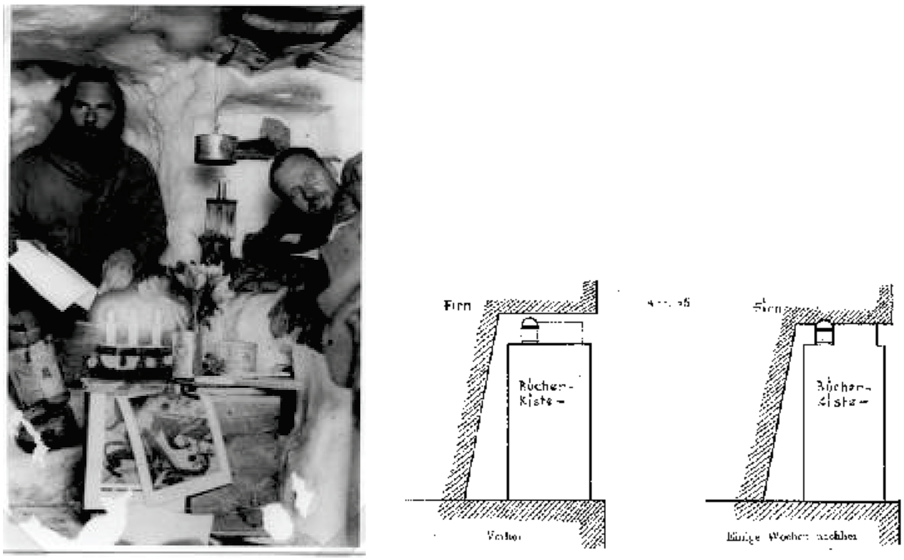

Figure 15.1 Left: Ernst Sorge and Fritz Loewe in the snow shelter at Eismitte (Stolen from Uni Melbourne) Right: Schematic stolen from Sorges report

15.2 Sorges law (Added by Henning)

A very interesting application of snow densification is Sorge's law. Ernst Sorge was a participant of the Alfred Wegener Expedition to Greenland, Eismitte 1930/31.

During the winter Sorge observed that a glass on top of the book shelf in the snow shelter got jammed due to the settlement of the ceiling (cf. Fig. 15.1)

Inspired by this observation Sorge invented a remarkable device, the so-called Firn-Schrumpfschreiber (Fig. 15.2) to measure the settling curves as a function of time. Sorges scientific observation drawn from this expedition can be summarized in the following statement:

> *If the accumulation rate is constant at a given location, the snow/firn density in a given depth below the snow surface is independent of time.*

This observation was credited by and rightly drawn attention to by H. Bader [1] and termed Sorge's law. Sorge's law applies whenever the climatic cycle does not appreciably change in the accumulation zone. This does not necessarily require continuous snowfall, it remains valid also in a temporarily averaged sense. Sorge's law has an interesting application, namely the estimate of the accumulation rate from density and settling measurements alone.

For illustration it is necessary to formulate the problem in the coordinate system in which the snow surface is at rest

(cf. Fig. 15.2). The 1d continuity equations in this frame of refererence has been discussed by e.g. by [10]. The mass continuity equation reads

$$\frac{\partial}{\partial t}\rho_s(z,t) + \frac{d}{dz}\big[\rho_s(z,t)v(z,t)\big] = 0 \qquad (15.2.1)$$

