

River ecomorphodynamics and bioengineering

(ENV-418, A.Y. 2025-26)

4ETCS, Master option

Prof. Paolo Perona

Platform of Hydraulic Constructions



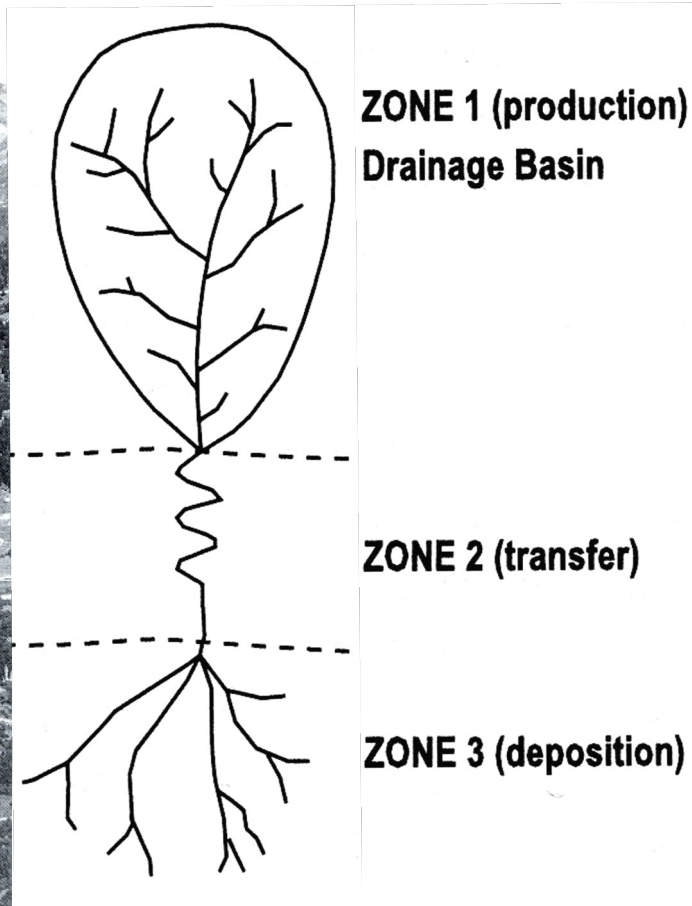
Lecture 10-1: Sediment

Objectives

- To understand the formation and geometry of river bedforms and related influence on hydro and morphodynamic processes.
 - Rivers and sediments
 - Small and meso-scale bedforms
 - Hydrodynamic conditions for bedform
 - Effects of bedforms on flow resistance
 - Partitioning methods for friction and form resistance and roughness

Rivers and sediment

Sediment production and fate



Course d'eau	Bassin 10 ⁶ km ²	Vit. Prod. mm/an
Rio delle Amazzoni (foce)	7	0.09
Mississippi (foce)	3.9	0.055
Congo (foce)	3.7	0.015
Nilo (delta)	2.9	0.015
Volga (foce)	1.5	0.01
Missouri (foce)	1.4	0.1
Gange (delta)	1.0	1.
Danubio (foce)	0.82	0.06
Hwang Ho (foce)	0.77	1.75
Reno (delta)	0.36	0.001
Po (delta)	0.07	0.15
Tevere (foce)	0.016	0.27
Waipapa (kanakahala)	0.0016	5

Questions in fluvial landscape morphodynamics

Sediment supply

how can we estimate sediment supply from a watershed ?

Sediment transport

what are the principles of water flow and sediment motion in rivers ?

IN THESE LECTURES

Channel stability

how does the channel adjust during water and sediment transport ?

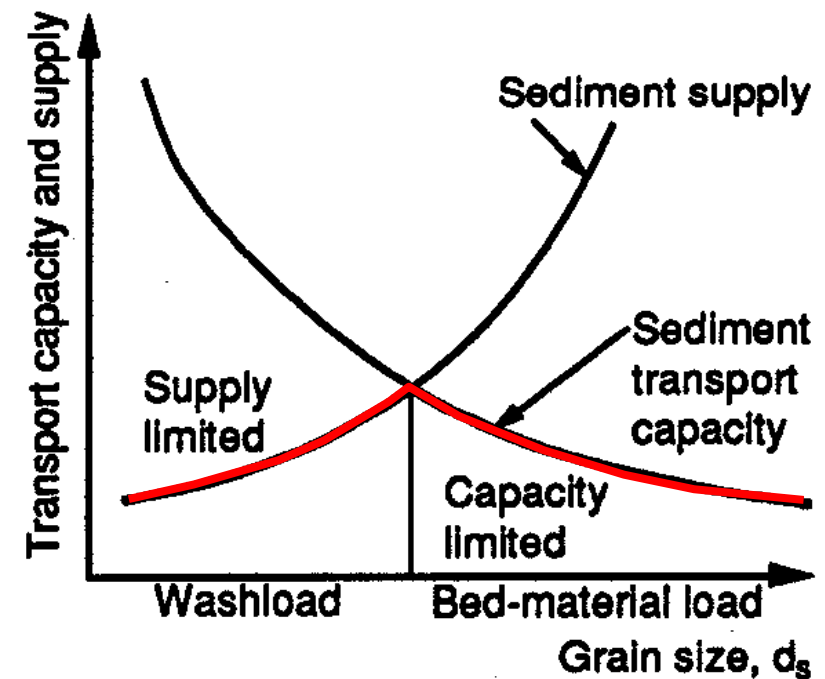


implications for river engineering and erosion control

- Sediment is supplied by upland erosion in the **watershed** (rill, gully erosion, mass movement) or by channel erosion within the **streams** and is carried by water flow

Sediment transport capacity is

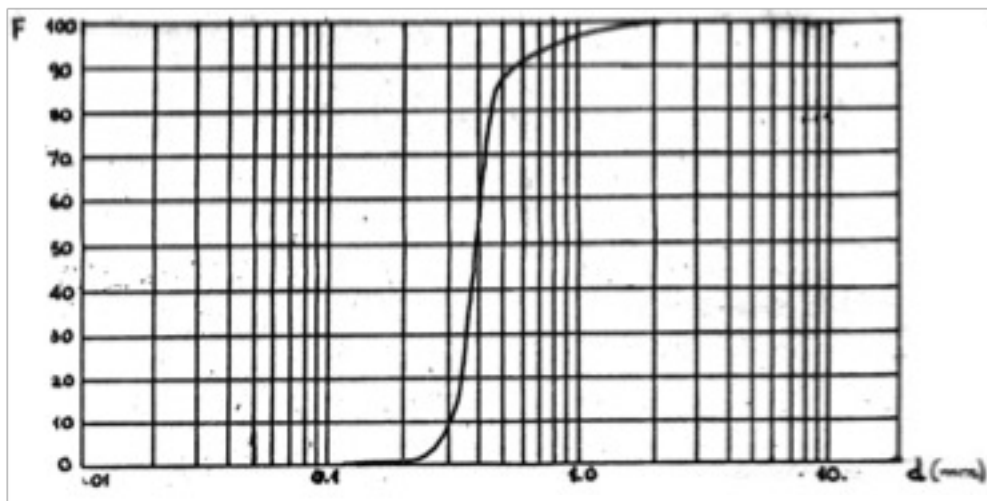
- **capacity limited**
 - how much water can transport
- **supply limited**
 - how much sediment is available
 - how much can be detached



Sediment transport (quick review)

Granulometric scale

$$d_g = d_r 2^{+\bar{\psi}} , \quad \sigma_g = 2^{\sigma_\psi} \quad (d_r = \text{reference diameter, 1 mm})$$



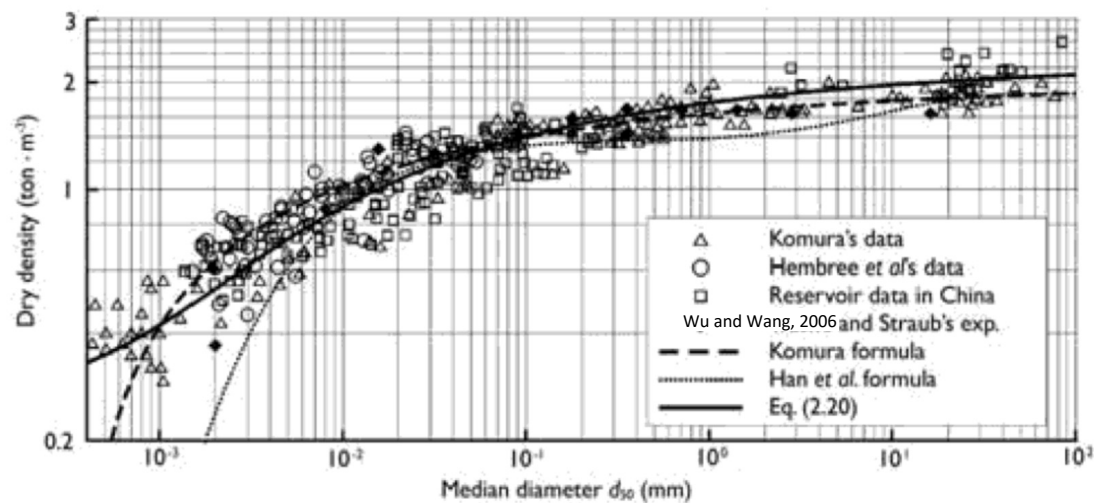
$$f(\psi) = \frac{1}{\sigma_\psi \sqrt{2\pi}} \exp - \frac{(\psi - \bar{\psi})^2}{2\sigma_\psi^2}$$

$$F(\psi) = \frac{1}{2} \left[1 + \operatorname{erf} \left(\frac{\psi - \bar{\psi}}{\sqrt{2}\sigma_\psi} \right) \right]$$

$$\left\{ \begin{aligned} d_{50} = d_g &= \sqrt{d_{84} d_{16}} && \text{Geometric mean} \\ \sigma_g &= \sqrt{\frac{d_{84}}{d_{16}}} && \text{Standard deviation} \end{aligned} \right.$$

Sediment transport (quick review)

Porosity



Wu and Wang, 2006

$$p = \frac{V_v}{V_v + V_s}$$

$$p = 0.245 + \frac{0.0864}{(0.1d_{50})^{0.21}}$$

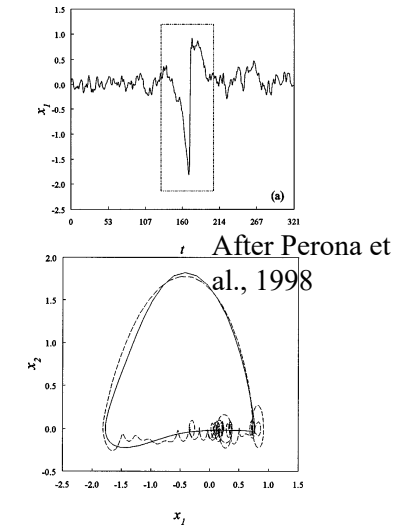
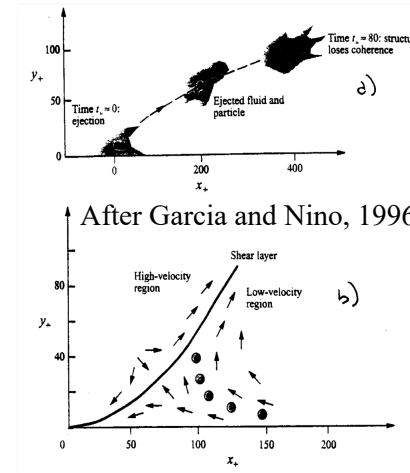
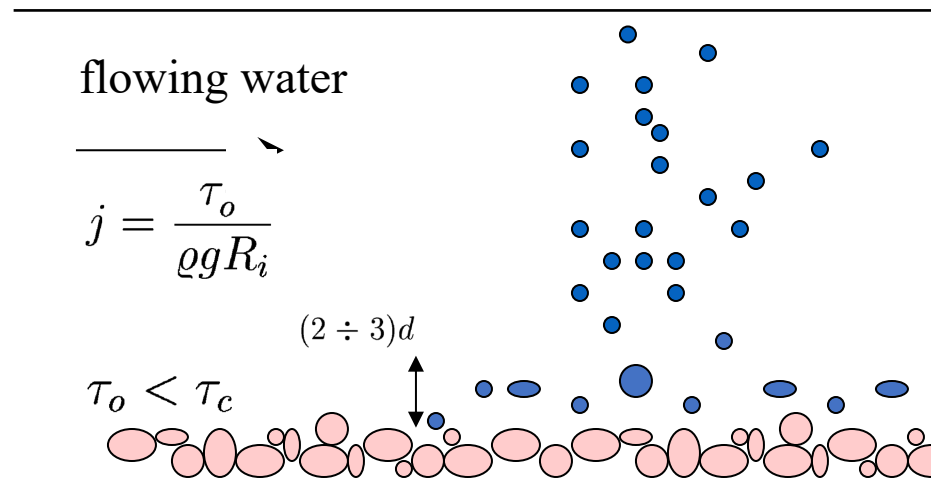
(Komura, 1963)

$$p = 0.13 + \frac{0.21}{(d_{50} + 0.002)^{0.21}}$$

Sediment transport (quick review)

Sediment transport (total load) is defined by

- type of movement (bedload and suspended load)
- method of measurement (measured and unmeasured load)
- source of sediment (washload and bed material load)



suspended load $\tau_0 > \tau_s$
transported in suspension

bed load $\tau_c < \tau_0 < \tau_s$
transported in contact with the stream bed (by saltating, rolling)

Incipient movement

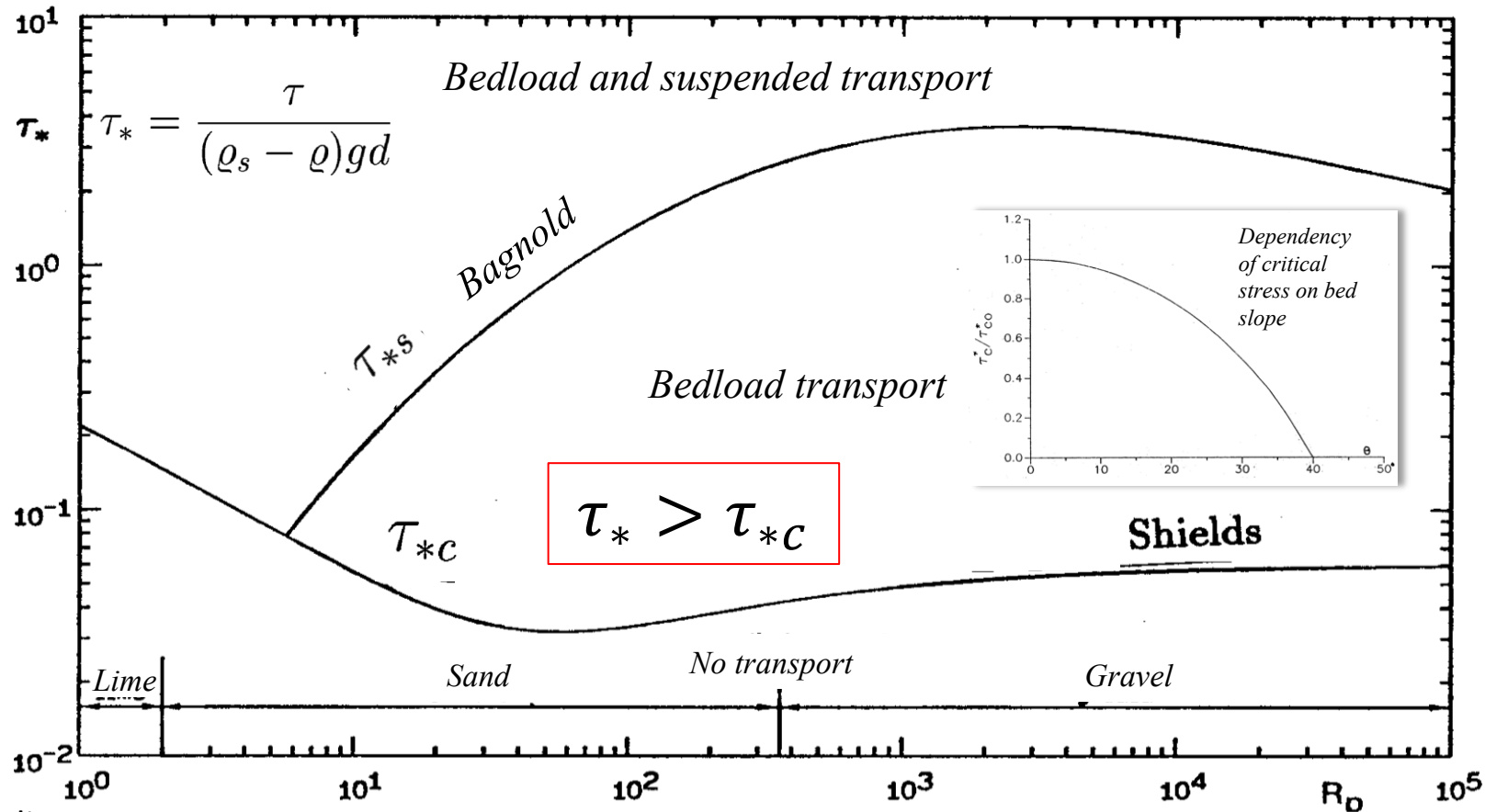
- The simultaneous movement of fluid and sediment constitutes a bi-phasic mixture
- The onset of sediment movement in a uniform, stationary stream can be supposed to depend on a critical shear stress τ_c

- By selecting $((\rho_s - \rho)), g, d$ as D.I. variables, then (PI-Theorem)

$$\tau_c = \tau_c((\rho_s - \rho), g, d, \nu, \psi, e)$$

$$\tau_* = \frac{\tau}{(\rho_s - \rho)gd} \quad \tau_{*c} = \tau_{*c}(R_p, \psi, e) \quad R_p = \frac{\sqrt{(s - 1)gd^3}}{\nu} \quad s = \frac{\rho_s}{\rho}$$

The Shields diagram

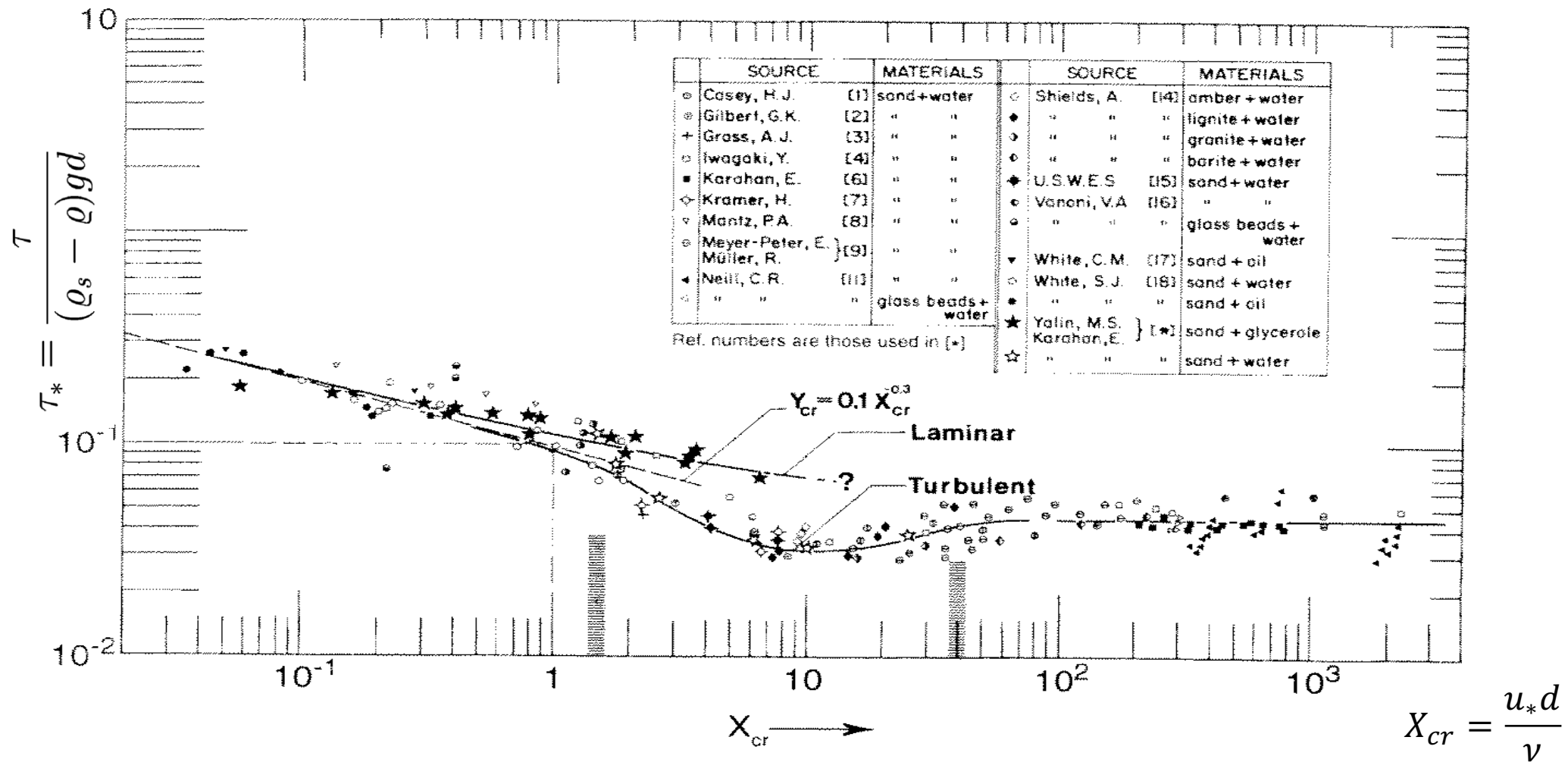


(Brownlie, 1981) $\tau_{*c} = 0.22R_p^{-0.6} + 0.06 \exp(-17.77R_p^{-0.6})$

$$R_p = \frac{\sqrt{(s-1)gd^3}}{\nu}$$

The original Shields diagram

- Incipient motion following Shields



Other effects on the critical shear stress

- Effect of slope on the critical shear stress



$$\tau_{ca} = \tau_{*c} \gamma (G - 1) \left[\sqrt{1 - \frac{\sin^2 \theta}{\sin^2 \varphi}} \right] d_m$$

φ = angle of repose

(e.g., Lane, 1970)

- Effect of grain size distribution (hiding function theory, Egiazaroff 1965)

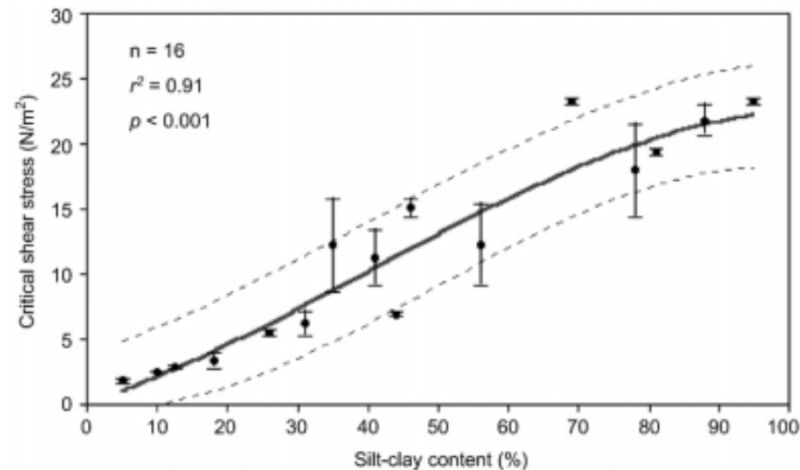
$$\tau_{*c}(d) = \tau_{*cg} G \left(\frac{d}{d_g} \right)$$

$$G = \left(\frac{d_g}{d} \right)^\beta \quad (\beta = 0.9)$$

(e.g., Parker, 1990)

