VIS, range (high albedo)
- Low reflectivity in NIR and IR (high emissivity)
- Anisotropic (forward) scatterer (directional albedo)
- Optical properties function of grain size and shape
- Consequences for remote sensing



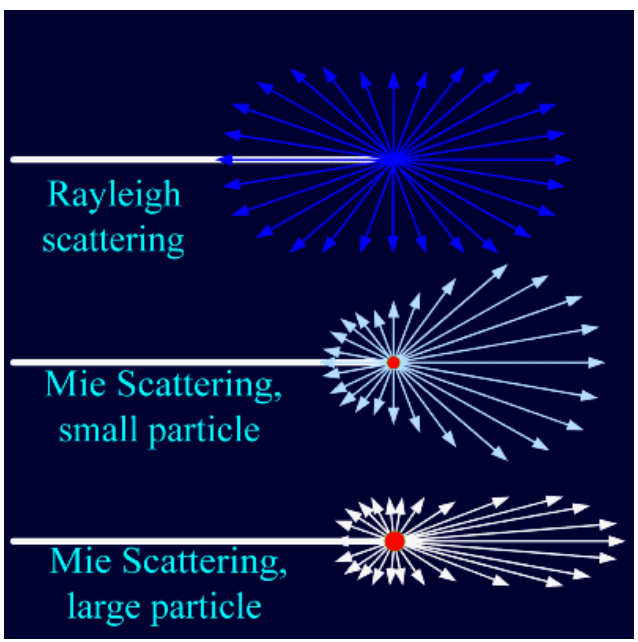
Forward scattering of snow



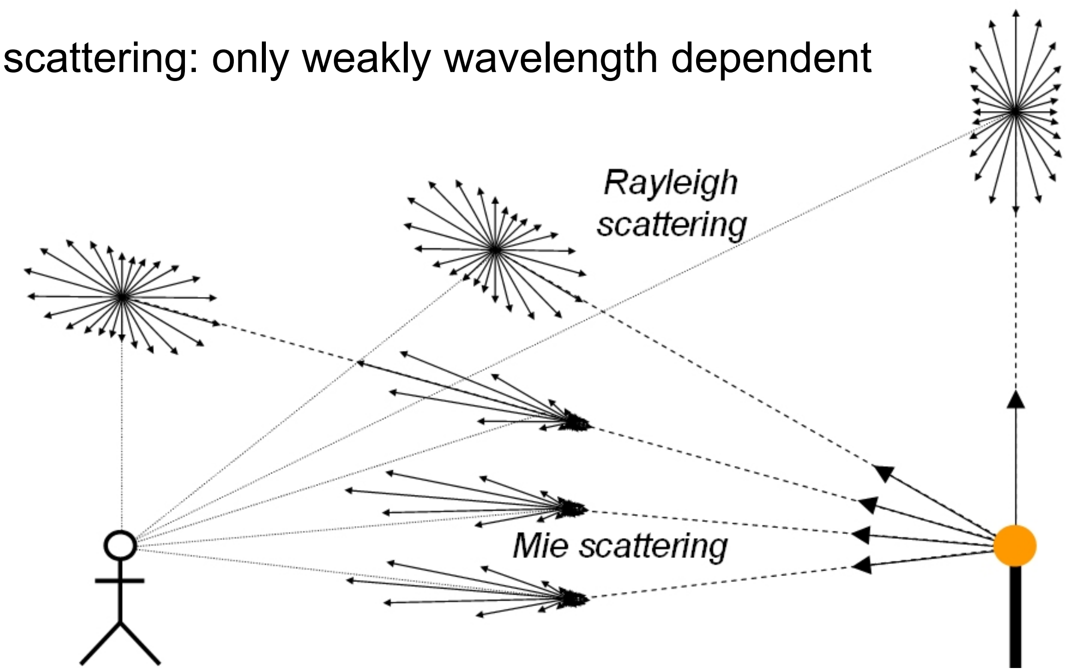
Consequences:

- Directional albedo
- Implications for
  - terrestrial
  - airborne
  - satellite