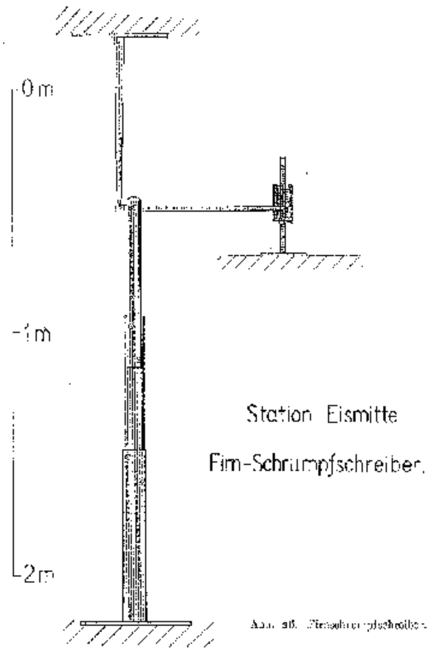

Figure 15.2 The firnschrumpfschreiber

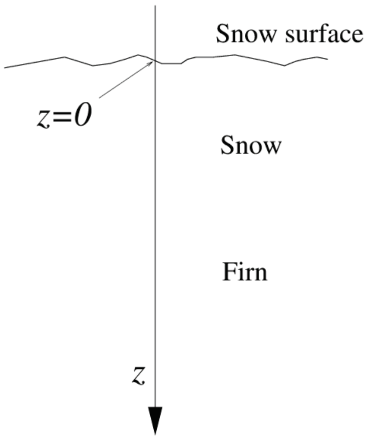

Figure 15.3 Coordinate system

with boundary conditions

$$\begin{aligned} \rho_s(0,t)v(0,t) &= q_{acc}(t) \\ \rho_s(0,t) &= \rho_s(t) \end{aligned} \tag{15.2.2}$$

and the static momentum equation (the force balance) reads

$$\frac{\partial}{\partial z}\sigma(z,t) = g\rho_s(z,t) \tag{15.2.3}$$

As indicated by [10], Sorge's law, i.e. a time invariant density profile $\rho_s(z,t) = \rho_s(z)$ is a direct consequence of a time-invariant boundary condition $q_{acc}(t) = q_{acc}$ and $\rho_s(t) = \rho_s$. In this case the age of the snow in a certain depth is directly related to the accumulation rate. To see this, we consider a snow/firn parcel at depth z with age $t(z)$. The total mass accumulated since the parcel's own deposition at $t = 0$ can be written in two alternative ways

$$q_{acc}\, t(z) = \int_0^z dz' \rho(z') \tag{15.2.4}$$

Solving for $t(z)$ yields the age of a snow/firn parcel at a certain depth in terms of the density profile and the accumulation rate

$$t(z) = \frac{1}{q_{acc}} \int_0^z dz' \rho(z') \tag{15.2.5}$$

Thus, the age can be estimated without reference to annual layers. By differentiating the age equation (15.2.5) gives

$$\frac{dt(z)}{dz} = \frac{\rho(z)}{q_{acc}} \tag{15.2.6}$$

and solving for the settling velocity $v(z) = dz/dt$ yields

$$v(z) = \frac{q_{acc}}{\rho(z)} \tag{15.2.7}$$

Here $v(z)$ is the downward velocity of a snow parcel at depth z relative to the surface, i.e. the same v which emerges in the mass continuity equation (15.2.1). Note that (15.2.7) follows also directly from the continuity equation (15.2.1), by using Sorge's law $\partial\rho_s(z,t)/\partial t = 0$ together with time invariant boundary conditions (**??**).

As a practical consequence, the mean accumulation rate q_{acc} can be estimated from velocity and density measurements without reference to annual precipitation layers. To estimate the mean accumulation rate Sorge measured

vertical velocities $v'(z_1), v'(z_2)$ and densities $\rho_s(z_1), \rho_s(z_2)$ for at least two positions z_1, z_2 in a snowpit. Since velocity measurements will be probably taken in an inertial frame of reference, i.e. a coordinate system where the surface is moving, a transformation of the velocities is required. These are indicated by primed velocities v' in contrast to the velocities v in Eq. (15.2.1). The surface velocity is unknown though, but velocity *differences* are equal in both coordinate systems. Hence

$$v'(z_2) - v'(z_1) = v(z_2) - v(z_1) \tag{15.2.8}$$

Combining this with Eq. 15.2.7 yields Sorge's method

$$q_{acc} = \frac{v'(z_1) - v'(z_2)}{\rho_s(z_2) - \rho_s(z_1)}\, \rho_s(z_1)\rho_s(z_2) \tag{15.2.9}$$

Measurements can be made at different depths and subsequently averaged. Thus density and velocity measurements alone allow to estimate the mean accumulation rate without any reference to annual precipitation layers. This is only possible in a frame of reference where the surface is at rest and requires the transformation of the governing equation in this coordinate system. Interstingly, the entire estimate is possible without having even touched densification mechanisms, constitutive equations and material behavior yet. (In the lecture I give this example after I touched the surface continuity equation for the mass balance for ice sheets and before I go to constitutive equations, so the section might also serve as a starter for the chapter or go somewhere else. But I am a huge fan of Sorge's law)