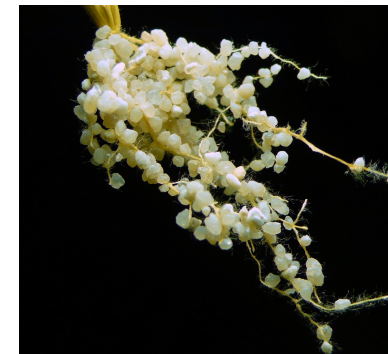
Other effects on the critical shear stress

- Cohesion (soil types)



(from Julian and Torres, 2006)

- Mechanical cohesion (vegetation roots)



Basal critical shear stress
(soil properties only)

Root
mechanical
strength
coefficient

$$\tau_c(z) = \tau_{c0} (1 + \sigma_1 r(z))$$

Root biomass density

Sediment flux (Bagnold hypothesis)

Du Boys (1879)

$$q_s = k'_s \tau_0 (\tau_0 - \tau_c)$$

Kalinske (1947)

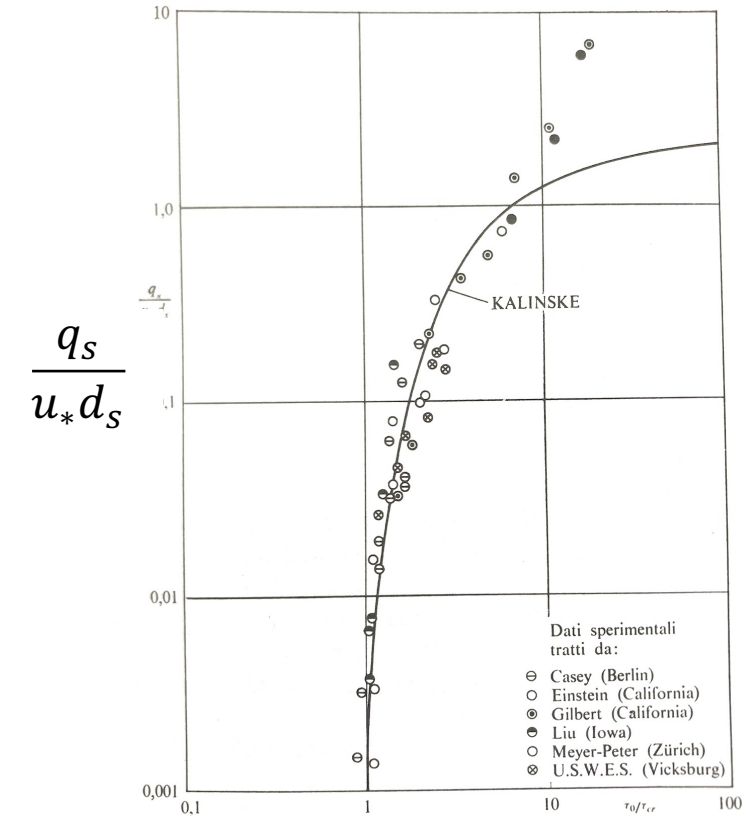
$$\frac{q_s}{u_* d_s} = f\left(\frac{\tau_0}{\tau_c}\right) = f(\eta_*)$$

Ashida and Michiue,
1972

$$q_s = \bar{c} \bar{V}_p$$

$$\phi = \frac{q_s}{\sqrt{(s-1)gd^3}}$$

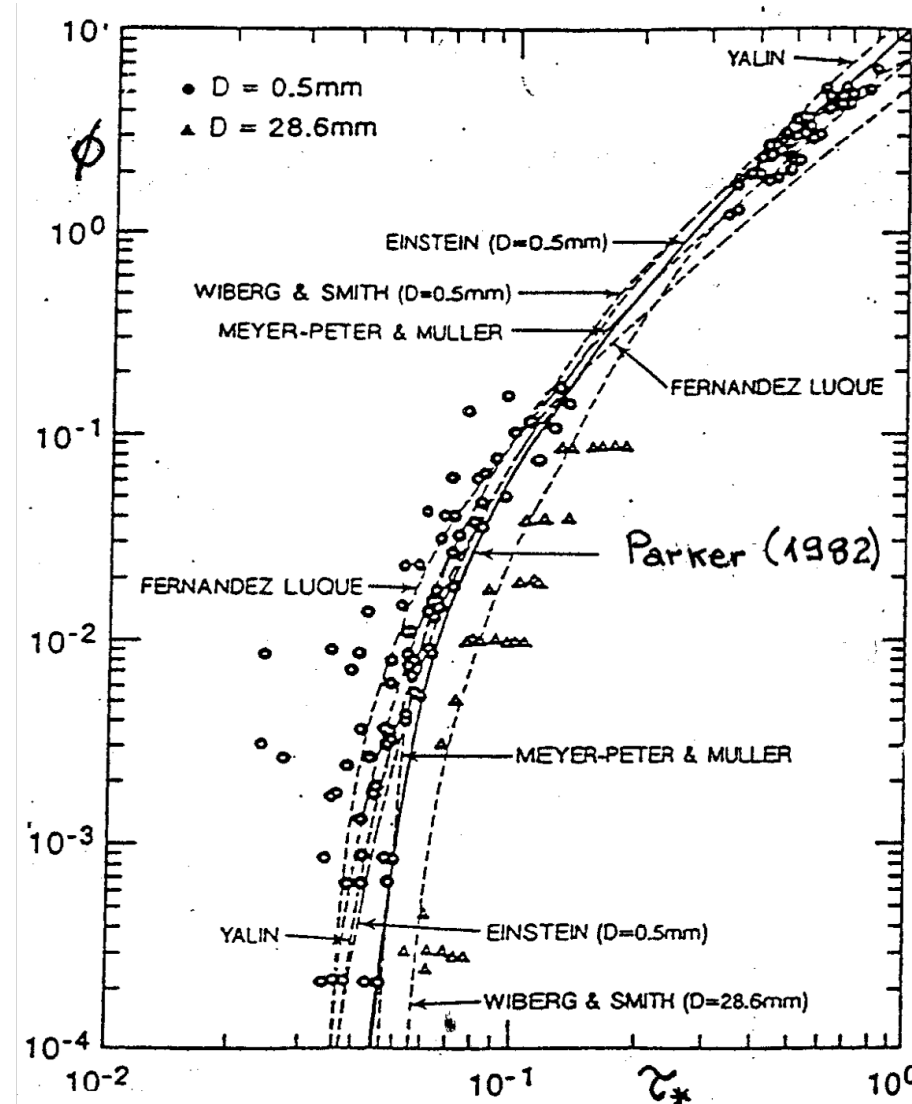
$$\phi = 17(\tau_* - \tau_{*c})(\sqrt{\tau_*} - \sqrt{\tau_{*c}}) \quad \frac{\tau_0}{\tau_c}$$



$$\phi = \frac{q}{\sqrt{(s-1)gd^3}}$$

Sediment flux due to advection

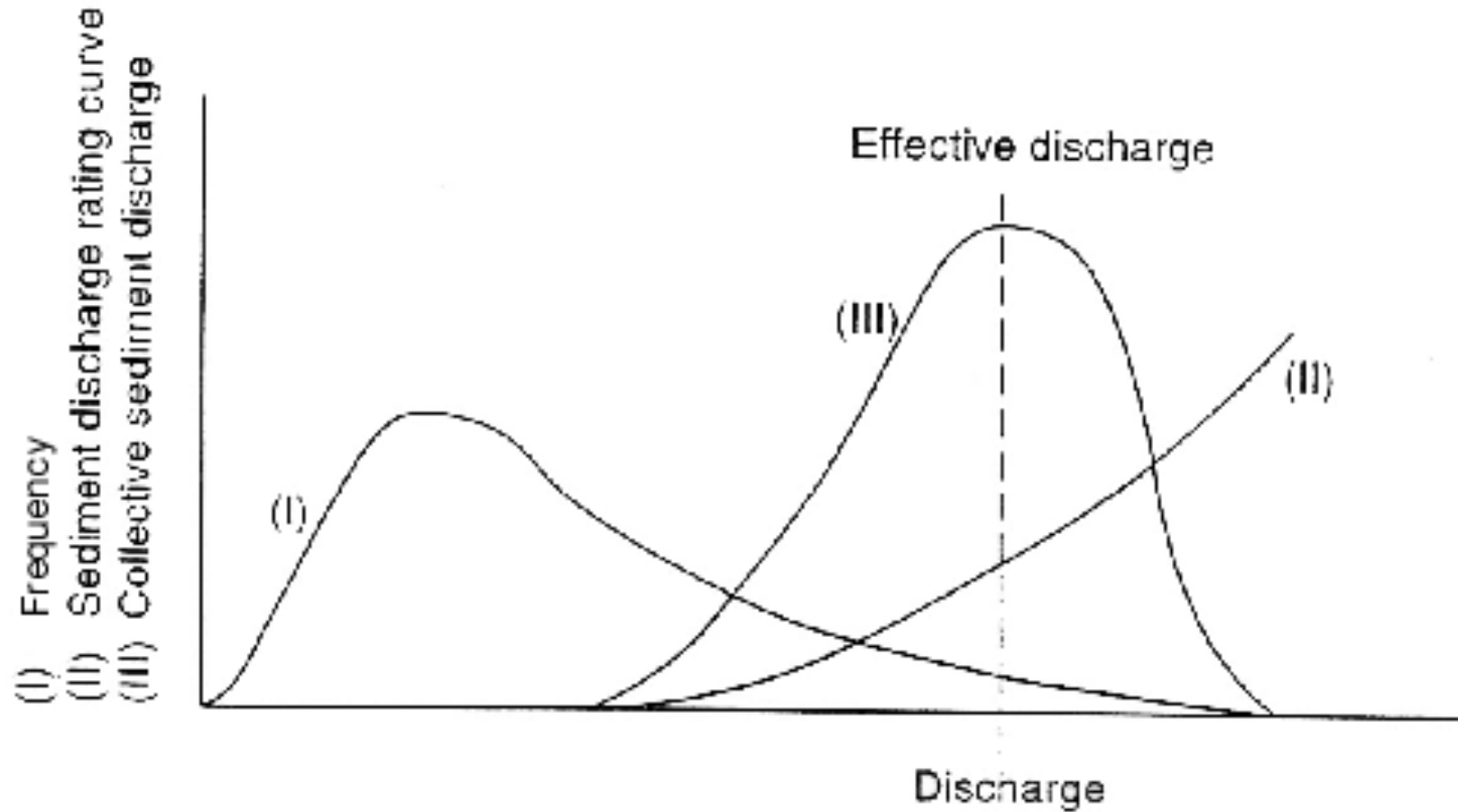
$$q = \bar{c}\bar{V}_p$$



Comparison among different bedload transport models for homogeneous sediment

$$\tau_* = \frac{\tau}{(\rho_s - \rho)gd}$$

Sediment transport capacity and effective discharge



Towards a unified scaling for gravel-bed and sand-bed rivers

Calvani et al., Geomorphology 2025



Problem: find a scaling relationship linking sediment size to hydraulic variables and channel geometry

Approach: introduce a modulating grain size diameter coarser than the mean grain size, D_{50} , unclear phenomena occurring in sand-bed rivers are smoothed and can be taken into account.

$$\hat{Y} = Y/D \text{ and } \hat{B} = B/D$$

$$\hat{Q} = \frac{Q}{\sqrt{gD^5}}, \quad R_p = \frac{\sqrt{\left(\frac{\rho_s}{\rho} - 1\right) gD^3}}{v}, \quad \hat{D}_{84} = \frac{D_{84}}{D},$$

$$\{\hat{\theta}, \theta_{cr}\} = \frac{\{\tau, \tau_{cr}\}}{(\rho_s - \rho)gD}, \quad S = \frac{\rho_s}{\rho}$$

$$\{s, Y, B\} = f_{s,Y,B}(Q, D, D_x, \tau, \tau_{cr}, \rho_s - \rho, g, \mu, \text{others})$$

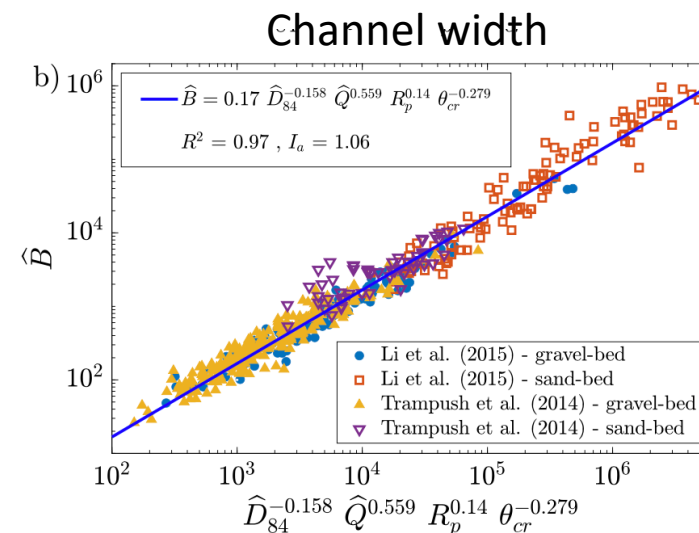
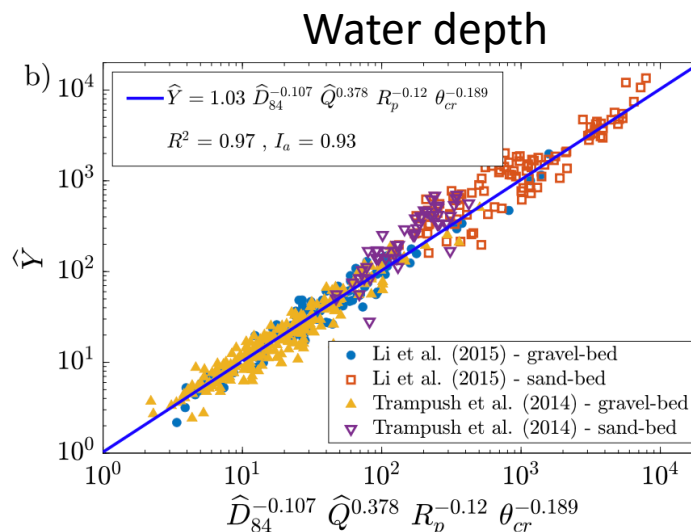
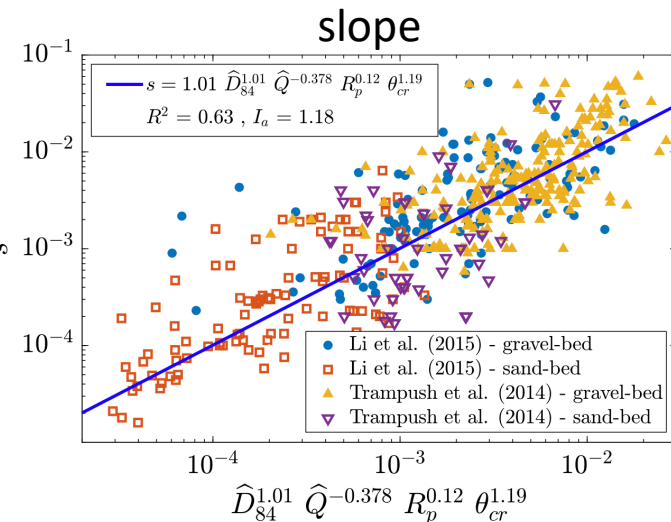
↓ Buckingham, Pi theorem

$$\{s, \hat{Y}, \hat{B}\} = F_{s,\hat{Y},\hat{B}}(\hat{Q}, R_p, \hat{D}_{84}, \hat{\theta}, \theta_{cr}, S - 1)$$

$$s = \hat{K}_s \hat{D}_{84}^{\frac{6\gamma - a + 10a\gamma}{6+7a}} \hat{Q}^{-\frac{6a}{6+7a}} R_p^4 \frac{1-a}{6+7a} \theta_{cr}^{\frac{3+5a}{6+7a}}$$

$$\hat{Y} = \hat{K}_Y \hat{D}_{84}^{\frac{1-3\gamma}{6+7a}} \hat{Q}^{\frac{6a}{6+7a}} R_p^4 \frac{a-1}{6+7a} \theta_{cr}^{-\frac{3a}{6+7a}}$$

$$\hat{B} = \hat{K}_B \hat{D}_{84}^{\frac{1-3\gamma}{6+7a}} \hat{Q}^{\frac{6}{6+7a}} R_p^{\frac{14}{3}} \frac{1-a}{6+7a} \theta_{cr}^{-\frac{3}{6+7a}}$$



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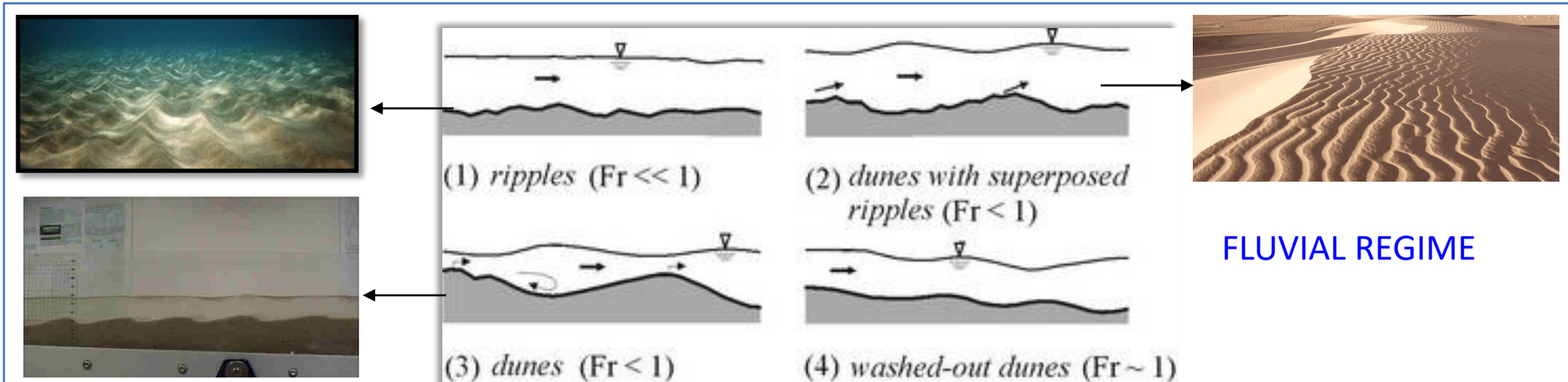
Photo: Hanna Schulz, <https://www.igb-berlin.de>

Lecture 10-2: Bedforms

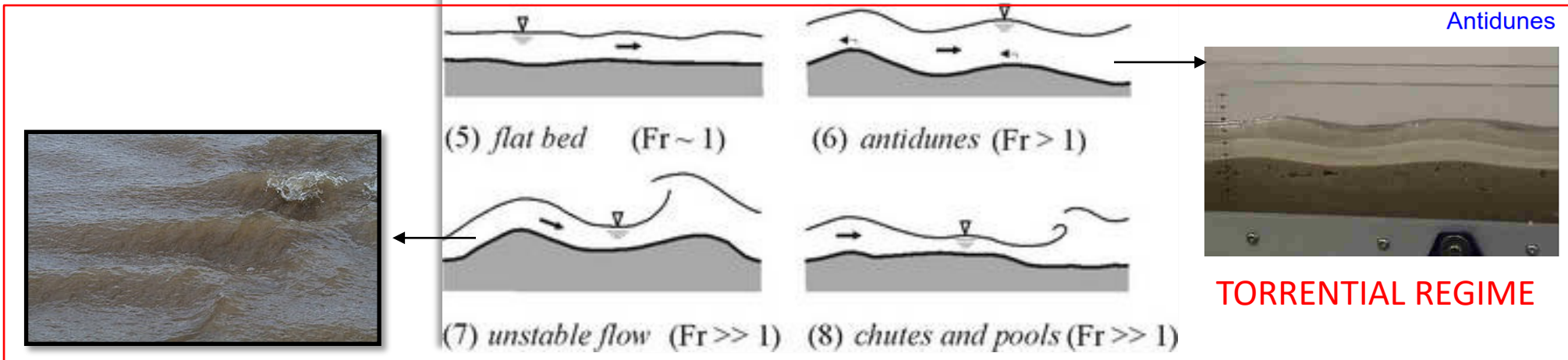
Concepts and definitions

- The mobile bed of fluvial streams is never really flat;
- Geometrical perturbations are usually present and can either be migrating or stationary;
- They can also have spatial size extending over a broad range of scales;
- We call small-scale bedforms those geometrical perturbations that have size comparable with the water depth; similarly, large-scale bedforms will have dimensions larger than the water depth;

Typical small-scale bedforms

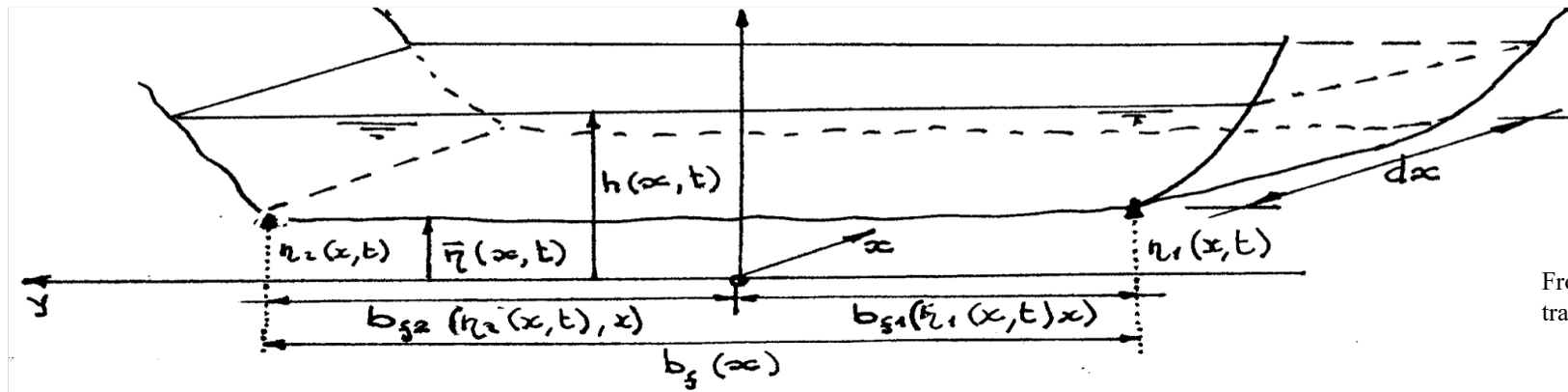


Dunes



Small scale bedforms

Morphodynamics: evolution (1D) equations



From Seminara G., Meccanica del trasporto dei sedimenti. Genova

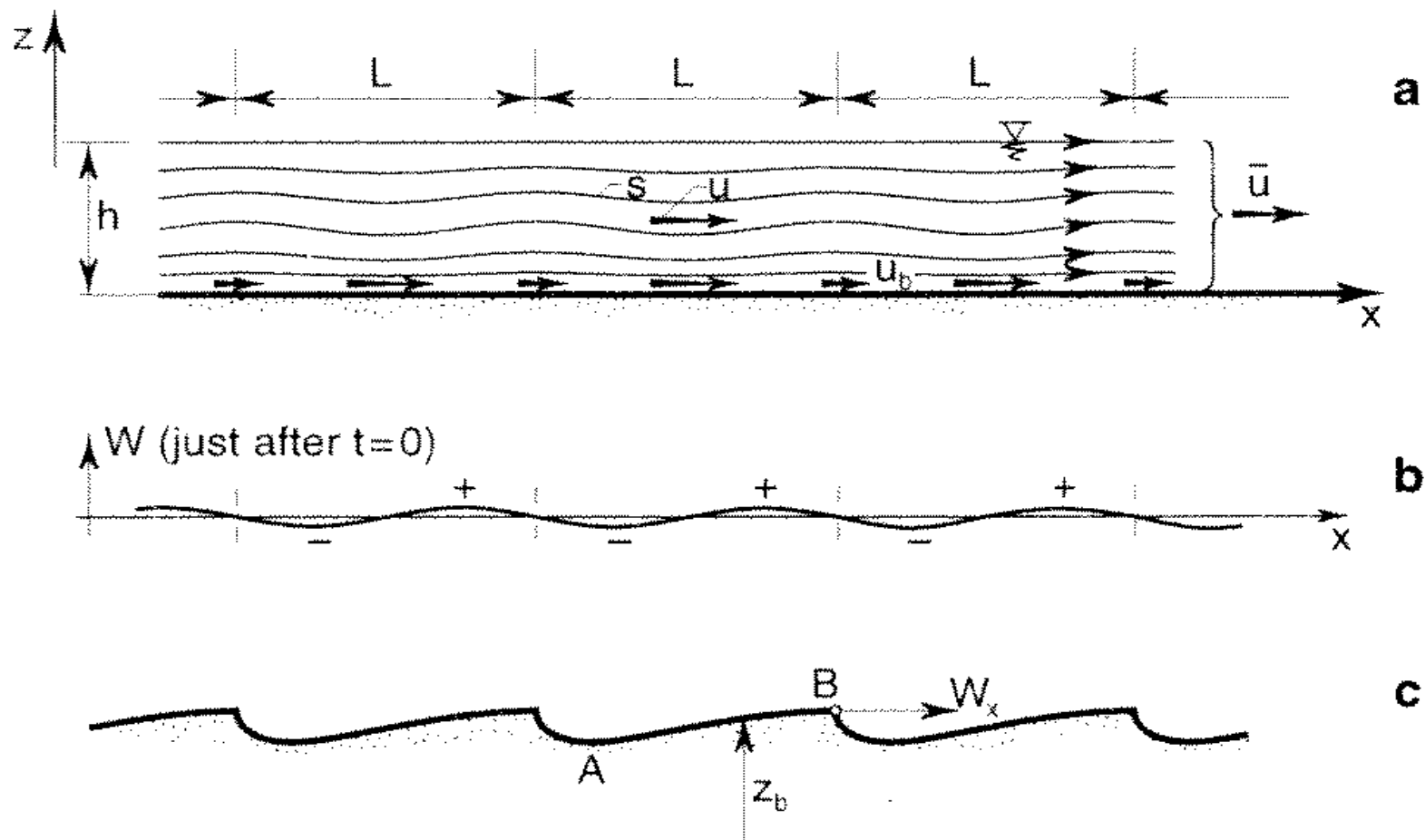
$$\left\{ \begin{array}{l} \Omega_{,t} + Q_{,x} = 0 \\ Q_{,t} + \left(\frac{\beta Q^2}{\Omega} \right)_{,x} + g\Omega h_{,x} + \frac{Q^2}{\Omega C^2 R} = 0 \\ (1-p)b_f \frac{\partial \bar{\eta}}{\partial t} + \frac{\partial}{\partial x} (b_f \bar{Q}_{sx}) = 0 \end{array} \right.$$