  remote sensing of snow



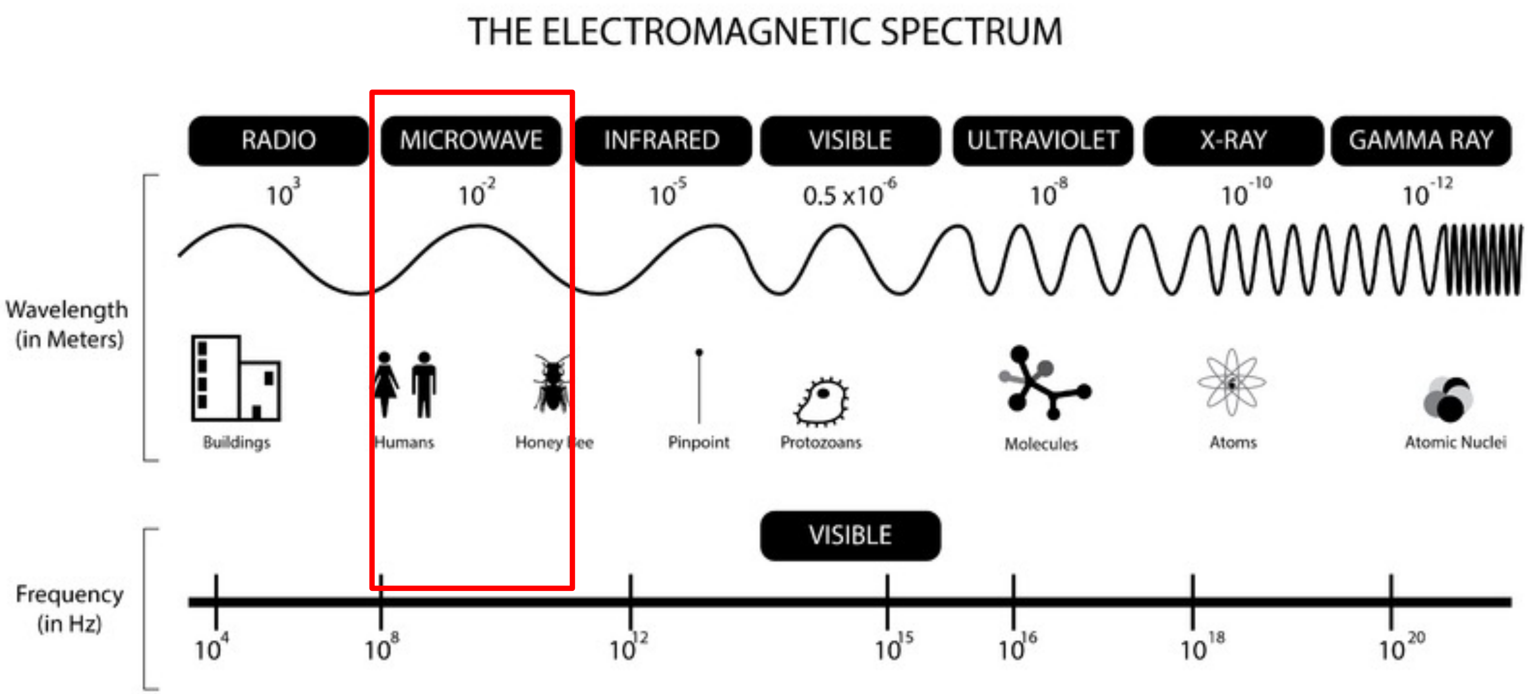
Mie scattering: only weakly wavelength dependent



Spectral reflectance & emission: microwave

Satellite remote sensing:
Global retrieval of snow properties (e.g. SWE):

- Microwaves: cm wavelength, GHZ frequency
- Compared to visible wavelength, MW
  - is not limited to the snow surface
  - penetrates through clouds
  - operates at darkness (polar night!)



Outline

- Physical properties of snow
- Thermal properties of snow
- Electromagnetic (radiative) properties of snow
- Mechanical properties of snow
- (Chemical properties: not discussed)

- Elasticity
- Stress (forces)
- Strain (Poisson ratio)
- Strength

Mechanical properties of snow

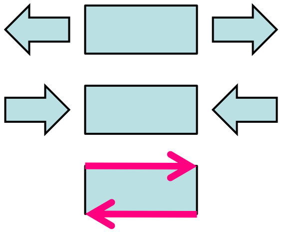
Acting force/s: weight (gravity), wind, people (skiers), vehicles (snow cat, …)
→ Deformation and change in matrix/structure and volume

- stress (force), $\sigma = F/A_0$ [Nm^{-2}] = [Pa]
- strain (deformation), $\varepsilon = \Delta L/L_0$ [m/m] or [-], [%]
- failure, yield stress: elastic, viscous/plastic behavior