Flow continuity

Flow momentum (conservative form)

Sediment continuity (Exner)

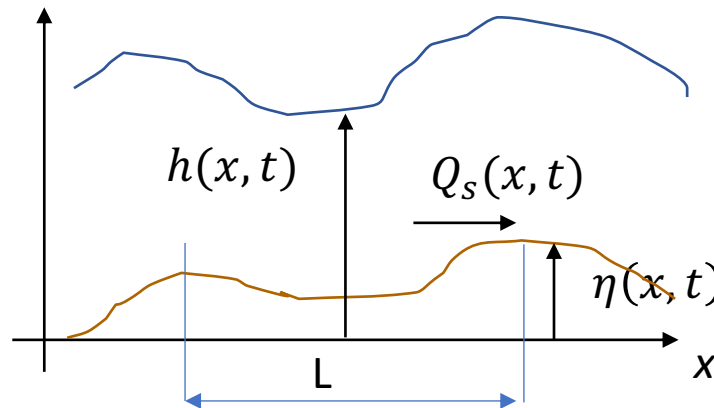
Bedform instability and waveform selection mechanisms



Bedforms mechanism: morphodynamic instability

- We use a simplified scheme to show why bedforms depend on flow regime via the Froude number

- Consider Exner's equation in the form $(1 - p) \frac{\partial \eta(x,t)}{\partial t} + \frac{\partial Q_s(x,t)}{\partial x} = 0$



Apply a small perturbation in the form of Fourier modes

$$\eta = \eta_0 + \eta_1 e^{-i\lambda(x-ct)} + b.c.$$

$$Q_s = Q_{s0} + Q_{s1} e^{-i\lambda(x-ct-\delta)} + b.c.$$

NOTE: c is a complex velocity;

$\lambda = \frac{2\pi}{L}$ = wave number

δ = phase between bedform and sediment flowrate

p = sediment porosity

Instability and small bedforms formation

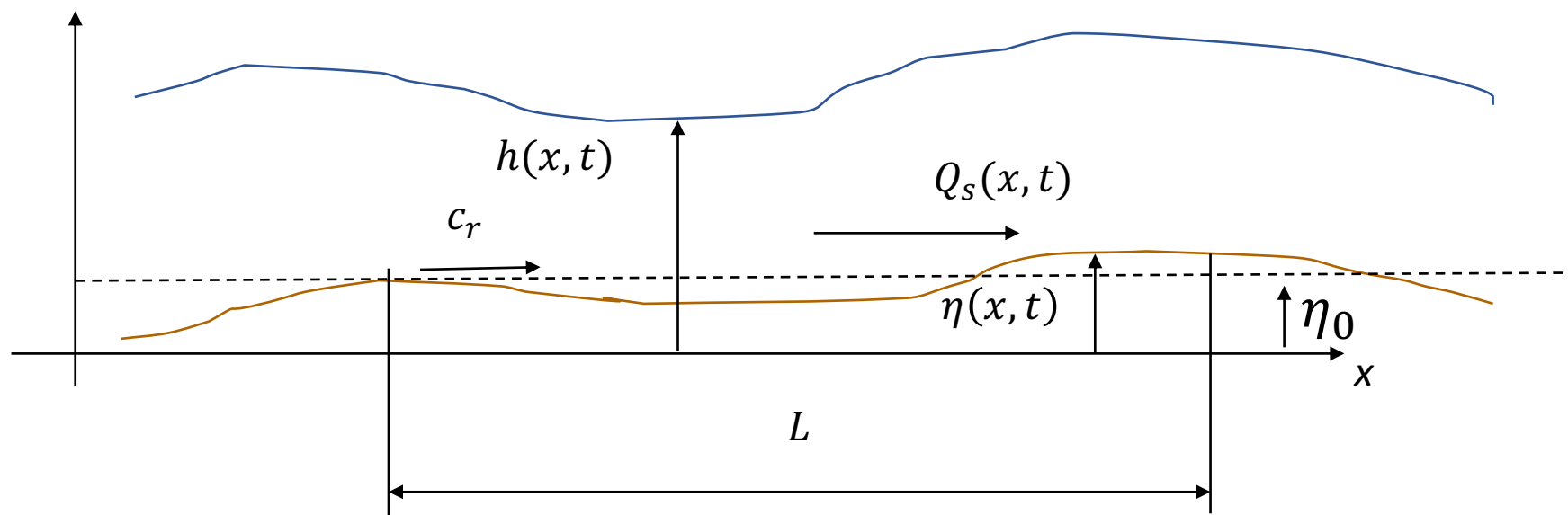
- Insert the perturbation into Exner equation, we obtain

$$-(1-p)c\eta_1 + Q_{s1}e^{-i\lambda\delta} = 0$$


$$c_r = \frac{Q_{s1}}{(1-p)\eta_1} \cos(\lambda\delta) \quad c_i = -\frac{Q_{s1}}{(1-p)\eta_1} \sin(\lambda\delta)$$

Migration celerity c_r depends on the spatial phase δ

Instability and small bedforms formation



$$c_r > 0 \text{ when } -L/4 < \delta < L/4$$

$$c_i > 0 \text{ when } -L/2 < \delta < 0$$

$$c_r < 0 \text{ when } L/4 < \delta < 3L/4$$

$$c_i < 0 \text{ when } 0 < \delta < L/2$$

Downstream vs upstream migration

Growth and attenuation

Instability and small bedforms formation

Assuming irrotational flow, it can be shown that there are three Froude regimes

- I – subcritical regime

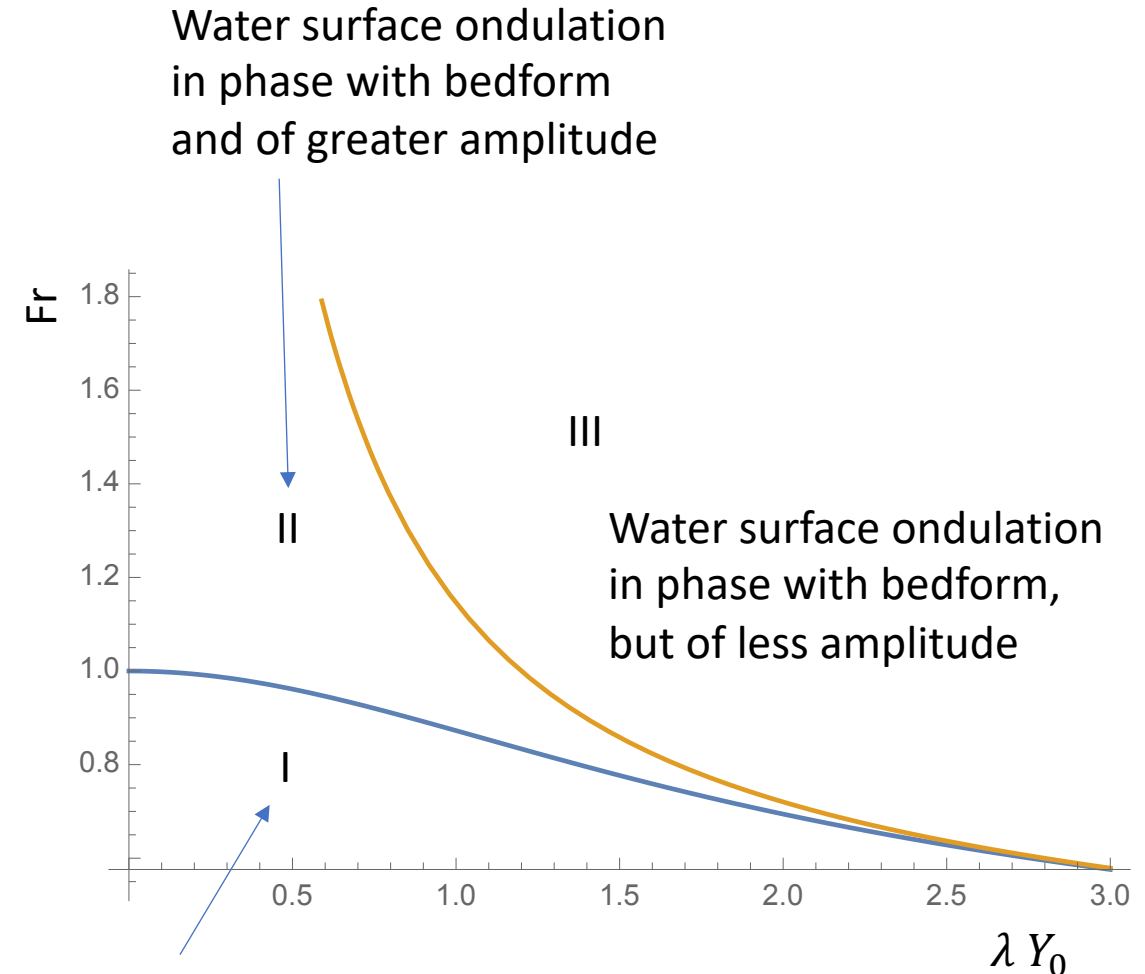
$$Fr^2 < \frac{\tanh \lambda Y_0}{\lambda Y_0}$$

- II – Lower super critical

$$\frac{\tanh \lambda Y_0}{\lambda Y_0} < Fr^2 < \frac{1}{\lambda Y_0 \tanh \lambda Y_0}$$

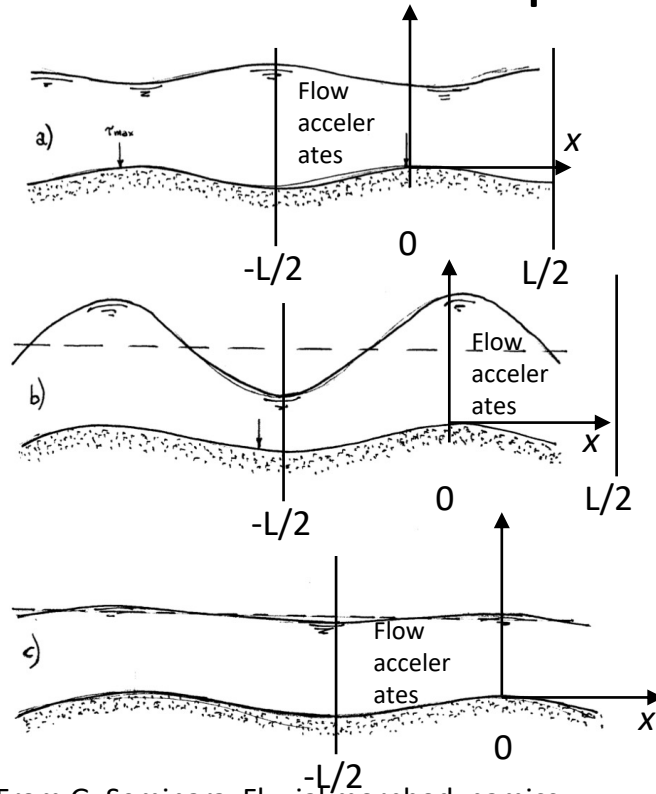
- III – Upper super critical

$$Fr^2 > \frac{1}{\lambda Y_0 \tanh \lambda Y_0}$$



Instability and small bedforms formation

- If the x-axis is on top of the (bed) crest, then



From G. Seminara, Fluvial morphodynamics

Regime I
(Dunes)

Small Froude, fine sediment, dominant bedload in phase with stress, i.e for $-\frac{L}{4} < \delta < 0$; $c_{r,i} > 0$ (downstream migration))

Regime II
(Antidunes)

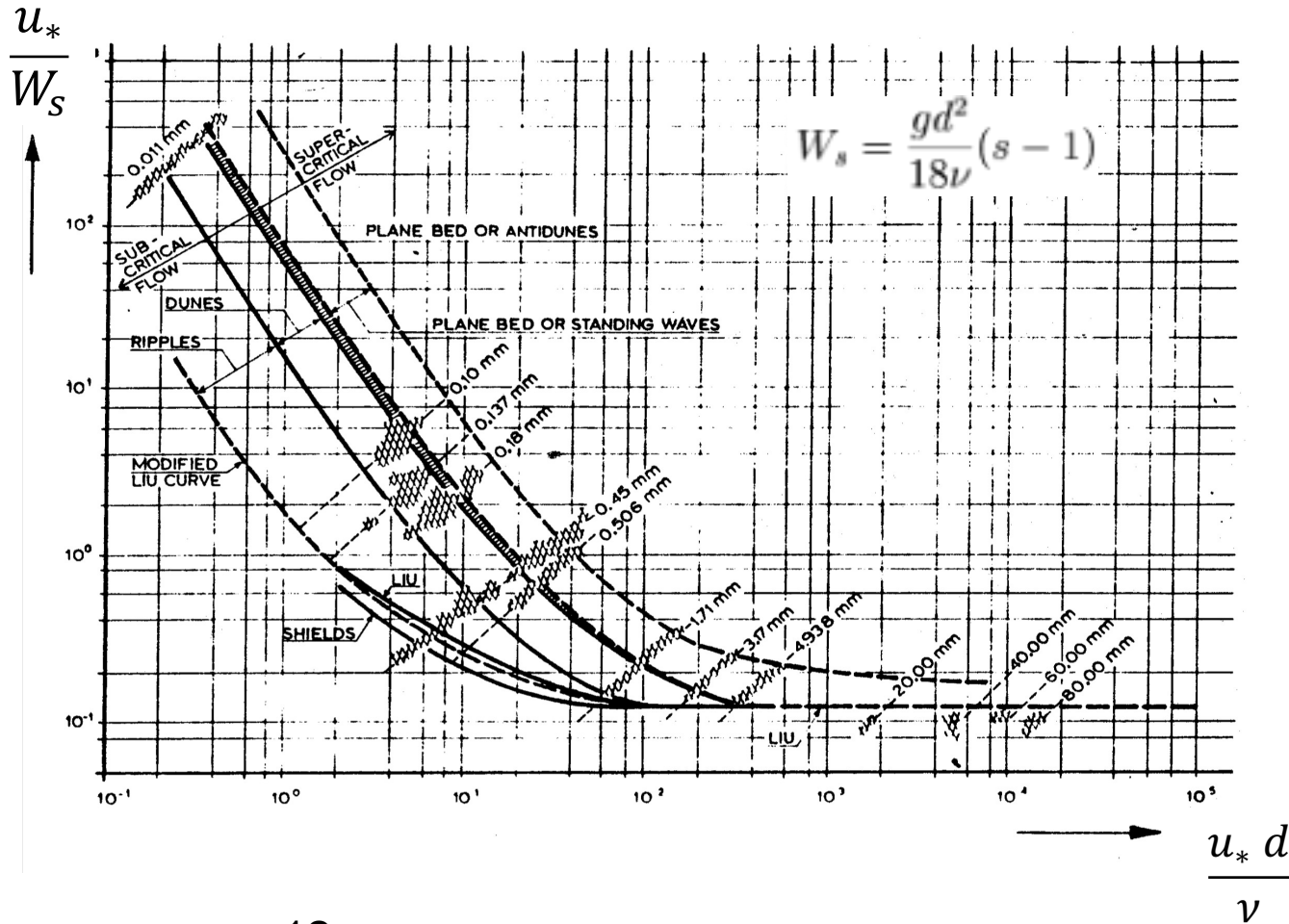
Coarser sediment: delayed transport due to sediment inertia, dominant bedload for $\frac{L}{2} < \delta < \frac{3L}{4}$; $c_i > 0$; $c_r < 0$ (upstream migration). Fine sediment: suspended load still out of phase with tension but both migration directions possible

Regime III

Transition with mechanisms depending on sediment size, formation of chute and pools, irregular regime. Stable only for coarse sediment

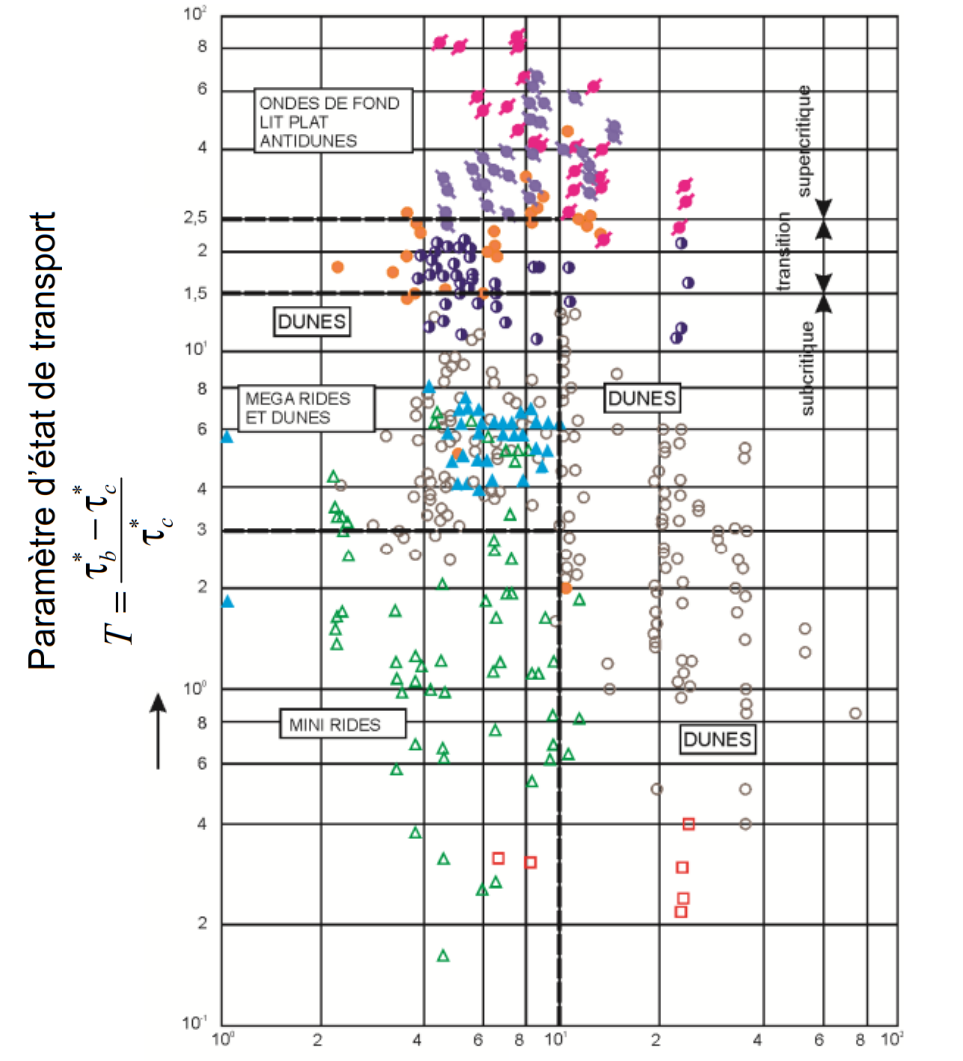
Regimes and (small) bedforms

(Simon and Richardson, 1961)



12

Van Rijn, 1984



Dunes $0 < T \leq 25$

Flat bed $T > 25$

Diamètre sédimentologique

$$d^* = d_{50} \left(\frac{g(s-1)}{\nu^2} \right)^{1/3}$$

Small-scale bedforms geometry

SUMMARY

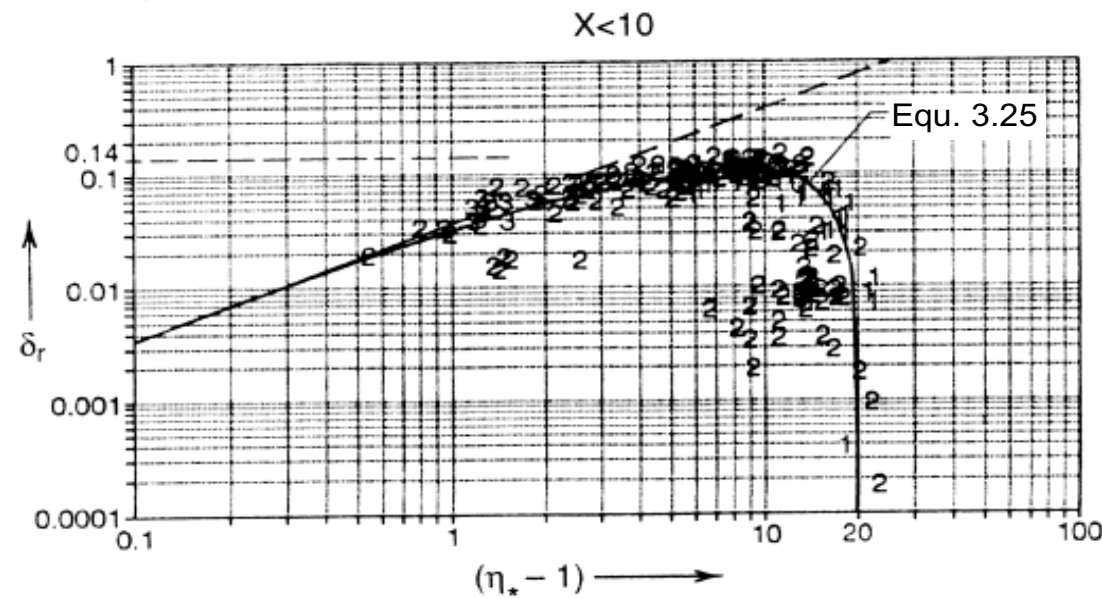
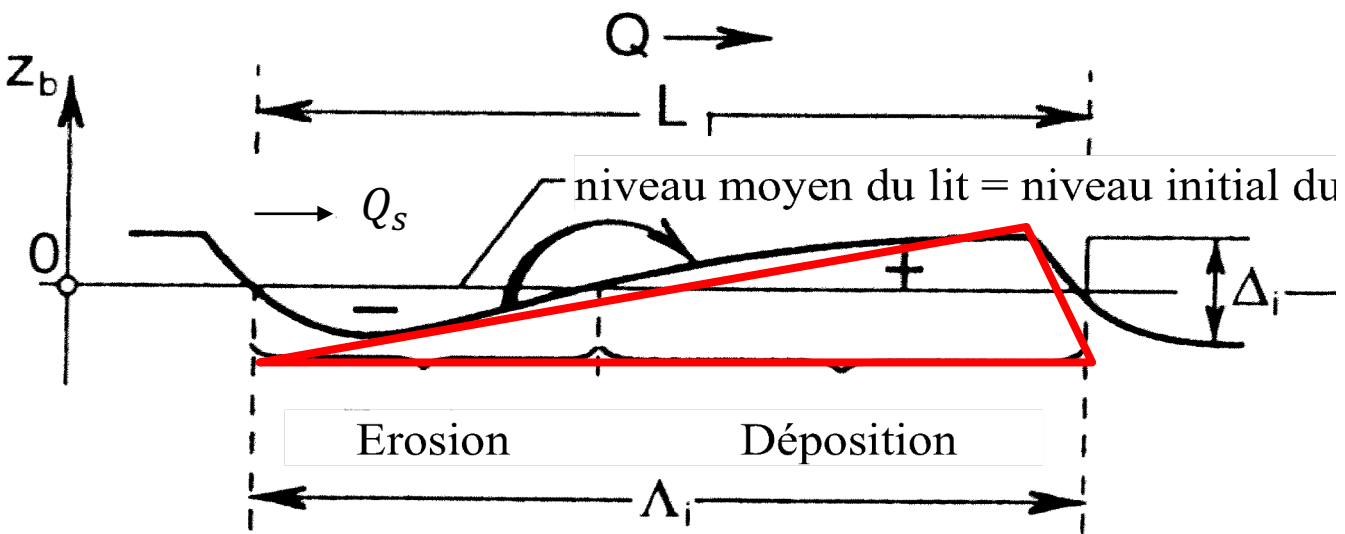
- Form at low Froude numbers in turbulent smooth-wall regime for $0.1 < Fr < 0.5$
- Waves of almost triangular shape;
- Migrate (invariably) downstream;
- Slightly asymmetric with steeper front and arcued apex;
- Amplitude smaller than water depth;
- Length of $O(10^1)$ cm



Geometric characteristics of Ripples

- Ripples form (Sumer & Bakioglu, 1984)

$$\frac{u_* d}{\nu} < 23 \div 25$$



- Shape factor $\delta_r = \frac{\Delta_r}{\Lambda_r}$

(ripples disappear for $\eta_* > 21$)

$$\eta_* = \frac{\tau_o}{(\tau_o)_{cr}} > 1$$

SUMMARY

- Dunes form in subcritical regime, i.e. $Fr < 1$
- They form under the condition $\tau_* > \tau_{*c}$
- Flat apex, invariably migrate downstream
- Amplitude $\Delta_d \approx \frac{1}{6} Y_0$
- Wavelength $\Lambda_d \approx 5 \div 10 Y_0$
- Notice that $\tau_* = \frac{Fr^2}{C^2(s-1)^d/Y_0}$ → dunes tend to disappear when d is $O(10^0)$ mm

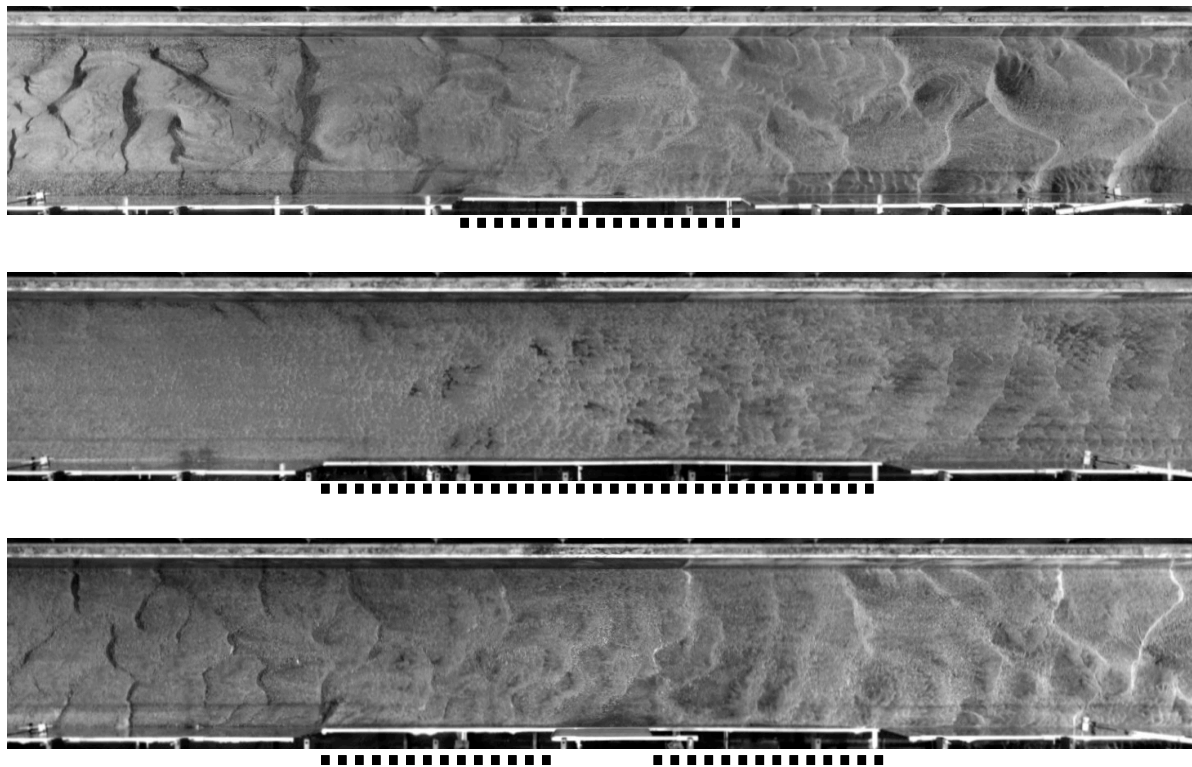


Almost rectilinear crest → 2D dunes

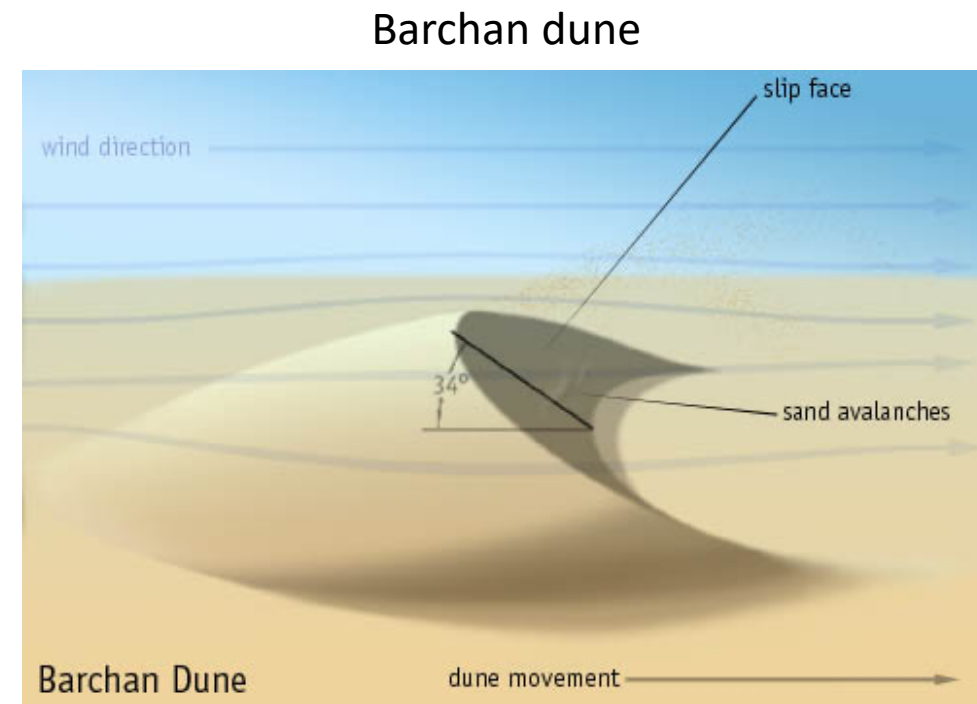


Dunes

Dunes degenerates into 3D structures



3D dunes show typically curved crest



Source: Wikipedia

Laboratory observations suggest that, in a sufficiently wide channel and provided the experiment is long enough, 2D dunes evolve invariably into 3D patterns. Theory unsettled so far.

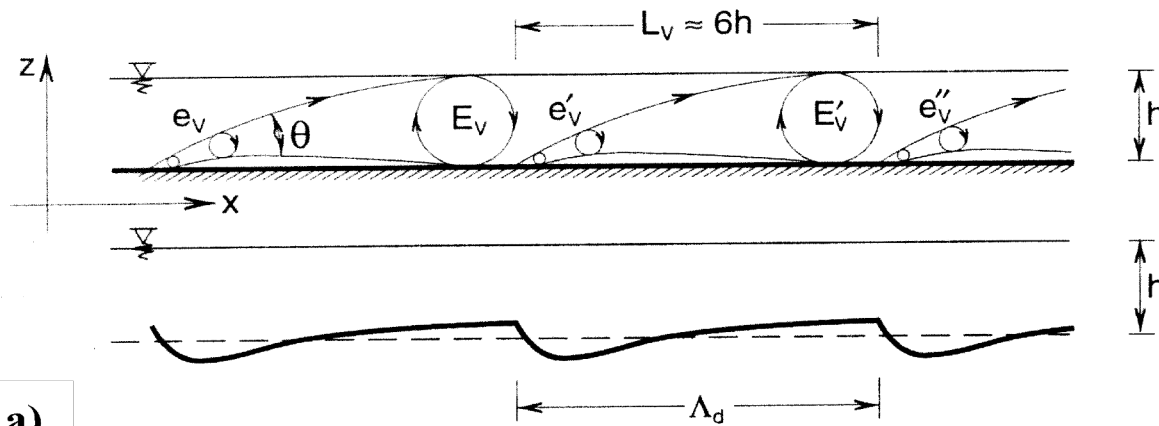
Dunes formation mechanism, x-z axes

- As Fr grows, so does turbulence intensity and vortices coalesce in gusts

$$\tan \theta \approx 1 / 6$$

$$\Lambda_d = L_V \approx 6h$$

Gusts corresponds to the evolution of the largest macro-turbulent vortex. In areas of high friction, close to the walls, stress fluctuations favour the formation of vortices. These are transported by the flow, they diffuse and combine by coalescence.

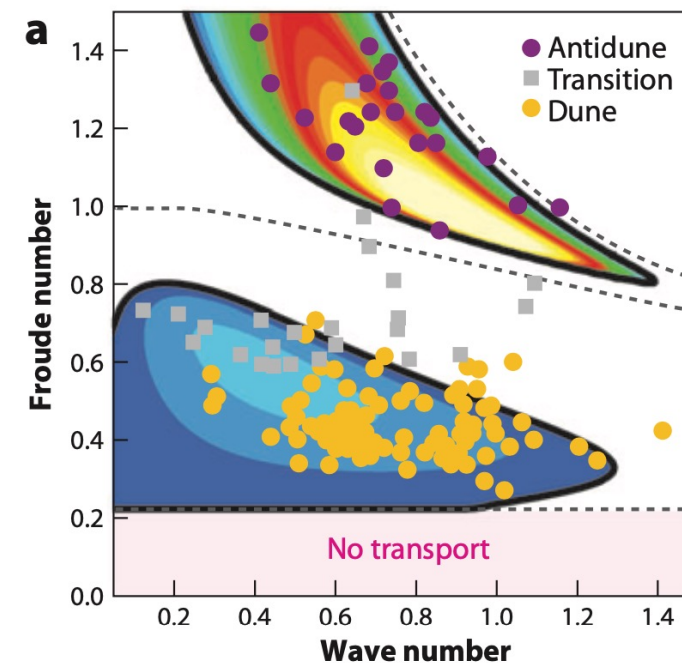


a)

$$\text{Wave number } \lambda_d = \frac{2\pi}{\Lambda_d}$$

- Large vertical turbulence

Dunes instability plot



Source: Seminara, Ann. Rev. Fluid Mech., 2010

SUMMARY

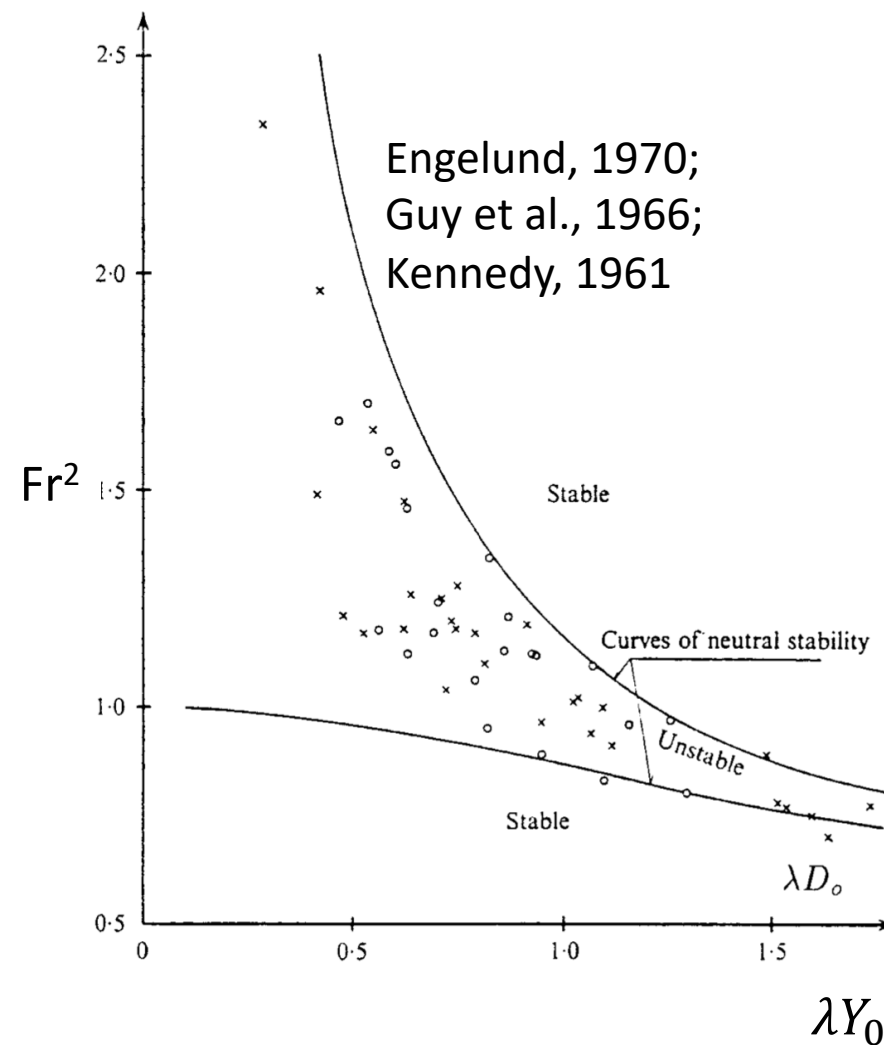
- Water surface is undulated and in phase with bed undulations
- Antidunes typically migrate upstream (not invariably)
- Quite typical sinusoidal shape for upstream migrating forms
- Flow field is not characterized by boundary layer separation

Antidunes



Antidunes formation

- Remember the regime domains, I, II, III
- Many authors have performed experiments to validate them
- Antidunes form in the unstable domain, i.e. Regime II



Large-scale (mesoscale) bedforms

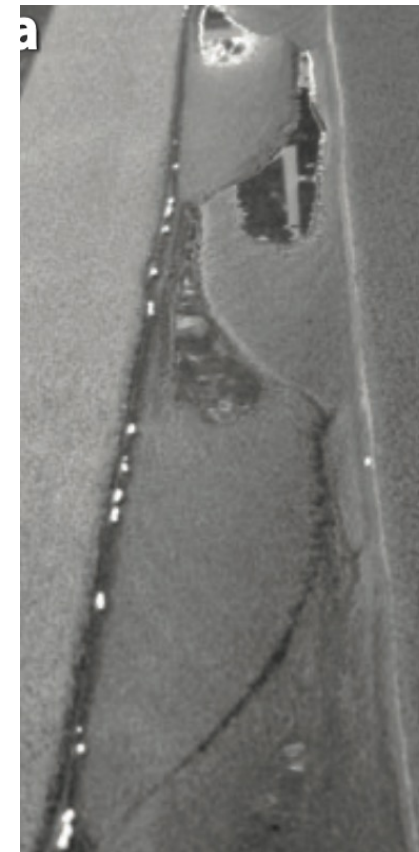
Sediment bars

SUMMARY

- Sand and gravel bars are the result of a free morphodynamic instability
- In straight channel (free) bars appear typically in alternated and very periodic patterns of well defined wavelength
- Free bars migrate downstream
- Free bars form in shallow streams
- In erodible banks free bars interact with bank erosion until they “resonate”



Rhine river

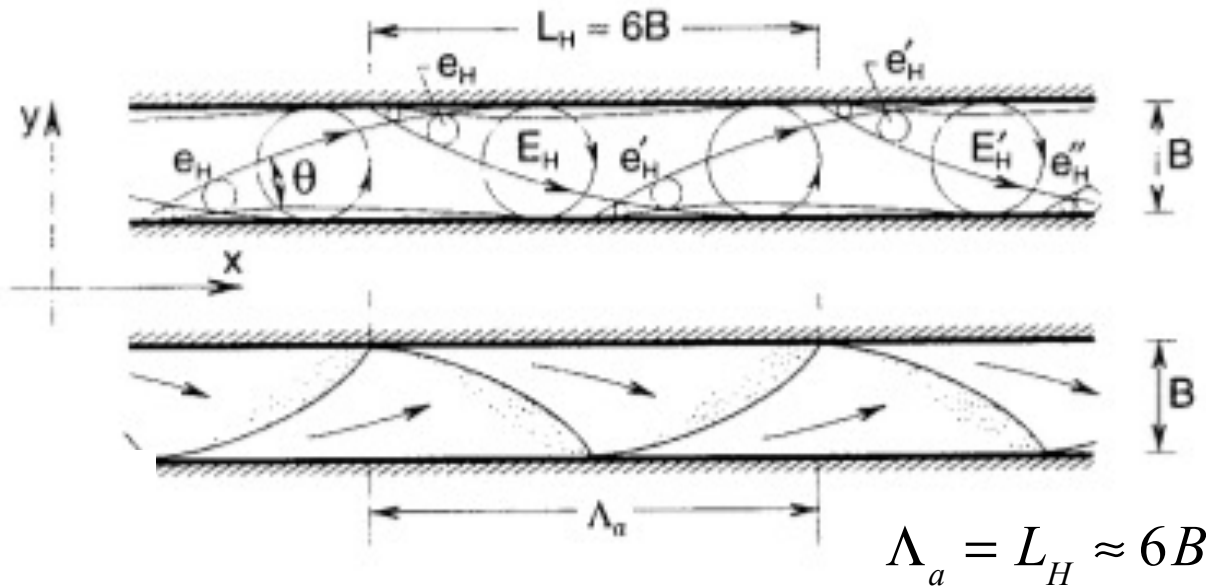


Laboratory

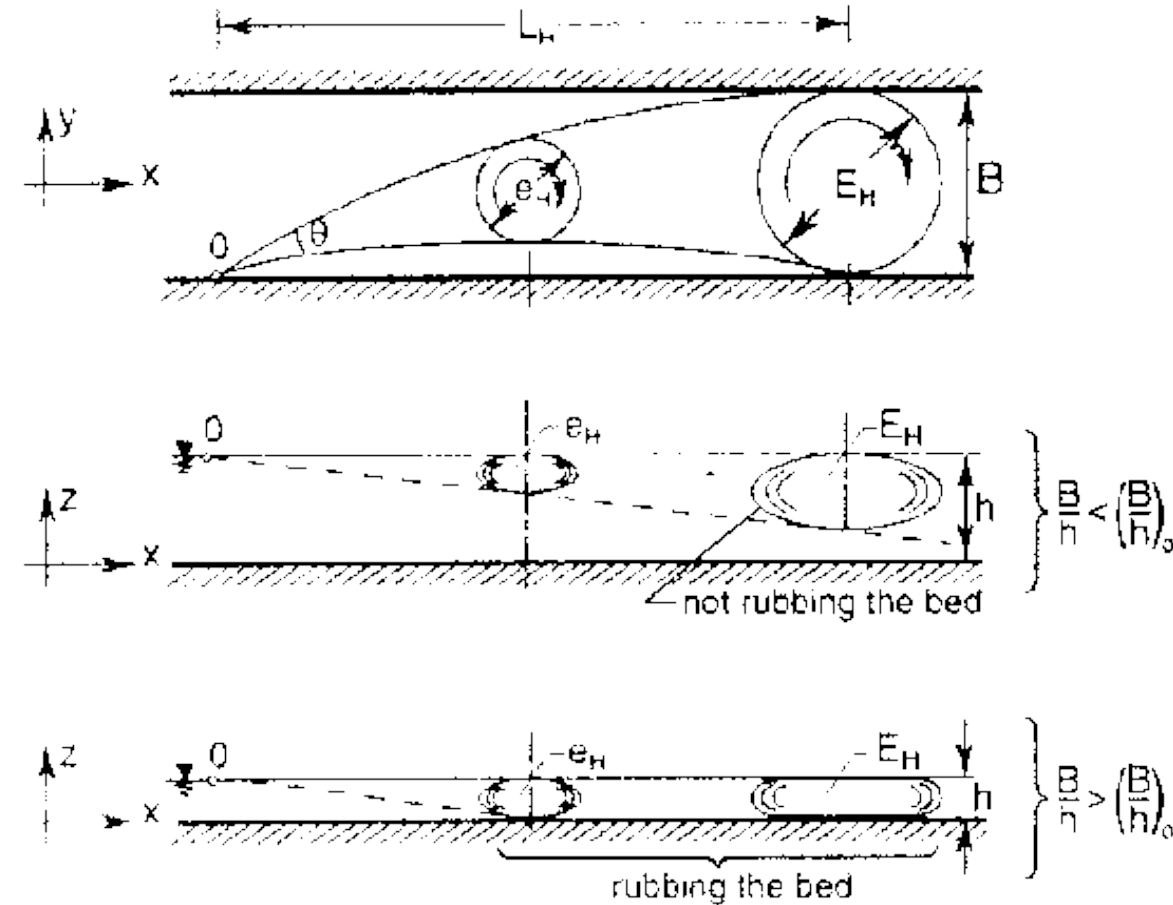
Phenomenological process of bar formation, x-y vs y-z planes