tension

compression

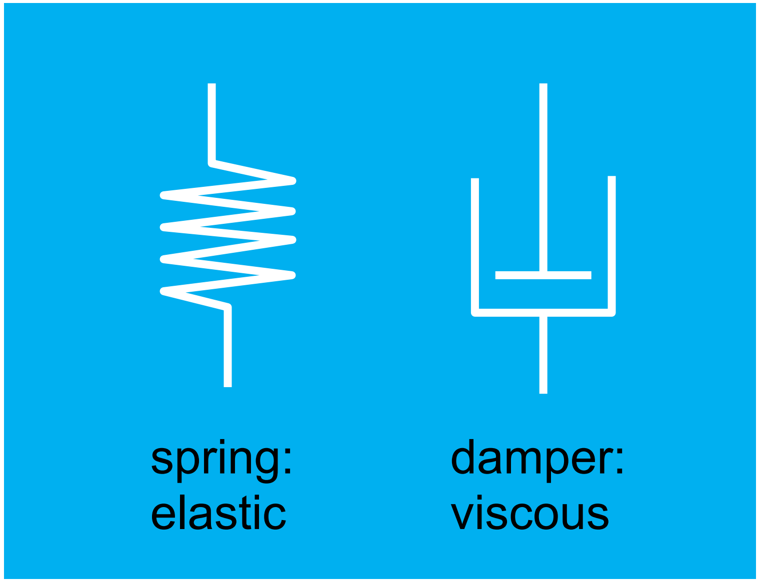
shear



Constitutive laws
Stress–strain relation: σ - ε
Failure criteria

a) elastic deformation
b) viscous deformation

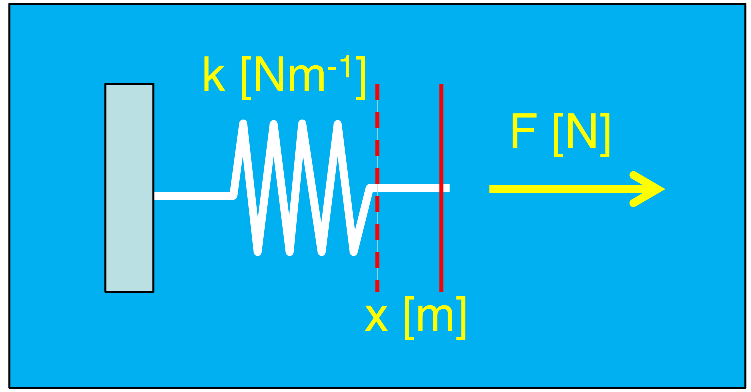
Models used:
a) spring
b) attenuator (damper)



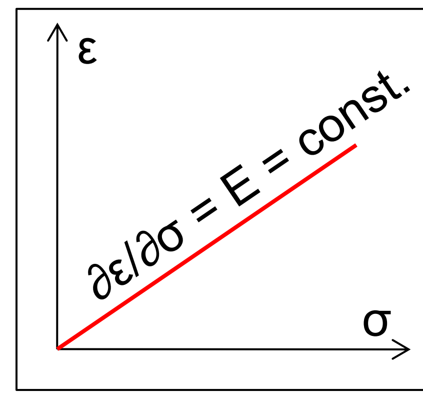
Elasticity

- Hooke's law: F = - k*x

  spring constant (k), displacement (x)



- Modulus of elasticity: $\lambda \equiv \sigma / \varepsilon$ general
- Young's modulus: $E = \sigma / \varepsilon$ tensile
- Shear modulus: $G \equiv \tau / \varepsilon$ shear
- Bulk modulus: $K = \sigma / \varepsilon$ volumetric