- Turbulent gusts in the x-y plane help explaining the formation of bars

$$\tan \theta \approx 1 / 6$$

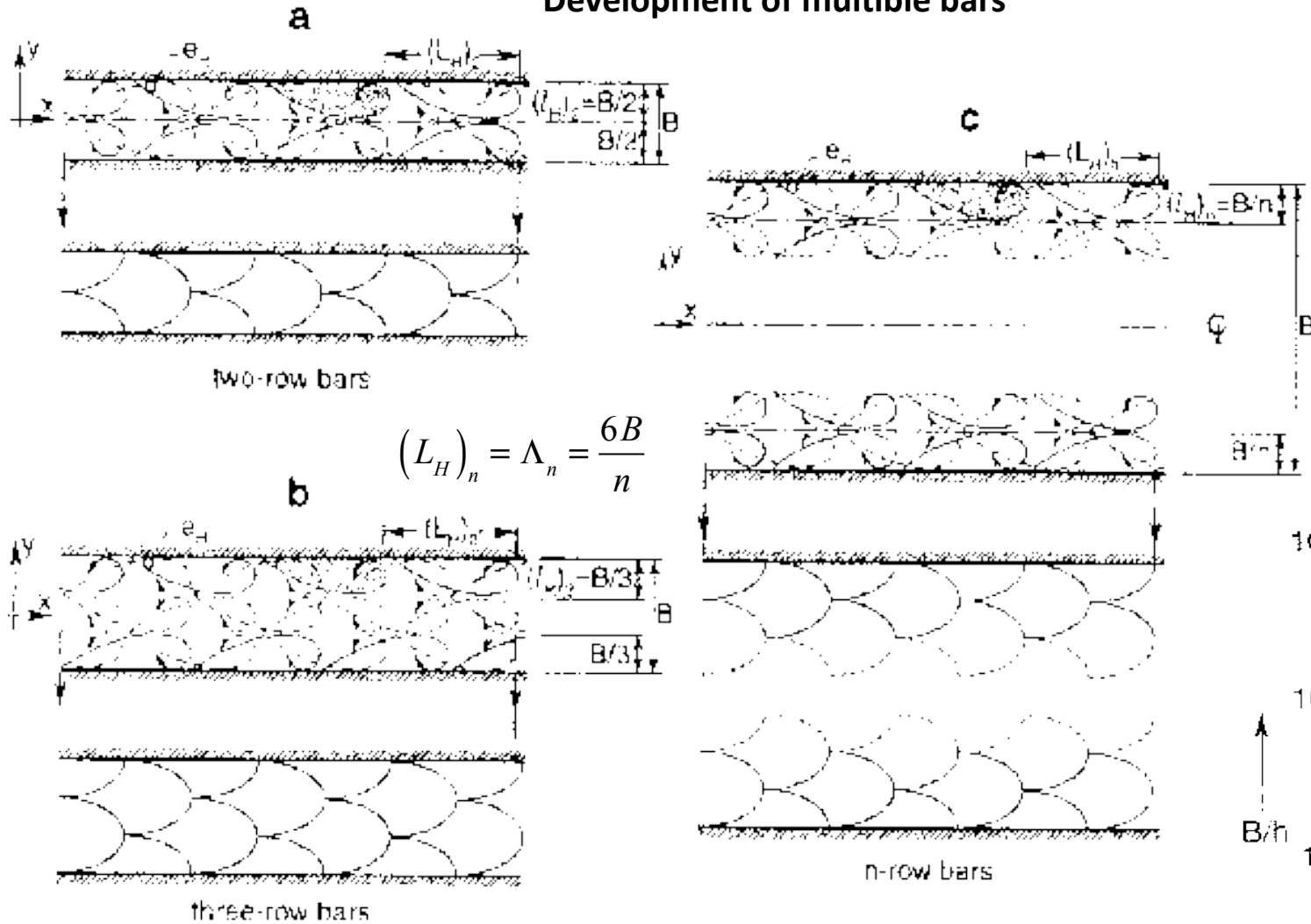


- The ratio B/Y_0 determines special horizontal turbulent vortexes



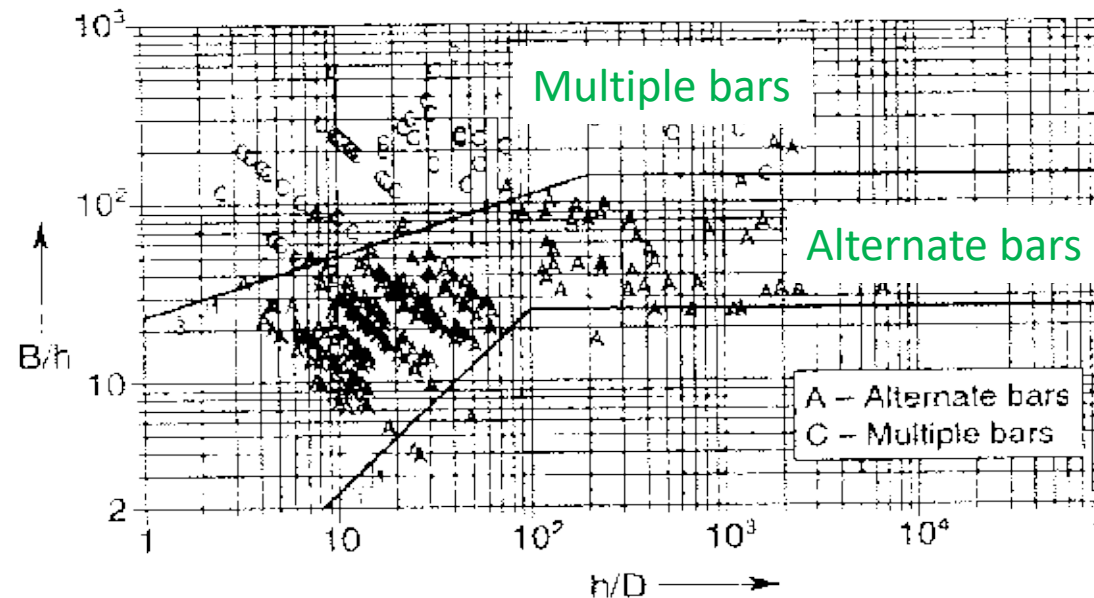
Phenomenology of multiple bars formation on the plan x-y

Development of multiple bars



When B/h is larger than $(B/h)_0$, but smaller than a certain value $(B/h)_1$, the eddies e_H touch the bed before they reach their size B . In this case, the eddies developed from both banks meet in the middle of the stream and double alternating banks are formed (Fig. a)

If $B/h > (B/h)_1$. The next increment of the ratio B/h leads to triple alternating banks, ..., n (Fig. b, c).



Morphodynamic stability analysis and free bar formation

$$\frac{\partial U}{\partial t} = -U \frac{\partial U}{\partial s} - V \frac{\partial U}{\partial n} - \frac{1}{F_0^2} \left[\frac{\partial Y}{\partial s} - \frac{\partial \eta}{\partial s} \right] - \beta \frac{\tau_s}{Y}$$

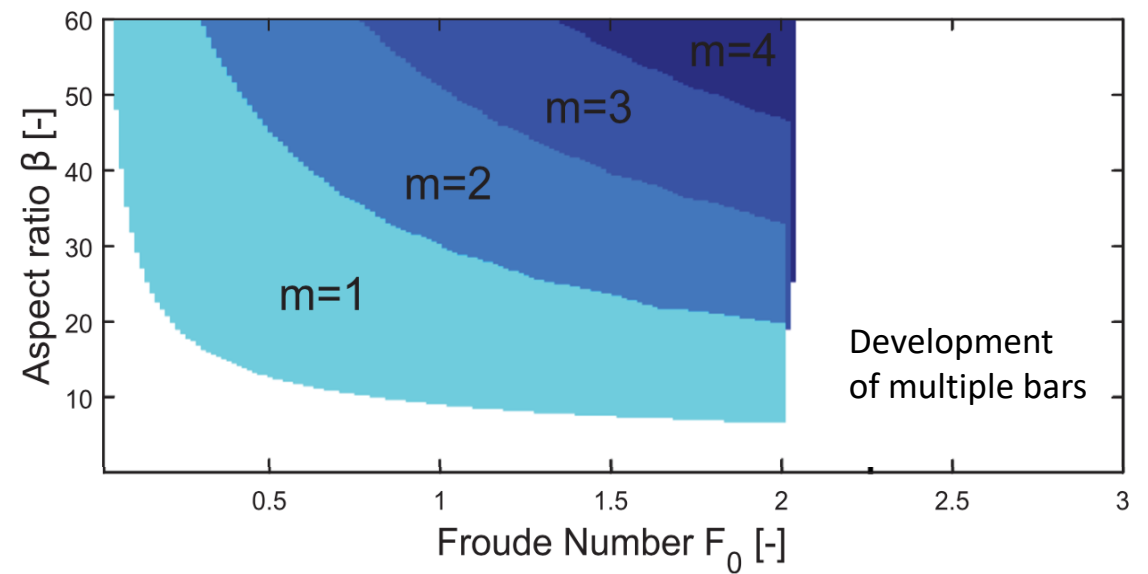
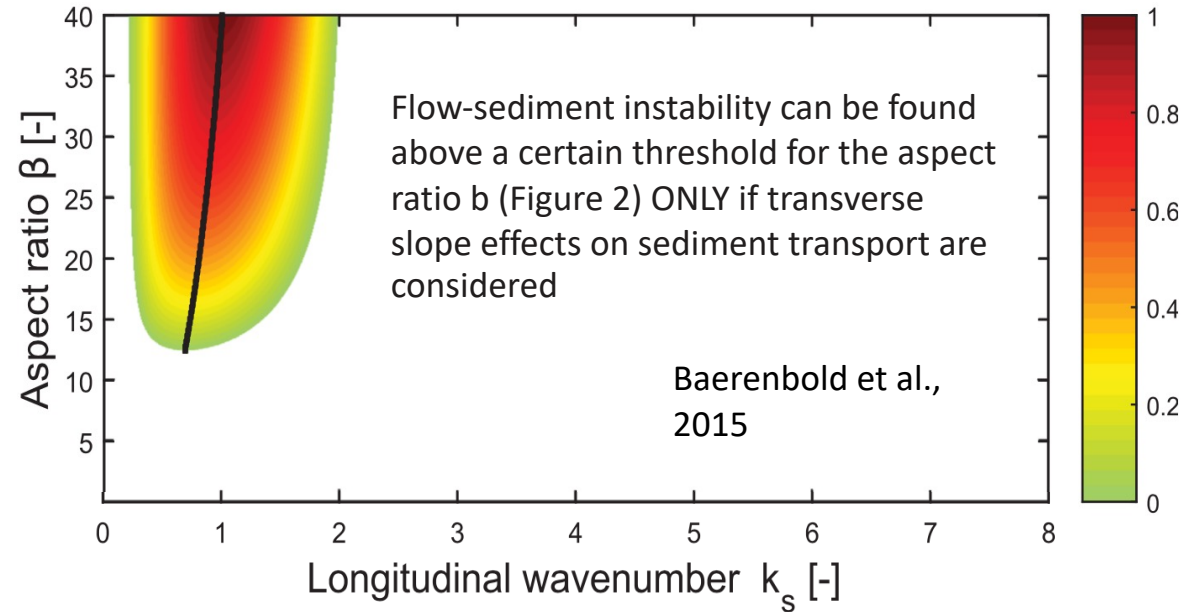
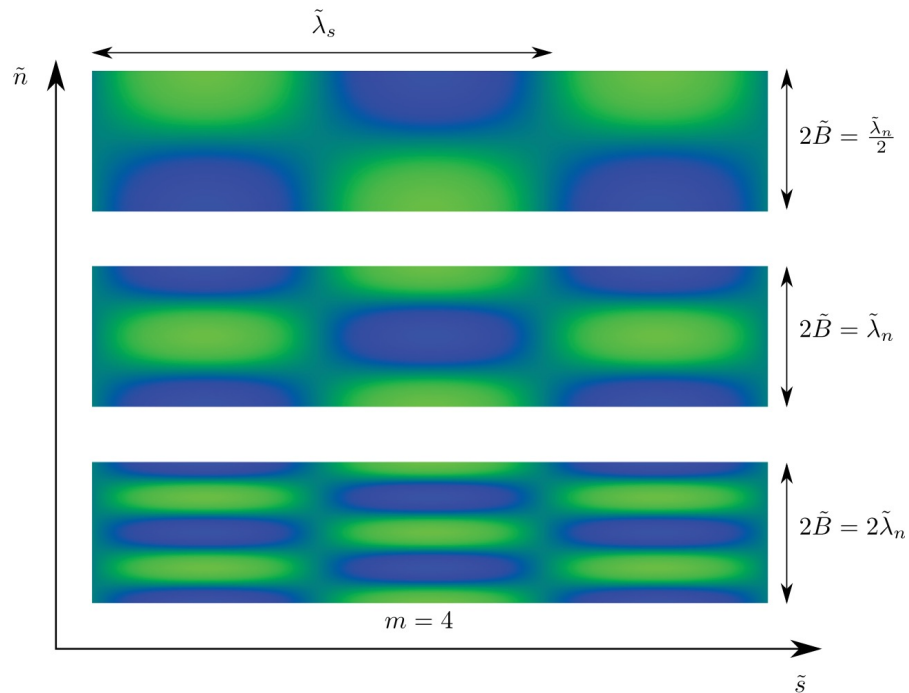
$$\frac{\partial V}{\partial t} = -U \frac{\partial V}{\partial s} - V \frac{\partial V}{\partial n} - \frac{1}{F_0^2} \left[\frac{\partial Y}{\partial n} - \frac{\partial \eta}{\partial n} \right] - \beta \frac{\tau_n}{Y}$$

$$\frac{\partial Y}{\partial t} = -\nabla \cdot (Y\mathbf{V})$$

Colombini et al., 1987

$$\frac{\partial \eta}{\partial t} = -\gamma \nabla \cdot (\|\mathbf{V}\|^3 \{\cos \delta, \sin \delta\})$$

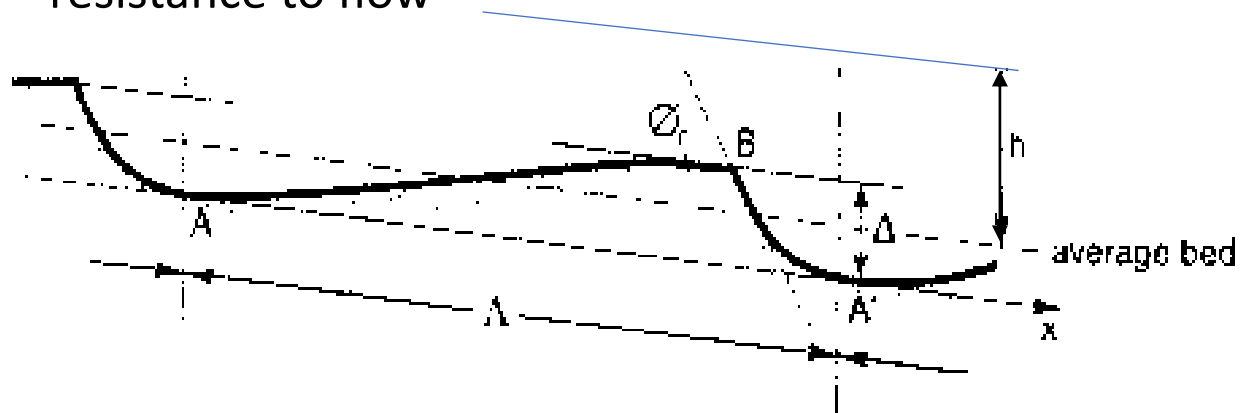
$$\beta = \frac{B_0}{Y_0}$$



Resistance to flow induced by bedforms

General considerations

- When the surface of the moving bed is marked by the presence of ripples, dunes or other periodic shapes in the x-direction of the flow, its surface (skin) roughness k_s is not sufficient to characterise resistance to flow



- One first defines $\bar{\tau}_0 = \frac{F_D}{B\Lambda} = \rho ghj$
- This fomulation is however not convenient for prompt use (j is unknown)

- Bed friction can better be decomposed in two parts, one due to (grain) friction and the other due to bed ondulation:

$$\tau_o = (\tau_o)_f + (\tau_o)_\Delta$$

- The idea is to express the friction coefficient c_f et c_Δ by adopting the friction velocity, u_*

$$u_* = \sqrt{\frac{\tau_o}{\rho}} = \sqrt{gSR_h}$$

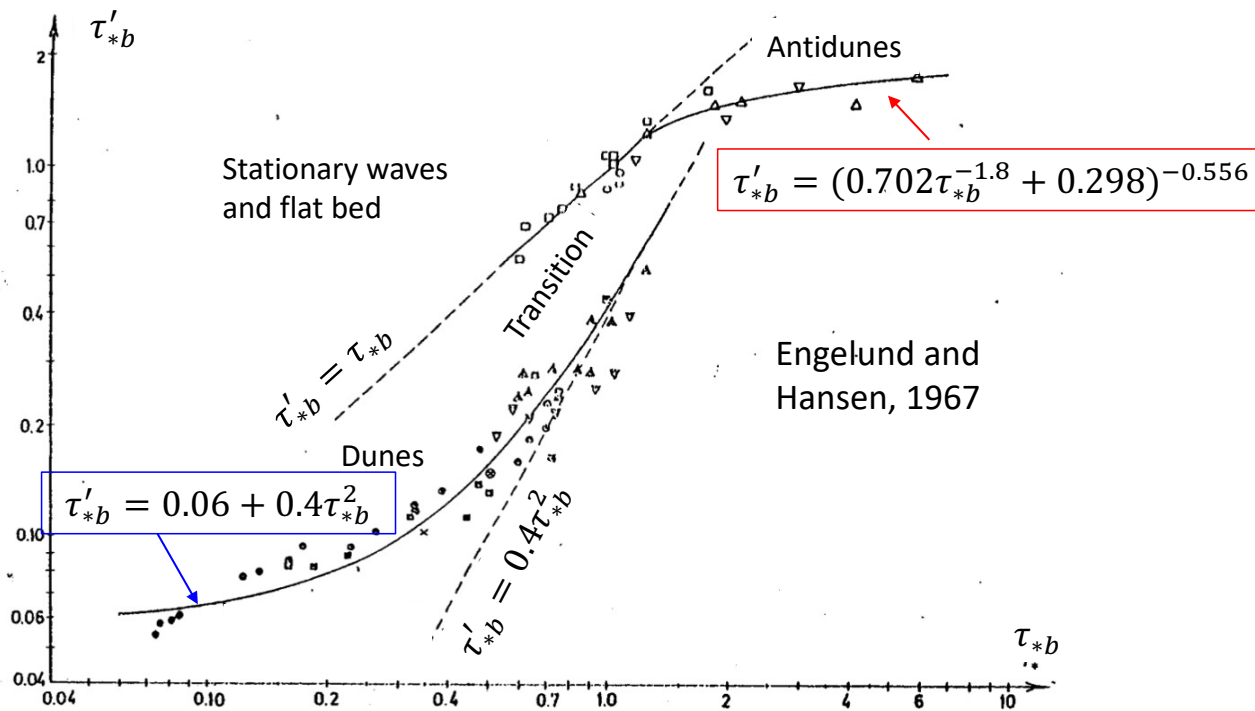
and the relationship between mean velocity and

$$\text{friction factor } c_f = \frac{U}{u_*} = \sqrt{\frac{8}{\lambda}}$$

Bedform resistance formulations

Several methods proposed:

- (Hans Albert) Einstein formulation (1950), we do not see it here
- Engelund and Hansen (1967)



- Van Rijn method (1984). This method is simpler and explicit as it bases on the relationships (amplitude and length) that characterize dunes' geometry

We compute c_f as for the flat bed and include bedform effect in the k_s .

If Λ_d and Δ_d are known then assess the equivalent dimensionless Chézy friction factor as

$$c_f = 2.5 \ln \left(\frac{12Y_0}{k_s} \right),$$

with

$$k_s = 3d_{90} + 1.1\Delta_d [1 - \exp(-25 \Delta_d / \Lambda_d)]$$

River ecomorphodynamics and bioengineering

(ENV-418, A.Y. 2025-26)

4ETCS, Master option

Prof. Paolo Perona

Platform of Hydraulic Constructions

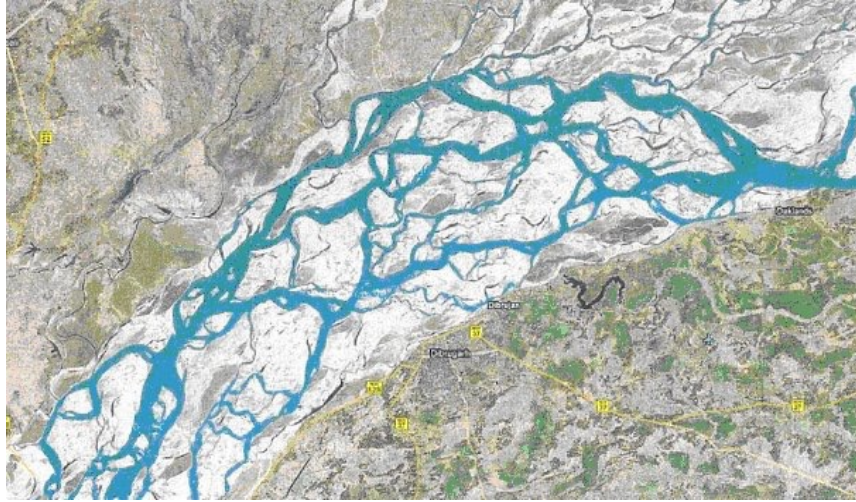


Lecture 10-3: River planforms
and regime theories

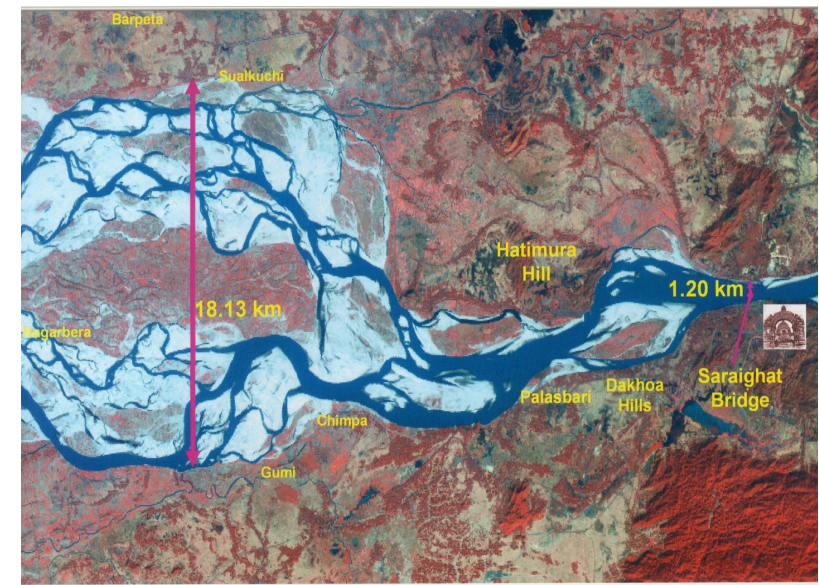
List of contents

- Examples of river planform styles or regimes
- River morphodynamics: stability and evolution
- River attributes
- Average hydraulic and geometrical quantities (equilibrium)
- The regime theory: empirical formulas
- Morphodynamic theory of river meandering
- The rationale morphodynamic approach
- Swiss approach to “the room for river” concept