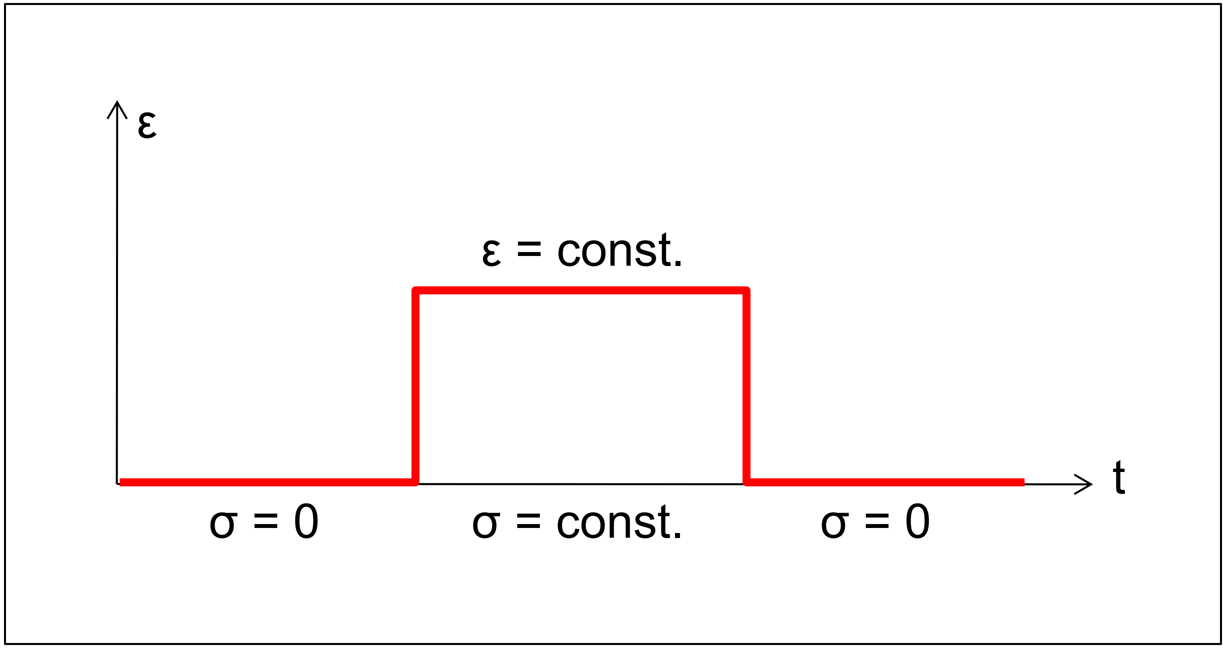


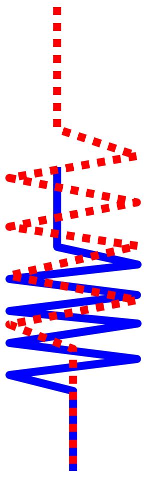
Note:
1/K ≡ compressibility

$$E = \frac{\sigma}{\varepsilon} = \frac{F / A_0}{\Delta L / L_0} = \frac{F L_0}{A_0 \Delta L}$$

Stress – strain relations

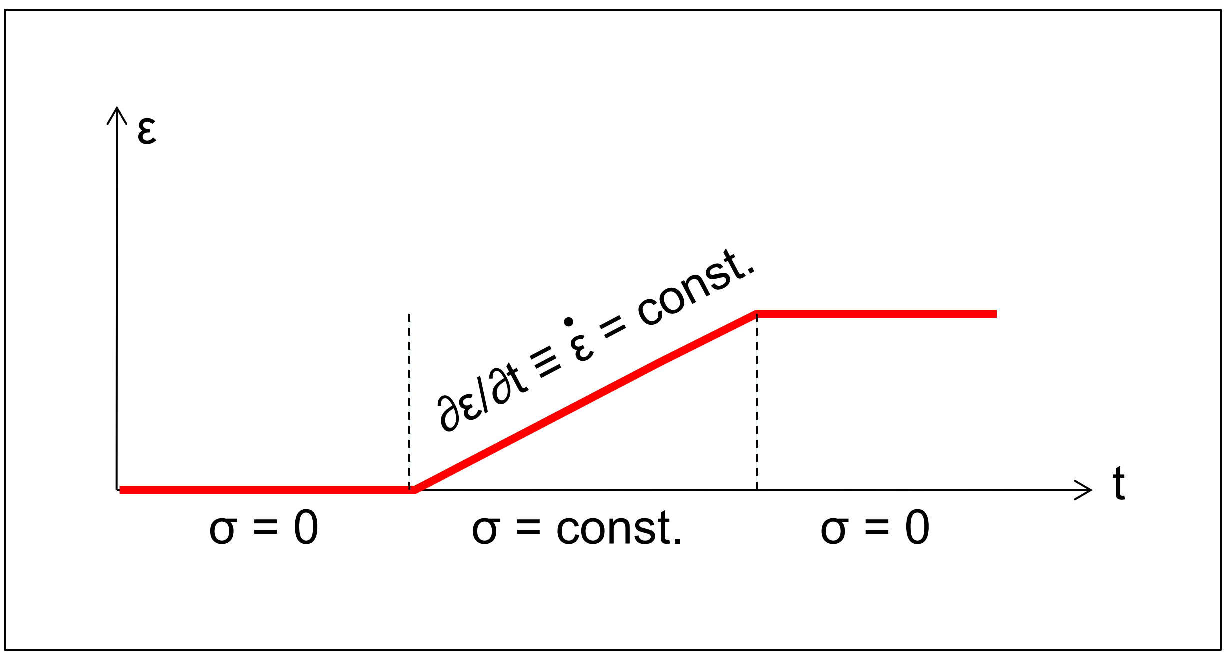
- Elastic model: e.g. spring
- $\sigma = E\,\varepsilon$, σ = [Pa]=[Nm^{-2}] , ε = [-], [%], [m/m]
- σ is an uni-axial stress

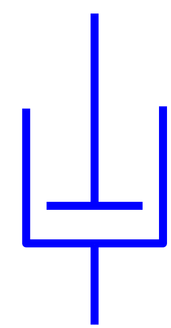




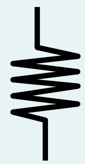
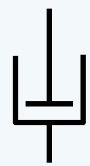
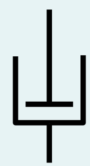
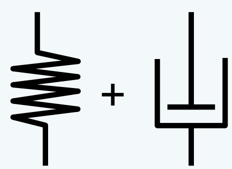
Stress – strain relations

- Viscous model: e.g. damper
- $\sigma = E\,\varepsilon$, σ = [Pa]=[Nm^{-2}] , ε = [-], [%], [m/m]
- σ is an uni-axial stress





Mechanical models of snow

Model for snow	Model elements	Time scale	Load
elastic	linear 	rapid (second/s)	small
viscous	linear 	long term	small
viscous	non-linear 	long term	large
visco-elastic	linear 	short term (minute/s)	small (<10kPa)

Snow: neither purely elastic nor purely viscous (→ visco-elastic)
→ Combination of elements; f [(magnitude + duration) of force]

Mechanical models of snow

linear-viscous behavior of snow is characterized by the material-‘constants’

- viscosity, η
- Poisson’s ratio, ν

however, ε = f (ρ, T, grain shape/size, humidity/LWC)

variable	deformation	
	small for:	**large for:**
density	high	low
grain shape	concave	dendritic
temperature	low	high
humidity	dry	wet

Poisson's ratio (ν)

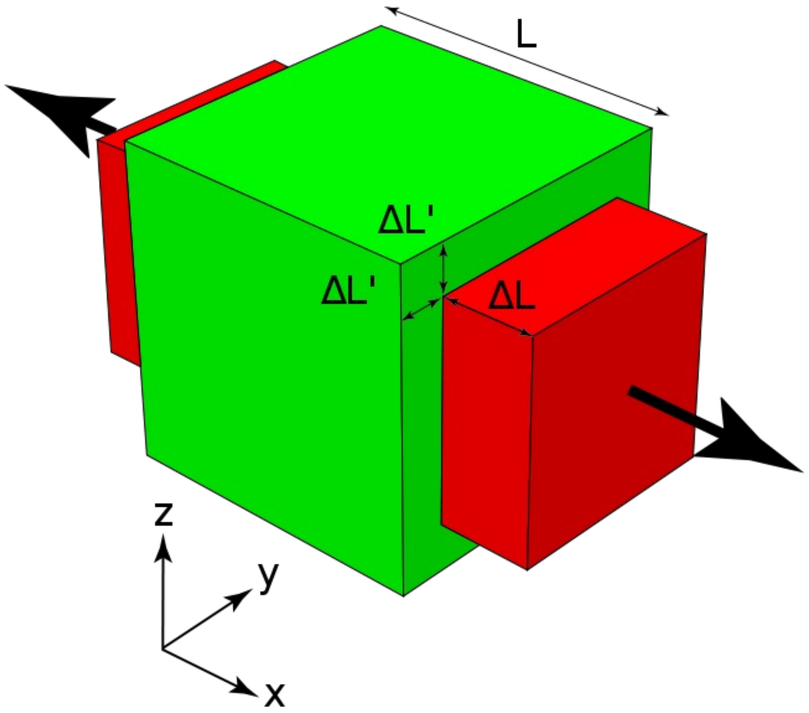
Uni-axial force → lateral contraction of expansion

$$\frac{\varepsilon_x}{\varepsilon_y} = -m, \quad where \ \frac{1}{m} \equiv \nu$$

$$with \quad 2 \le m \le \infty,$$

$$resp. \quad 0.5 \ge 1/m \ge 0.$$

$$\nu = -\frac{d\varepsilon_{trans}}{d\varepsilon_{axial}} = -\frac{d\varepsilon_y}{d\varepsilon_x} = -\frac{d\varepsilon_z}{d\varepsilon_x}$$



ε_{trans} = transverse strain (<0 for axial tension (stretching), >0 for axial compression)
ε_{axial} = axial strain (positive for axial tension, negative for axial compression)

Strain rate and shear stress

Strain rate [s^{-1}], deformation per time unit

$$\dot{\varepsilon} = \frac{d\varepsilon}{dt} = \frac{d}{dt}\left(\frac{l - l_0}{l_0}\right) = \frac{1}{l_0}\frac{dl}{dt} = \frac{v}{l_0}$$

Shear stress = force per unit area, $\tau = \frac{F}{A}$

In straight, uniform and parallel flow (fluid): $\tau = \mu \frac{du}{dy}$

u is a shear velocity [m s^{-1}]
µ is a dynamic viscosity [Pa*s]

(Note: kinematic viscosity, $\eta = \frac{\mu}{\rho}$ [m^2 s^{-1}])

Snow strength *ψ*

strength = f {
- snow type (grain shape, structure, density)
- condition of snow pack (temperature, humidity)
- type of load (stress, loading velocity)