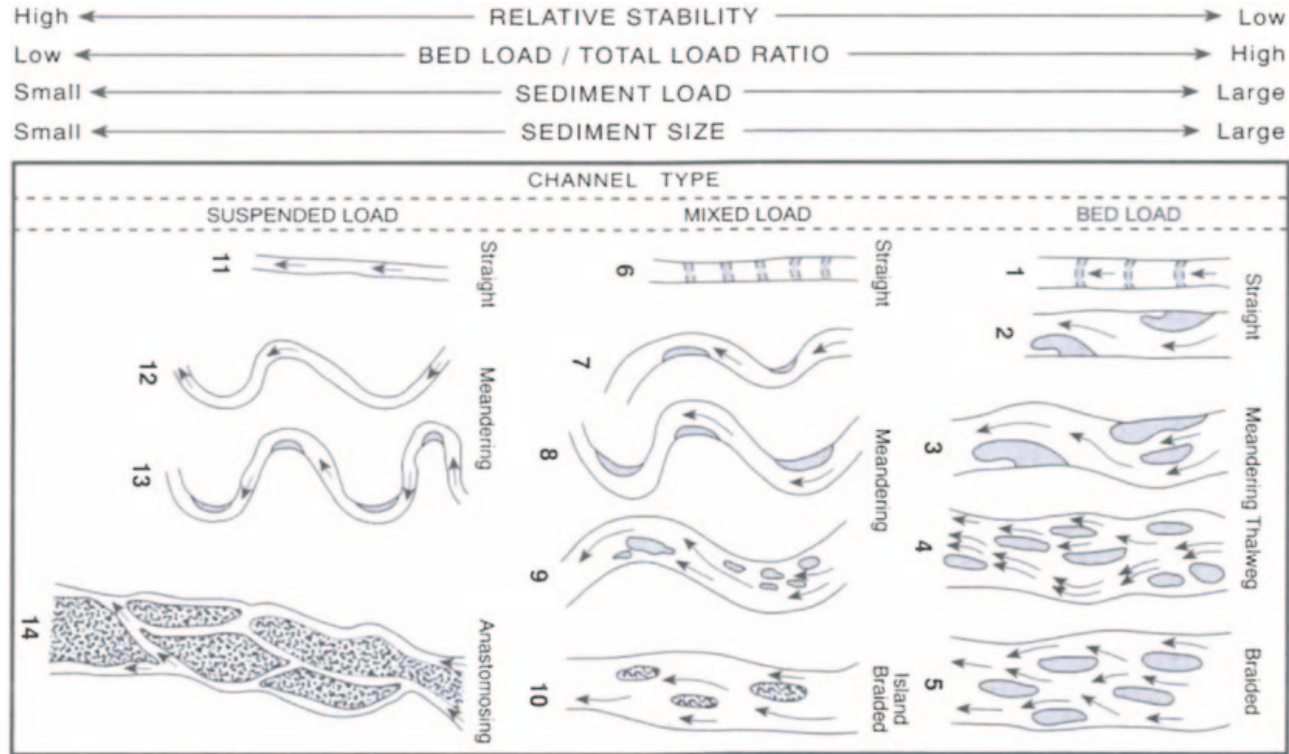
River planform styles (or regimes)



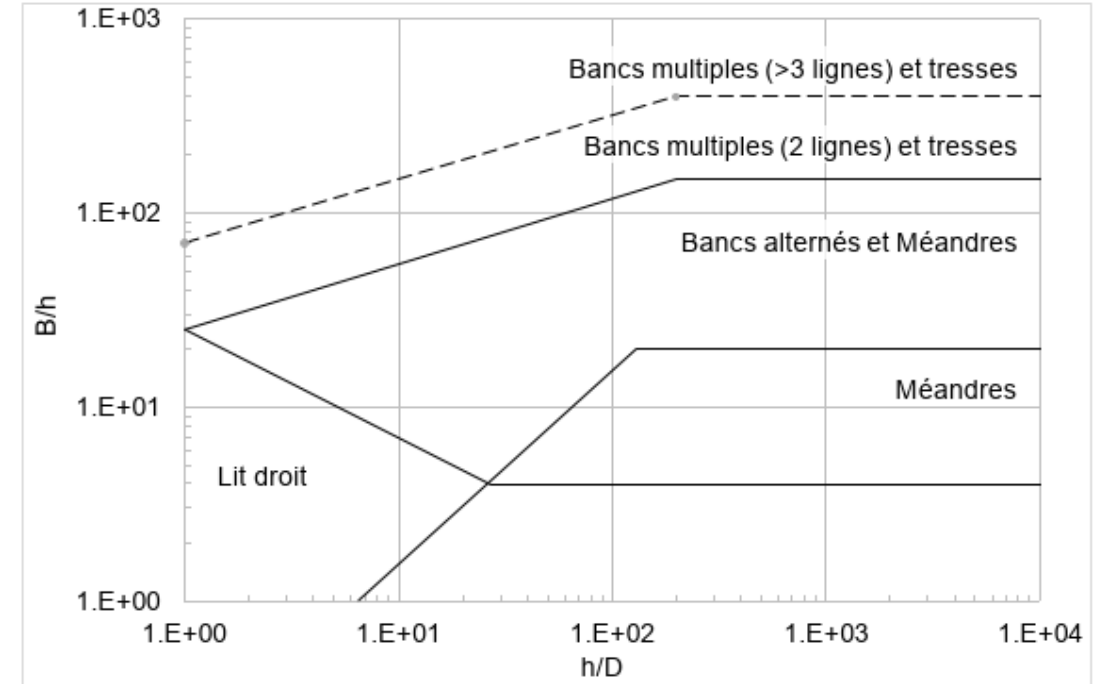
- Alluvial watercourses tend to develop their own morphology, which is called a regime or 'style'. (Regime is here not to be confused with 'hydrological regime' though the two are related);
- At this stage of development, the watercourse has reached a statistically stable state which, in theory, does no longer varies over time in average.
- The regime situation thus requires minimal intervention for its maintenance, but may change if the fluvial system is perturbed (e.g., by changing hydrological flow regime).



Planimetric styles or forms



Source: Ahmari et Da Silva (2011)

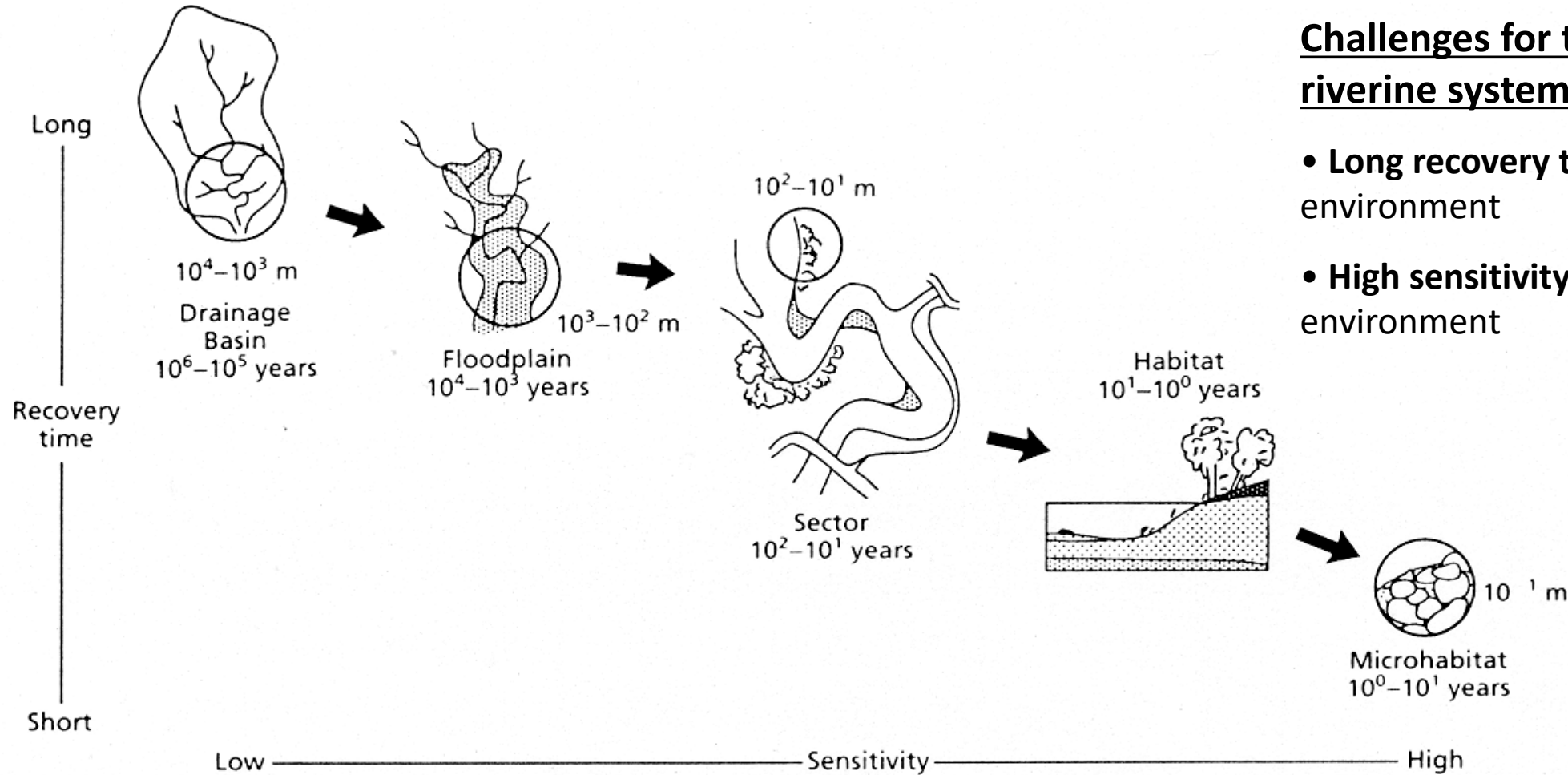


B = natural riverbed width or regime width

h = average water depth

D = characteristic sediment size (d_m)

Morphological changes: recovery time and sensitivity

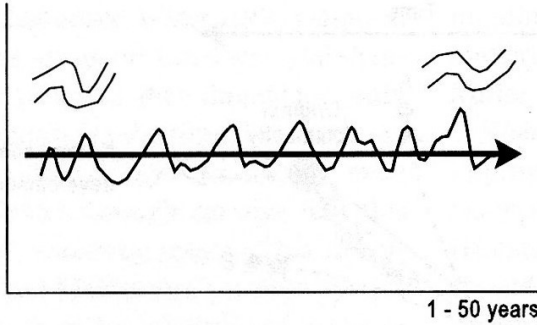


Challenges for the management of riverine systems:

- Long recovery time of the physical environment
- High sensitivity of the biological environment

Physical environment + biological environment = river "health"

Dynamic equilibrium in nature

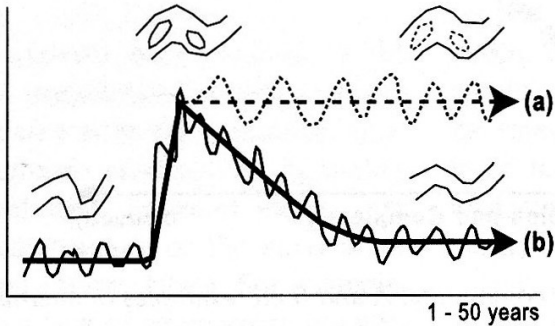


Channel in equilibrium with water and sediment load. Minor morphological change (scour/fill)

A stream is in **equilibrium** (*graded stream*) when over a period of time, its slope and channel characteristics adjust so that the available water discharges have **just the energy** required for the transport of the sediment load from the drainage basin

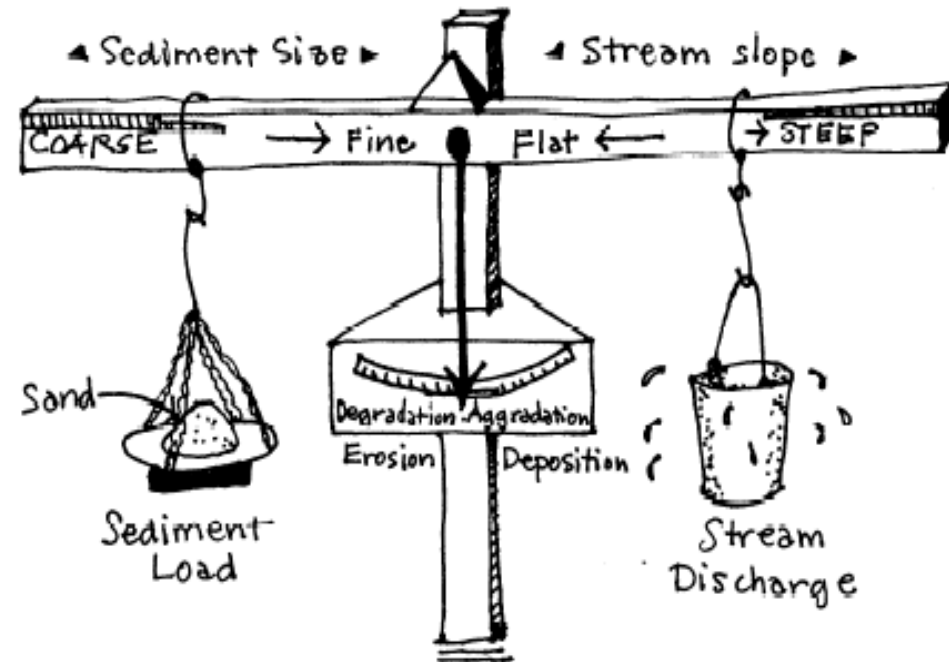
Stability is a condition in which **sediment** sizes and loads, **water** discharges, and **channel** geometry and slope are **in balance**

CHANNEL MORPHOLOGY

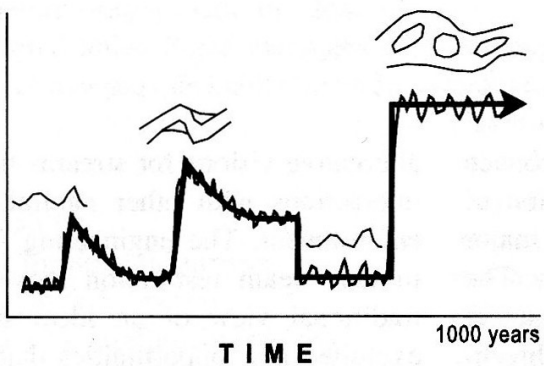


Threshold effects:
 (a) No return to original state - morphology changed to new equilibrium
 (b) Adjustment to previous equilibrium

Lane's equation (1955): $Q_s d_s \propto QS$



This equation is by no means a quantitative equation and should not be used for design purposes!



Longer term planning of river renaturalization must expect dynamic adjustments

Evolution of a water course as a function of $f(Q_{s,in})$

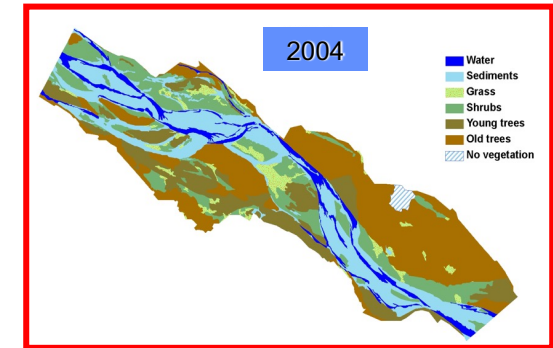
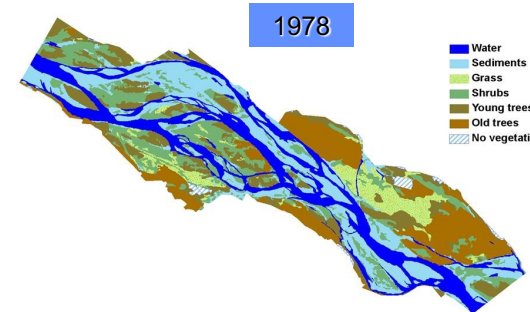
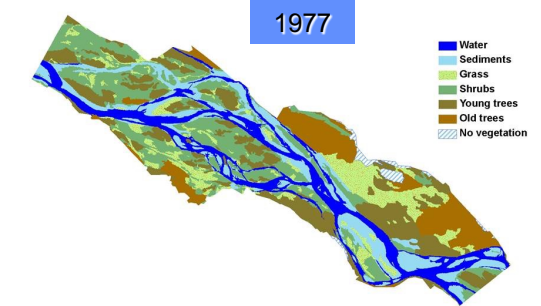
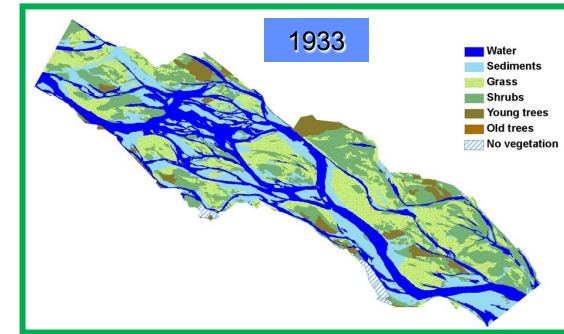
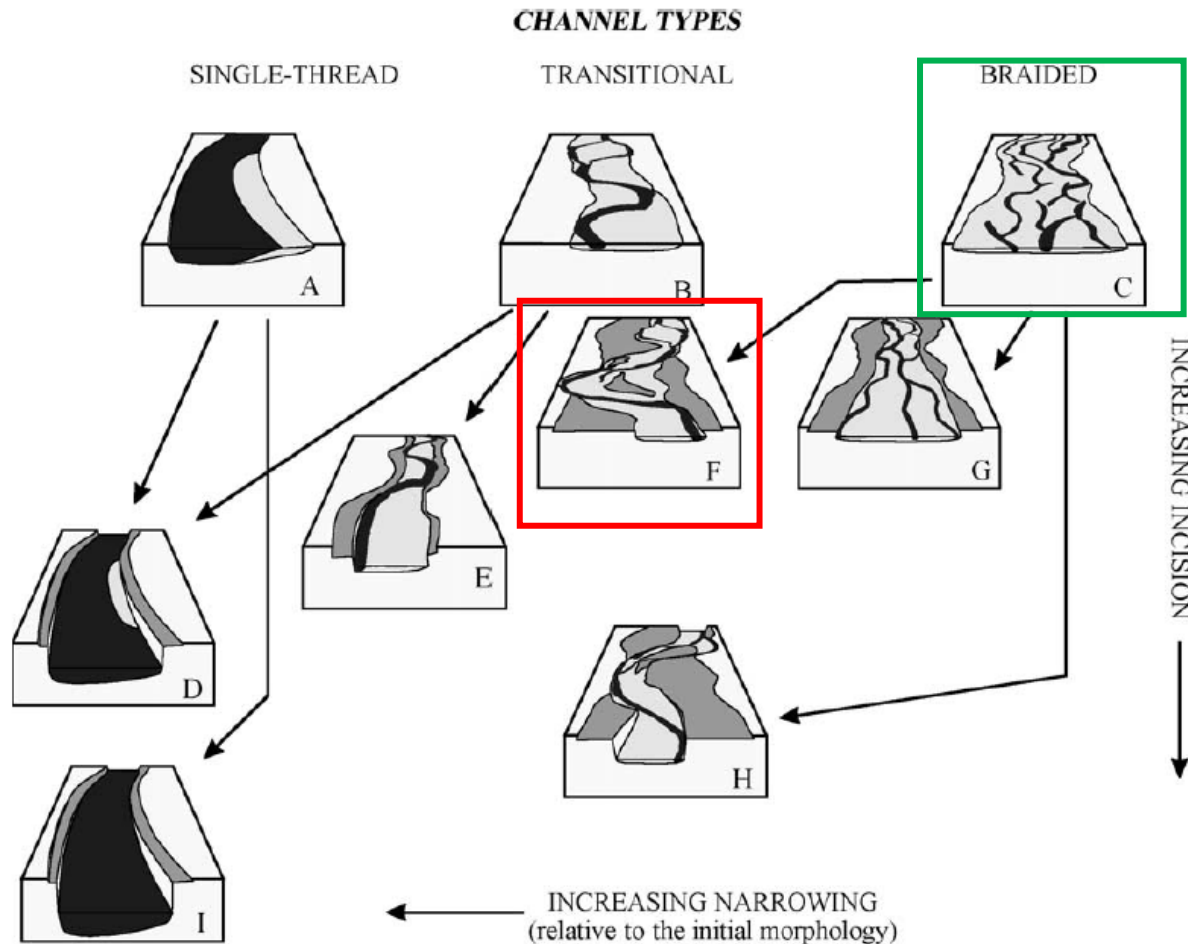
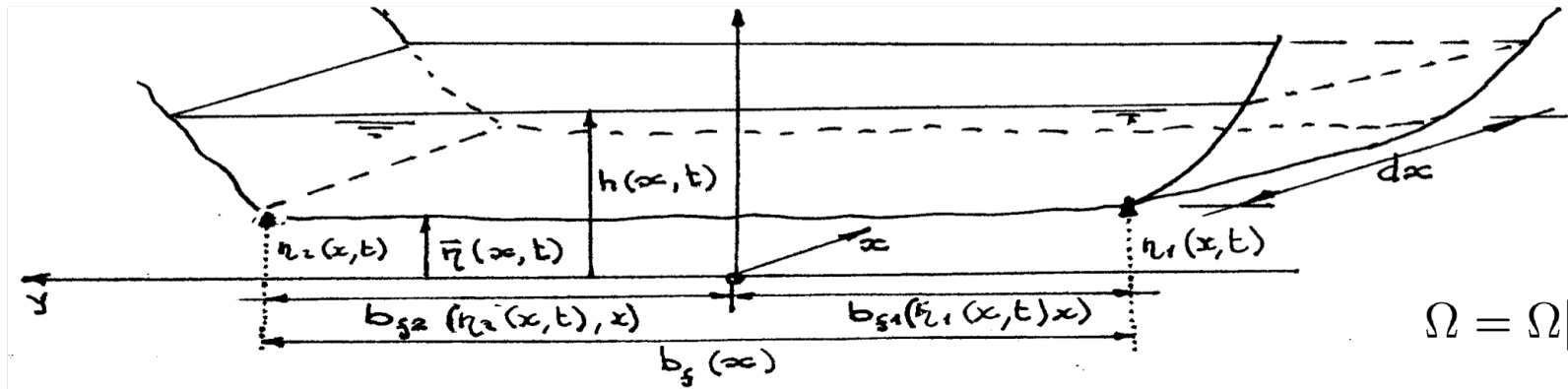


Figure 1. Ajustements de morphologie causés par une diminution des apports solides, à partir de trois conditions initiales (A : lit unique « *single-thread* », incluant les rivières rectilignes et rivières sinueuses, en méandres ; B : lit mixte transitoire « *transitional* », entre A et C, incluant les bancs alternés ; C : lit en tresses « *braided* »), en fonction du degré d'incision et de rétrécissement du lit (Surian and Rinaldi, 2003). Différents degrés d'incision et de rétrécissement (« *narrowing* ») relatifs à la morphologie initiale peuvent avoir lieu.