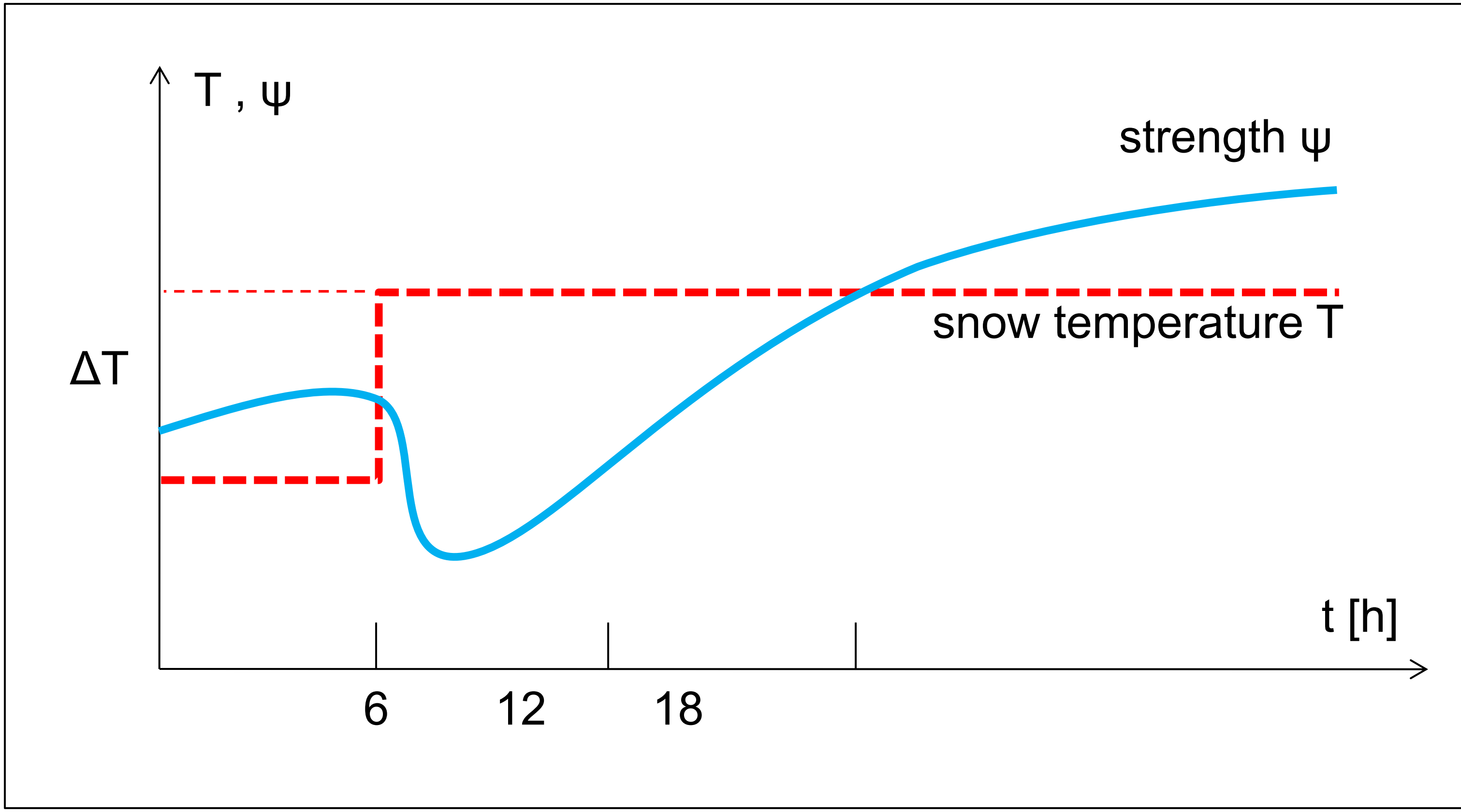
Strength increases with
> new bonds (compaction)
> larger contact areas (sintering)
> destructive metamorphism

and decreases with
> dendritic → round gains (metamorphism)
> increasing LWC
> constructive metamorphism

Influence of temperature:

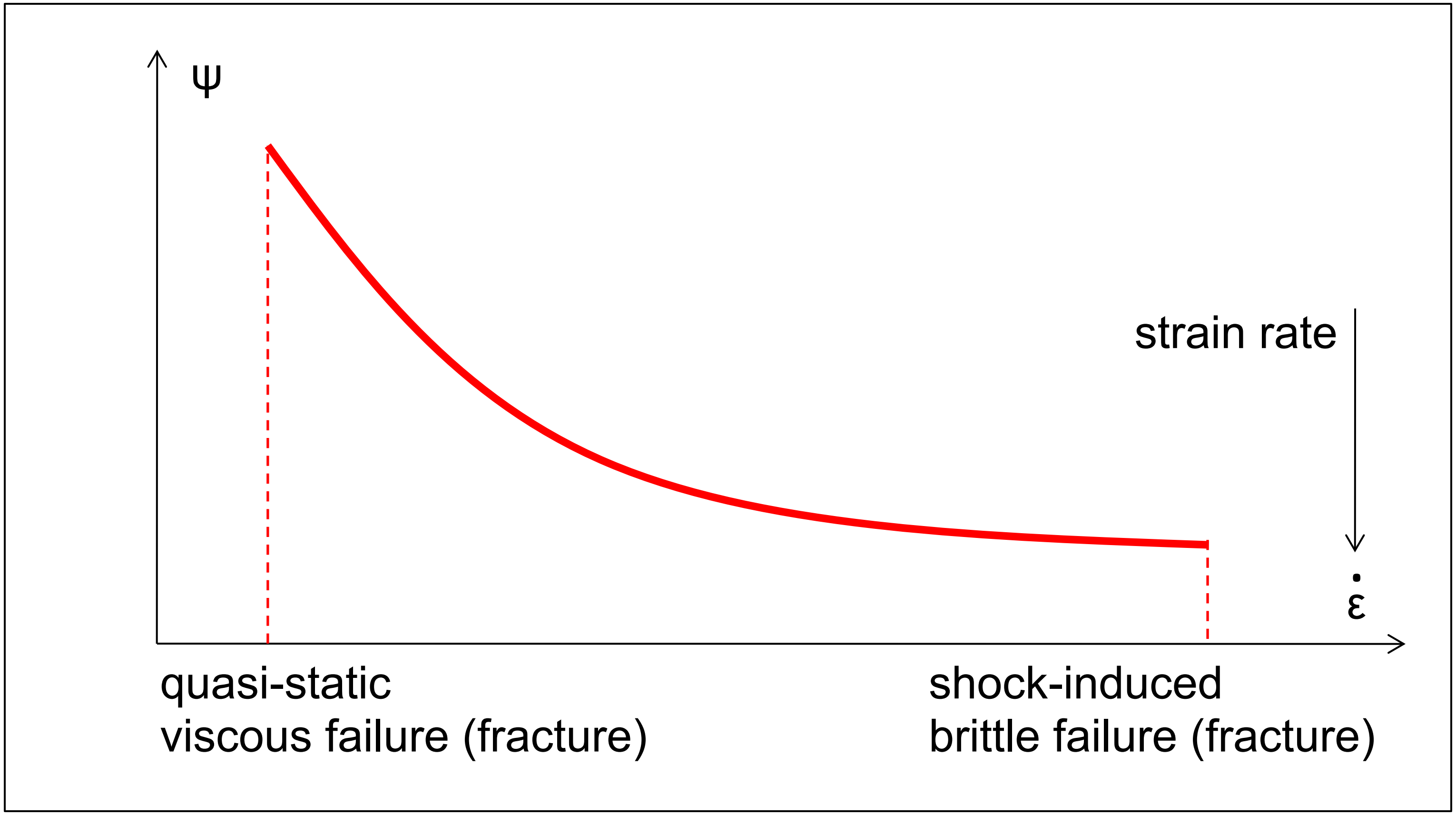
ΔT without change of structure → reversible process
ΔT plus densification (compaction) → irreversible process

Snow strength



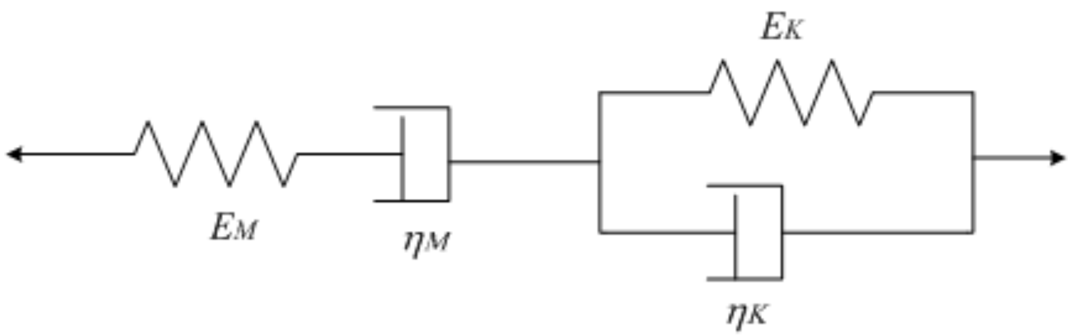
Evolution of snow strength for a step change in T with an associated change of snow structure