Mathematical morphodynamics



From Seminara G., Meccanica del trasporto dei sedimenti. Genova

$$\Omega = \Omega[h(x, t), \bar{\eta}(x, t), x]$$

Equilibrium conditions provide already interesting hints about riverbed morphology between two sections 1 and 2 (prismatic channels)

$$(1 - p)b_f \frac{\partial \bar{\eta}}{\partial t} + \frac{\partial}{\partial x} (b_f \bar{Q}_{sx}) = 0$$

Exner

$$Q = \text{constant}$$

$$\Omega_{,t} + Q_{,x} = 0$$

Flow continuity

At equilibrium



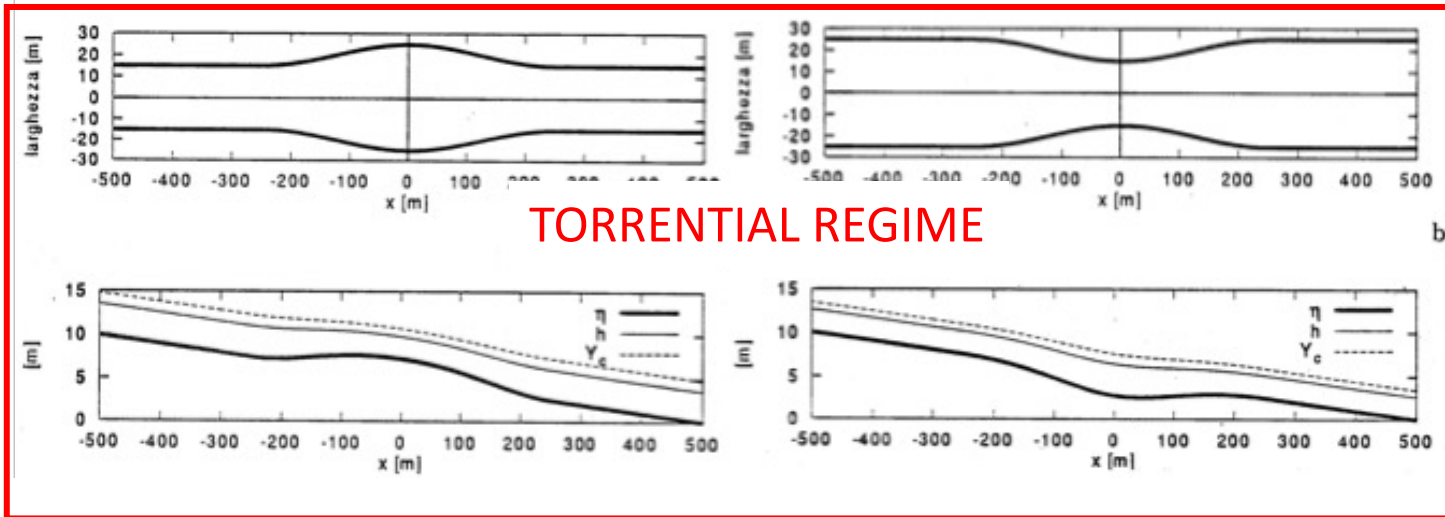
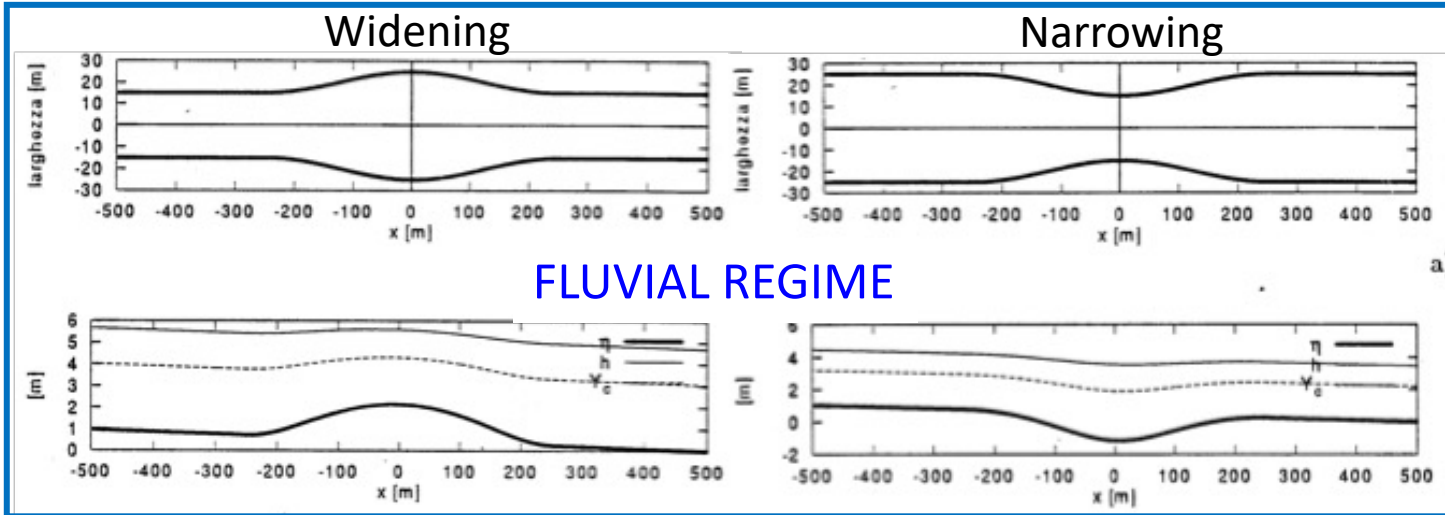
$$\frac{d}{dx} (b_f Q_s) = 0$$

$$Q_{,t} + \left(\frac{\beta Q^2}{\Omega} \right)_{,x} + g\Omega h_{,x} + \frac{Q^2}{\Omega C^2 R} = 0$$

Flow momentum

$$\frac{d}{dx} \left(\beta \frac{Q^2}{\Omega} \right) + g\Omega \frac{dh}{dx} + \frac{Q^2}{\Omega C^2 R} = 0$$

Channel widening/narrowing



$$\left(\frac{Y_1}{Y_2}\right) = \left(\frac{b_2}{b_1}\right)^{8/11}$$

$$\left(\frac{i_{f2}}{i_{f1}}\right) = \left(\frac{b_2}{b_1}\right)^{14/33}$$

- Channel widening: $b_2/b_1 > 1$
 - $Y_1 > Y_2$
 - $i_{f2} > i_{f1}$
 - Water depth decreases; slope increases
- Channel narrowing: $b_2/b_1 < 1$
 - $Y_1 < Y_2$
 - $i_{f2} < i_{f1}$
 - Water depth increases; slope decreases

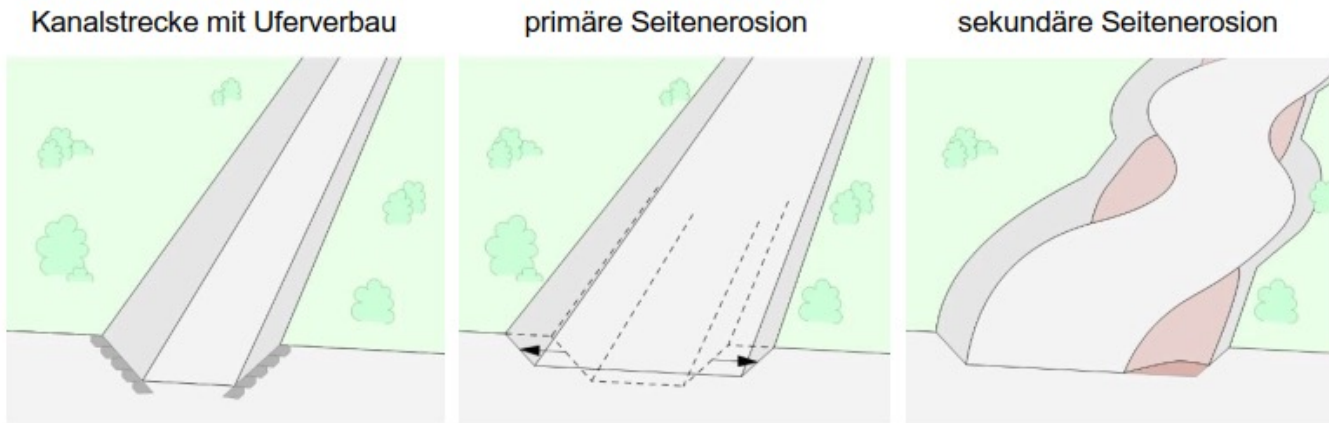
From Seminara G., Meccanica del trasporto dei sedimenti. Genova

Planimetric evolution of water courses

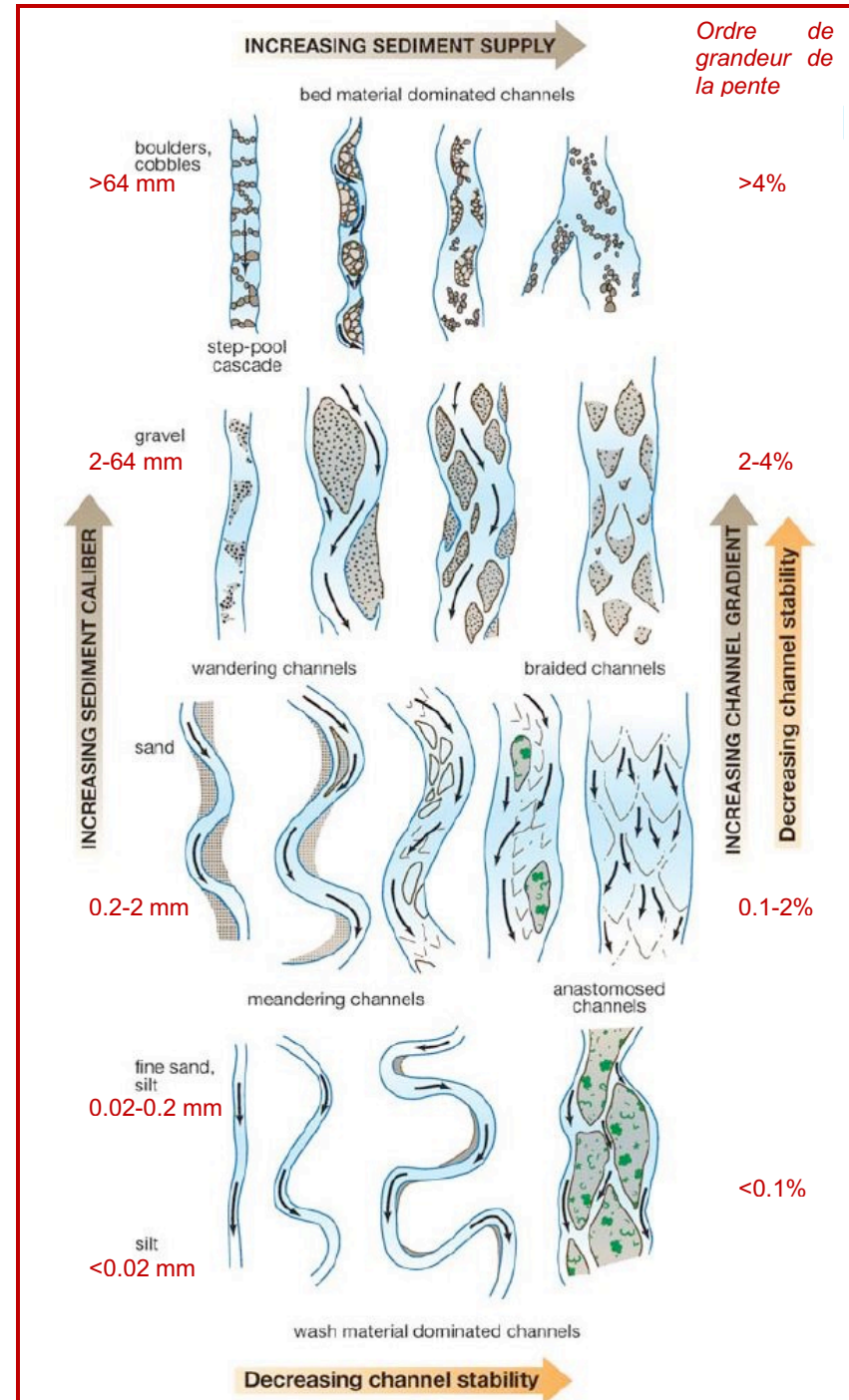


Planimetric evolution

- Primary vs secondary erosion
 - Primary erosion: widening up to regime river width
 - Secondary erosion: creation of different morphologies/styles



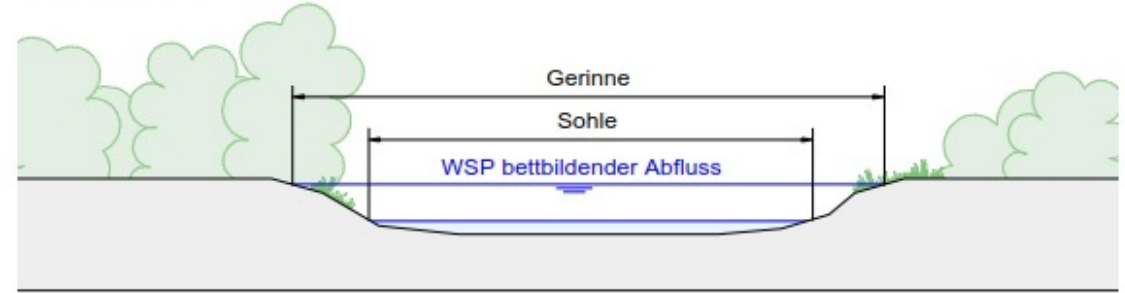
- A first interesting discussion arises already here
- There is no uniform consensus about this definition:
 - Regime width (Yalin et da Silva, 2001)
 - Equilibrium width
 - Natural river (bed) width (art. 41a OEaux)
- The Swiss approach is to consider the natural riverbed width (quite a big issue as it requires to know pristine conditions, i.e. historical data, maps, etc.)



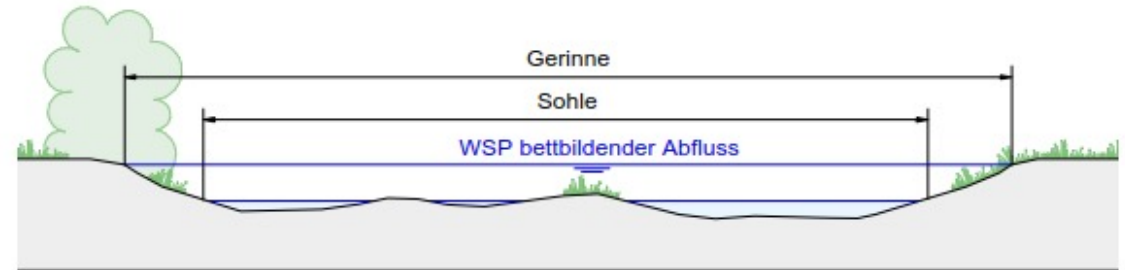
Natural riverbed width

- Natural riverbed width (here: «Sohle») and full channel width (here: «Gerinne»)
- Geomorphic relevant discharge: Floods HQ_2 à HQ_5

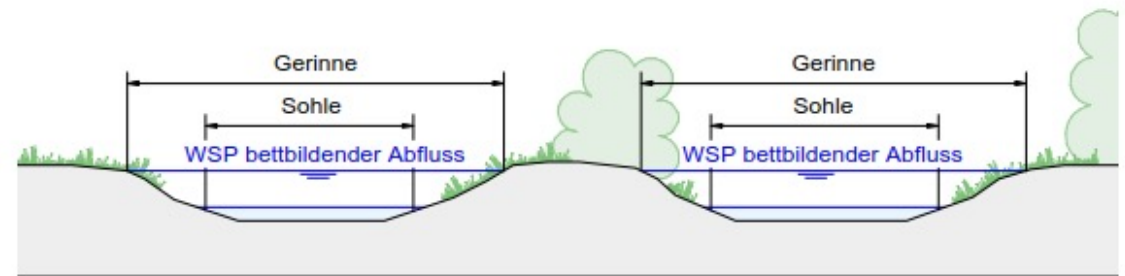
Einarmiges Gerinne:



Einarmiges Gerinne, verzweigte Struktur:



Mehrarmliges Gerinne:



Natural riverbed width

- How to determine it
 - Historical documents
 - Spatial references (e.g., max erosion, etc.)
 - Topographic analyses
 - Empirical formulas
- Homogeneous reaches
 - Similar hydraulic conditions (discharge, granulometry, substrate, etc.)

1. Historische Dokumente

Analyse historischer Karten, Luftaufnahmen, Gemälde, Fotos und Berichte



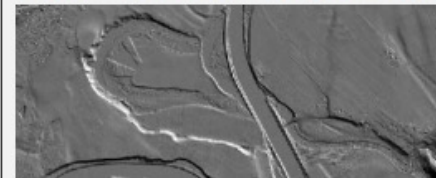
2. Naturnahe Vergleichsstrecken

Analyse von aktuellen Karten und Luftbildern, Begehung im Feld



3. Terrainanalysen

Analyse von historischen Flussläufen mit Hilfe von Terrainmodellen



4. Empirische Ansätze

Anwendung der Regime- und der Schleppekrafttheorie

Regime theory: empirical formulas

Natural river width

- Some empirical relationships

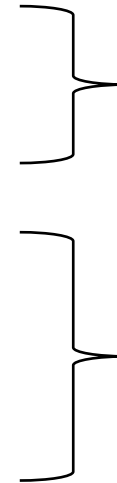
Gleichgewichtsbreite nach Yalin¹⁰: $SB = 1.5 \cdot Q^{0.5} / dm^{0.25}$

Gleichgewichtsbreite nach Parker: $SB = 4.4 \cdot Q^{0.5} / ((s-1) \cdot g \cdot dm)^{0.25}$

Grenzbreite nach Ashmore: $SB = 0.0098 (\rho_w g Q J)^{0.777} / dm^{-0.7}$

Grenzbreite nach Millar: $SB = 16.5 Q^{*0.7} J^{0.6} \mu^{-1.1} dm$

mit $Q^* = Q / ((s-1) g dm)^{0.5} dm^2$

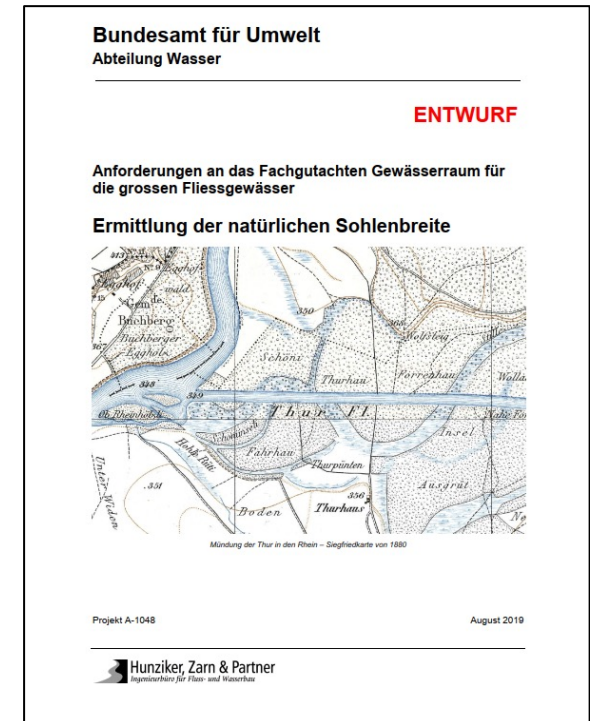


Riverbed width after primary erosion
 → Good for monocoursual channels

Riverbed width after secondary erosion
 → Good for braided and anabranching rivers

Where

- SB = natürliche Sohlenbreite
- ρ_g = Dichte Wasser = 2650 kg/m³
- ρ_w = Dichte Wasser = 1000 kg/m³
- g = Erdschwerebeschleunigung = 9.81 m/s²
- Q = bettbildender Abfluss [m³/s]
- J = Bruttogefälle [-]
- dm = mittlerer Korndurchmesser [m] des Sohlenmaterials
- s = ρ Geschiebe / ρ Wasser



Structure of empirical regime formulae

- The first attempts to formulate the regime characteristics B_R , h_R and S_R (Regime Width, Depth and Slope) are to be credited to the English engineers in India in the early 20th century (Kennedy, Lindley, Lacey, Inglis, Blench).
- The purely empirical regime formulae produced by these authors are invariably of the form (A is a generic variable, Q is the discharge):

$$A_R = \gamma_A Q^{n_A}$$

Riverbed width $B_R = \gamma_B Q^{n_B}$

Water depth $h_R = \gamma_h Q^{n_h}$

Mean slope $S_R = \gamma_S Q^{n_S}$

Mean channel velocity $V_R = \gamma_V Q^{n_V}$

Regime formula parameters

- B_R is proportional to the square root of Q ($n_B = 1/2$) practically in all formulas

$$B_R \approx Q^{1/2}$$

- The exponent of Q in the relationship for the water depth may depend on sediment size D

$$h_R \approx Q^{n_h}$$

for sand $n_h \approx 1/3$

for gravel $n_h \approx 0.43$

- The exponent of Q in the relationship of the slope at regime (n_s) does also depend on the characteristic grain size D

$$S_R \approx Q^{n_s}$$

for sand $n_s \approx -0.1$

for gravel $n_s \approx -0.43$

- The exponent of Q in the relationship for the velocity may also depend on sediment size D (roughness effect)

$$V_R \propto Q^{n_V}$$

for sand $n_V = 0.2$

for gravel $n_V = 0.07$

Yalin¹⁰: $SB = 1.5 \cdot Q^{0.5} / dm^{0.25}$

Parker: $SB = 4.4 \cdot Q^{0.5} / ((s-1) \cdot g \cdot dm)^{0.25}$

Notice how in all the other cases, V_R , B_R et h_R increase with Q , whereas S_R decreases when Q increases

Although the regime formulas are empirical they show a surprising degree of universality.

➤ Moreover, for simple continuity reasons, notice that

$$Q \propto V_R \cdot h_R \cdot B_R \propto Q^{n_V + n_h + n_B}$$

i.e., it must be $n_V + n_h + n_B = 1$

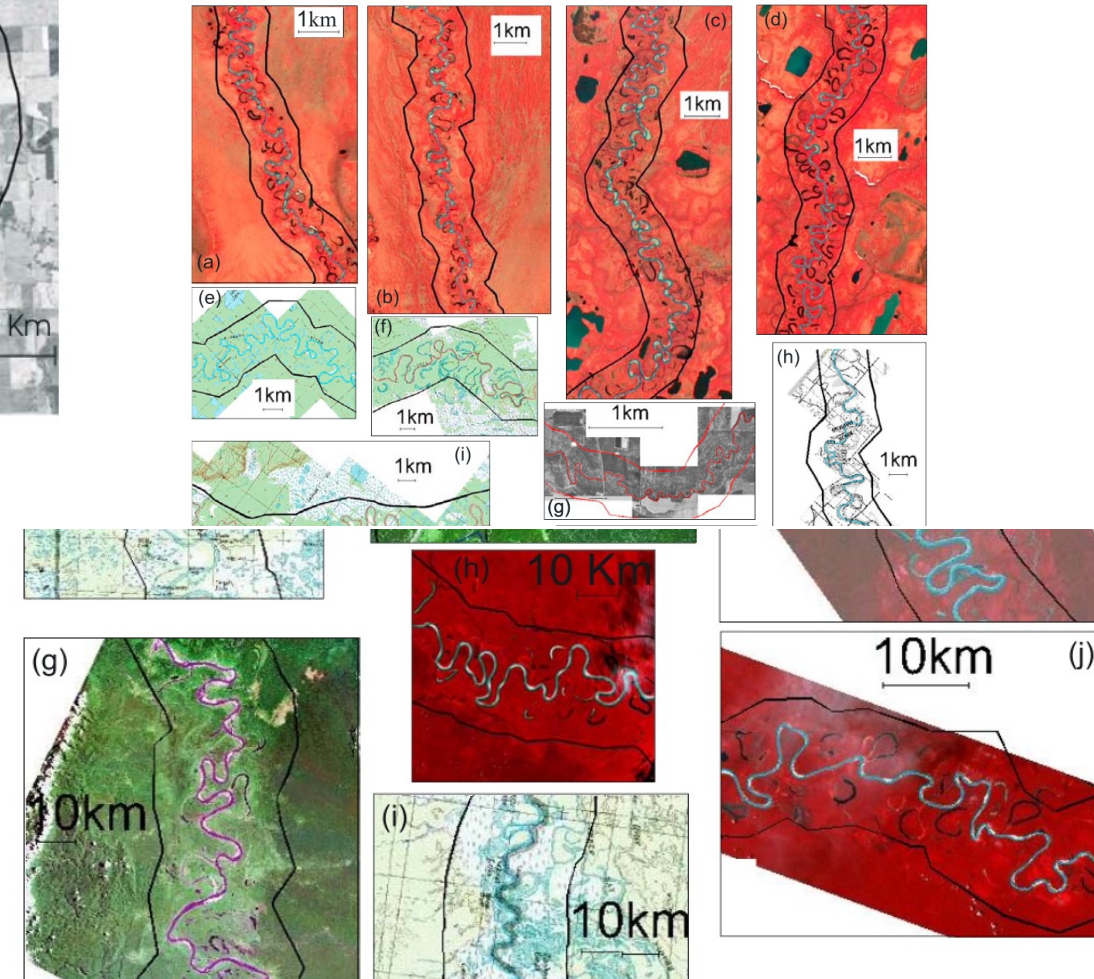
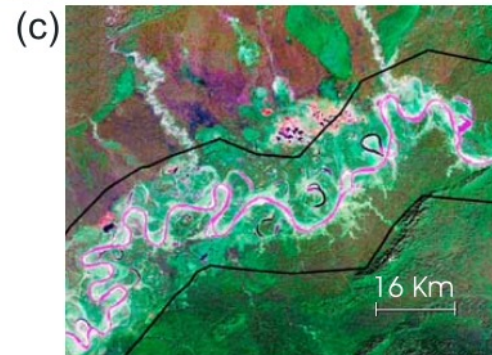
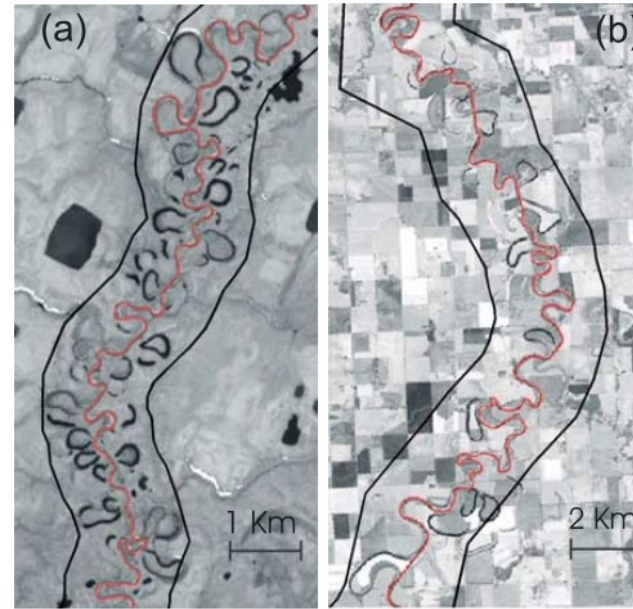
Morphodynamic theory of river meandering



Colville River (Alaska)

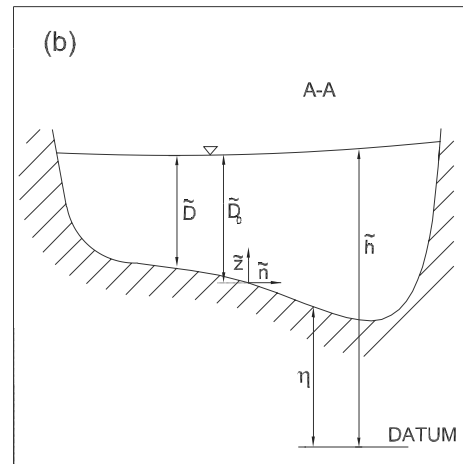
Meandering channels

- Meandering is the result of a planimetric (bed-bank) instability of monocrsual rivers whereby erosion on the outer bank is compensated by deposition (and sediment stabilization at the inner bank)
- Main characteristics:
 - Bend amplitude growth
 - Cutoff
 - Meander belt
- Meandering channels are scale invariant

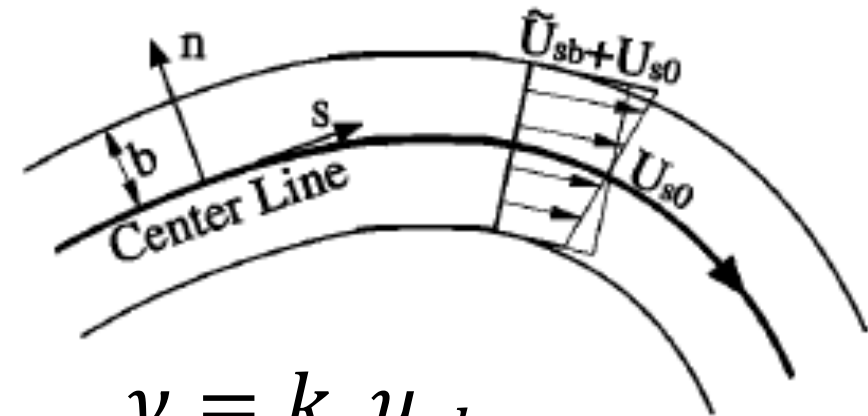


Meander evolution

- The instability leading to bar formation “interacts” with the tendency of the stream to form bends, which force the bars to localize at the inner bend (point bars)
- The two mechanisms “resonate” to determine a typical selected wavelength with which the meander evolves



- Empirical models
- Morphodynamic models (see Camporeale et al. 2005 for a Review)
 - Analytical determination of the excess bank velocity u_{sb}
 - Curve evolution and renormalization problem



$$v = k_e u_{sb}$$

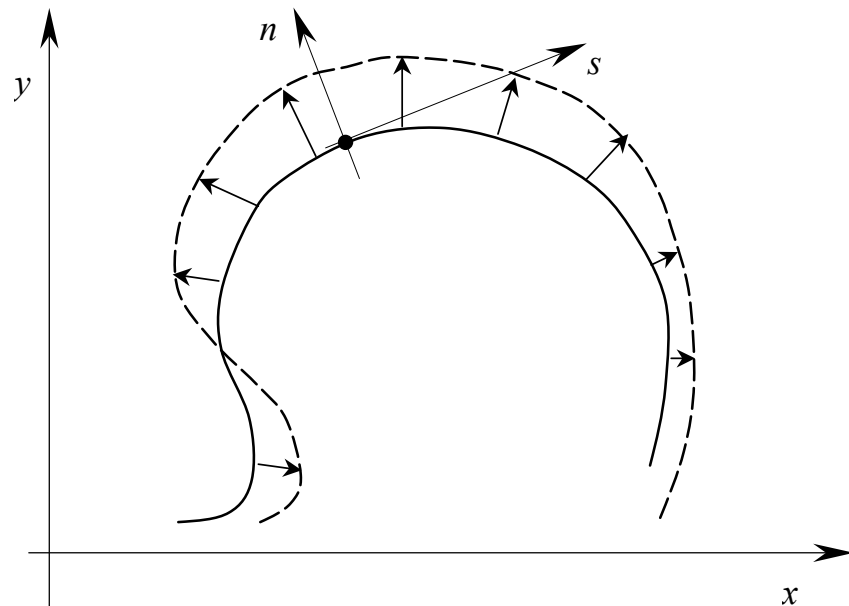
$$v = -\frac{kD^2}{T} + \frac{D}{T} Pa \int_0^s e^{-s'/D} k(s-s') ds'$$

The Ikeda et al. (1981) model

Short-term and long-term evolution

- Bend hydro- and morphodynamic are key to obtain the correct bend evolutive dynamics before a cutoff occurs because of the “spatial memory” effect
- The erosion rate is made of two terms: a local one and a non-local one

$$v = -\frac{kD^2}{T} + \frac{D}{T} Pa \int_0^s e^{-s'/D} k(s-s') ds'$$



- For long-time analyses, the cutoff introduces “noise” in the system
- This is due to the lost of part of the bend
- The non-local term suddenly change
- average quantities description is enough

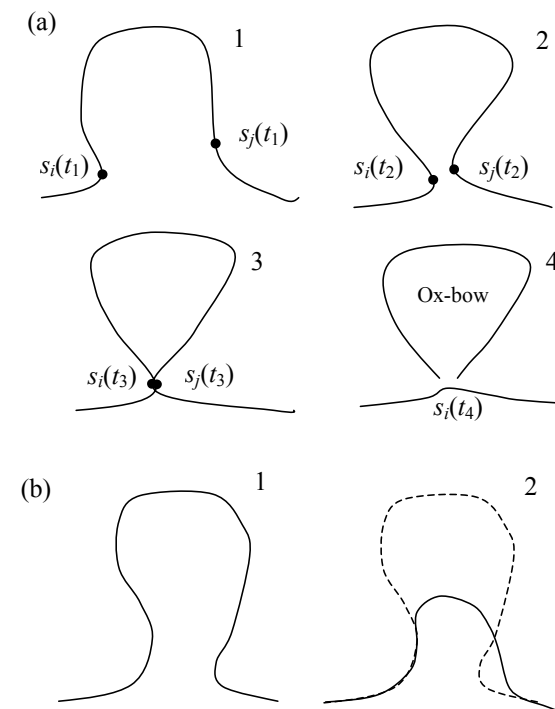
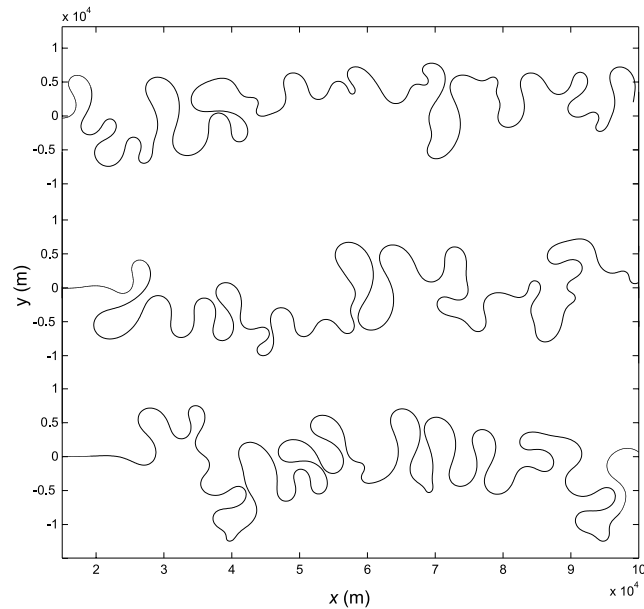


Figure 3a,b. Basic mechanisms of different types of cutoff: (a) Four steps of a neck cutoff and (b) chute cutoff rounding off a bend reach.

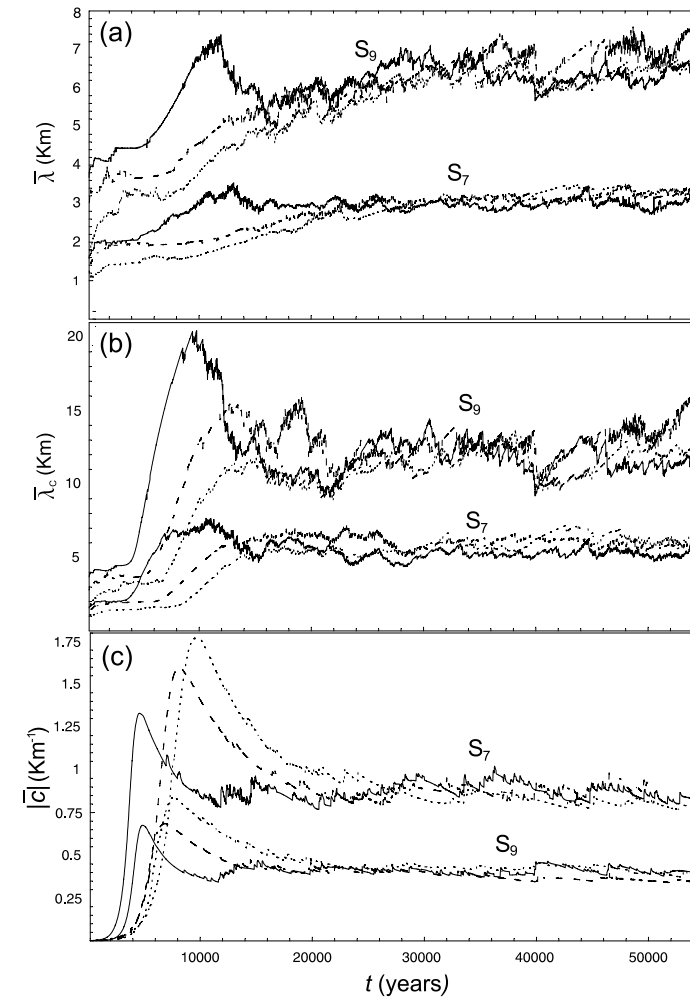
Long-term statistics

From Camporeale et al., 2005



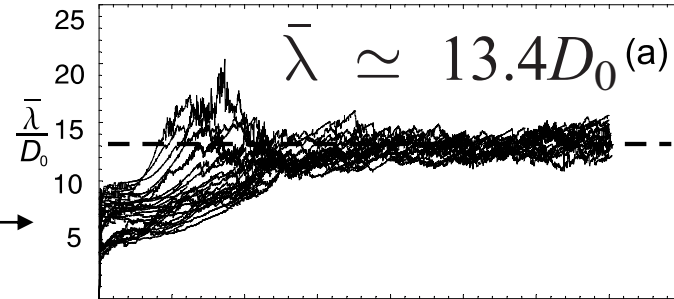
At long-terms, the detail of the hydrodynamic model being used is irrelevant

The cutoff process determines alone self-confinement of the belt

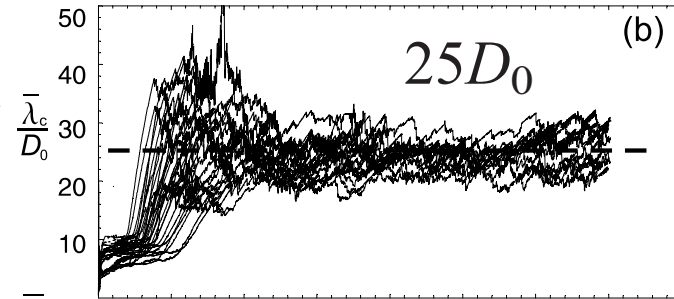


From Camporeale et al., 2005

Mean wavelength

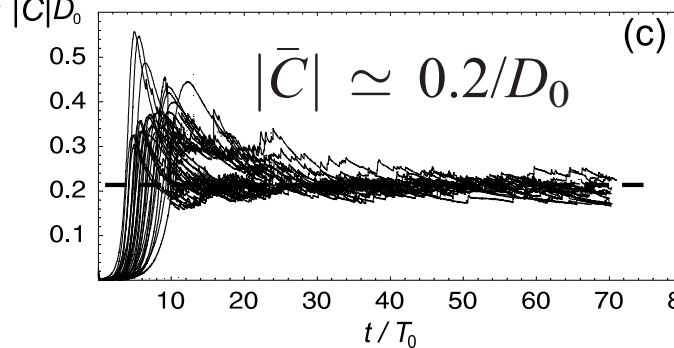


Mean curvilinear wavelength



Mean curvature

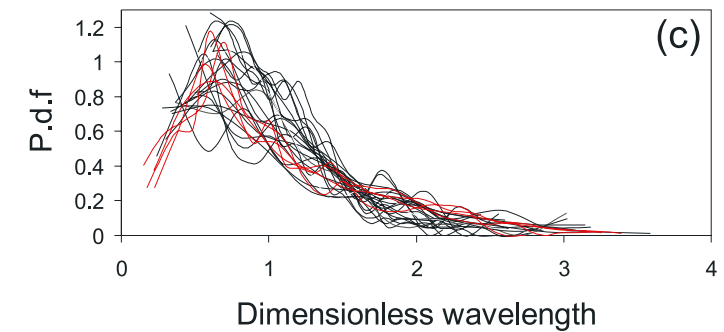
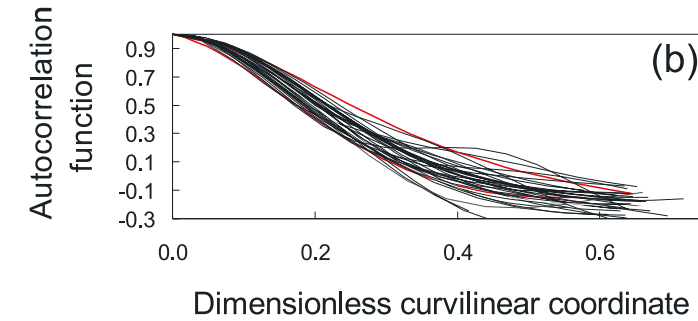
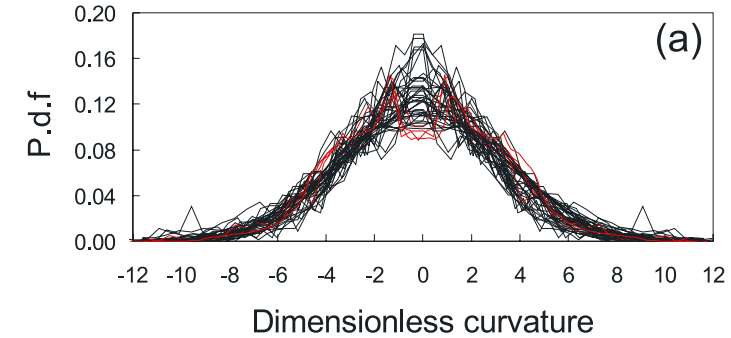
$$D_0 = H_0/2C_{f0}$$



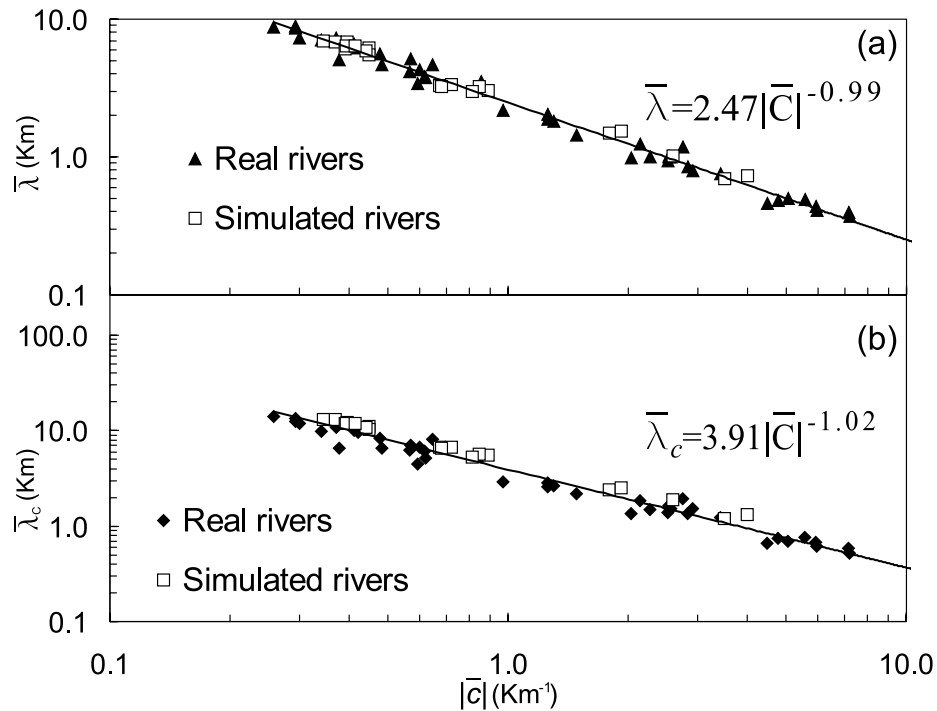
Regime theory

$$\bar{\lambda}/D_0 = 14$$

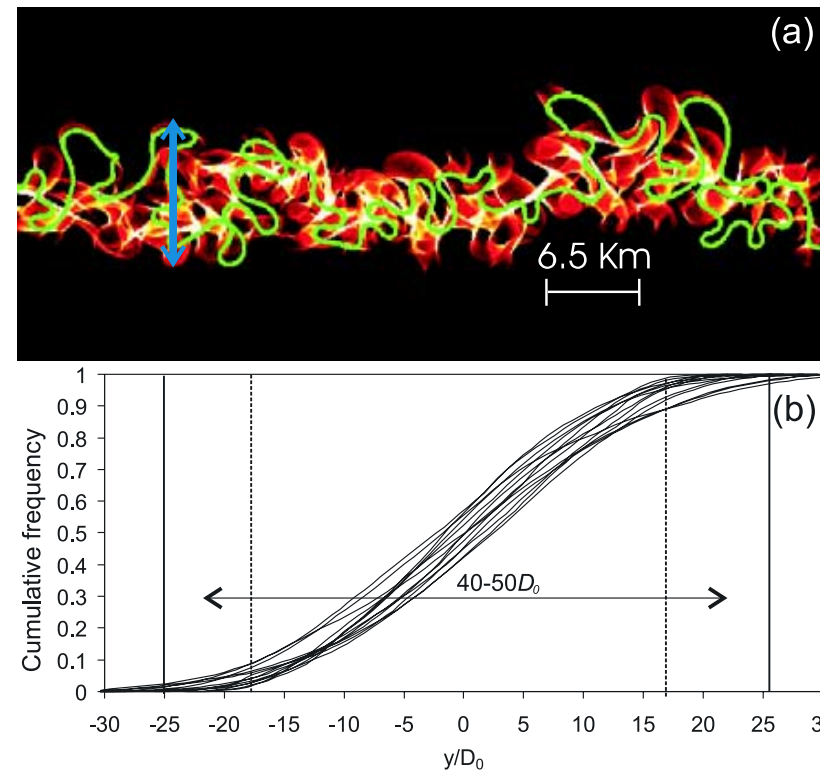
24



Comparison with data



Camporeale et al., 2005



Meander belt width

The «Room for rivers» concept

- Espace réservé aux eaux: couloir de divagation et habitats

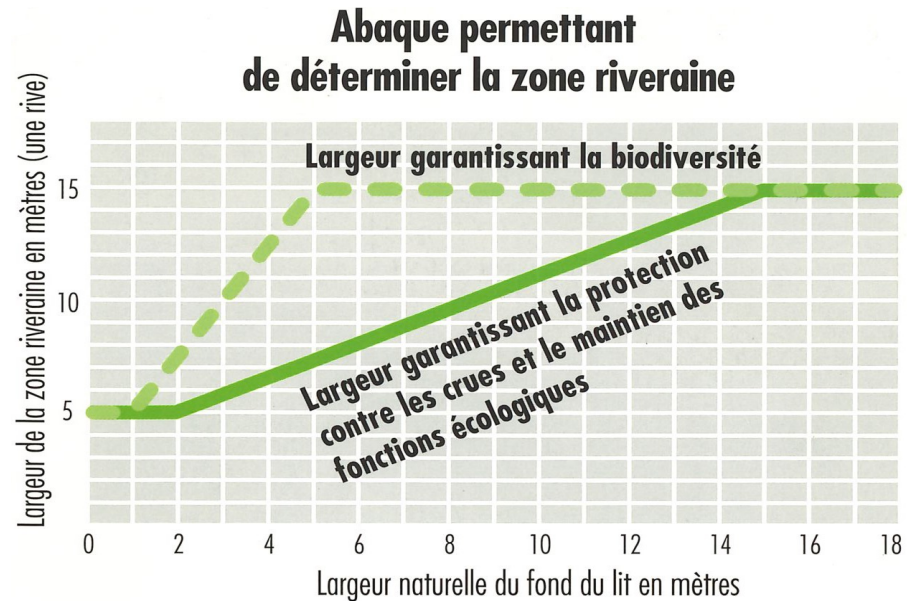
- art. 36a LEaux

- Art. 36a¹ Espace réservé aux eaux

¹ Les cantons déterminent, après consultation des milieux concernés, l'espace nécessaire aux eaux superficielles (espace réservé aux eaux) pour garantir:

- leurs fonctions naturelles;
- la protection contre les crues;
- leur utilisation.

- art. 41a OEaux



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