Snow strength

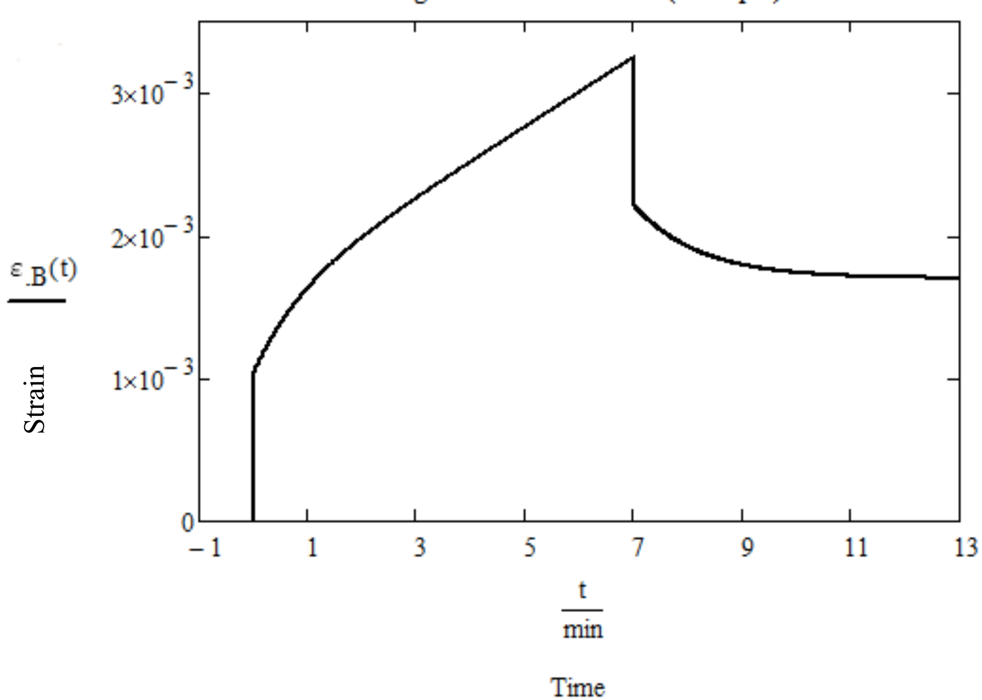


Behavior of snow strength as a function of strain rate

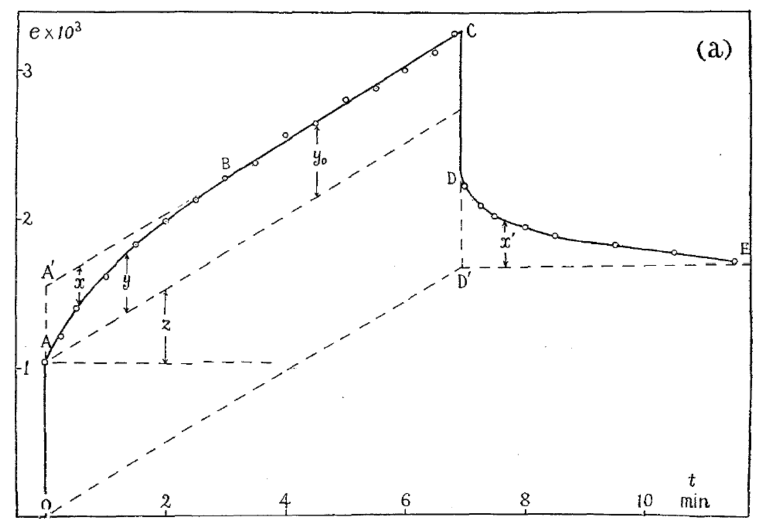
Burgers Model for Snow



Burgers Model for Snow (Example)



Real Snow Sample



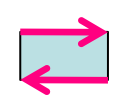
Snow strength (qualitative)

variable	snow strength	
	small for:	large for:
density	low	high
grain shape	concave, spiky	round
grain size	large	small
temperature	≈ 0	< 0
humidity	wet	dry

compressive strength > tensile strength > shear strength

4 – 400 kPa

2 – 200 kPa



0.3 – 100 kPa

Tensile (σ) and shear (τ) strength

snow type	density [kg/m^3]	grain size [mm]	tensile strength [kPa]
felted snow	127	0.5-1.5	3
concave grains	279	2.0	0.2
round grains	357	1.0-1.5	41
edged grains	408	1.0-1.5	27
round grains	472	0.5-1.0	162

Measured tensile strength for different snow types

Shear strength (τ):
min. 0.3-1 kPa (fresh snow, 100-200 kg/m^3)
avg. 2-20 kPa (mean snow, 300 kg/m^3)

$$\tau = \sigma \tan \varphi + c$$ (Coulomb–Mohr law)

τ = shear strength, σ = normal stress, φ = internal friction angle (30-40^o), c = cohesion [Pa],

Illustration of snow mechanics

Details in
"Snow Mechanics and Avalanches" lectures

Illustration of snow mechanical properties

Next:


Snow
Energy Balance
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