

We will look into moist atmospheric processes

→ Write combined 1st + 2nd laws for a system that contains (dry + moist) air and liquid water. (saturated air).

→ We will then understand changes in thermal state associated with formation and dissipation of clouds.

- Typical processes that lead to cloud formation
 - isobaric cooling (radiative fog)
 - adiabatic isobaric process
 - adiabatic expansion (clouds)
 - adiabatic isobaric freezing

→ Most clouds are a mixture of the above, but studying each process is useful.

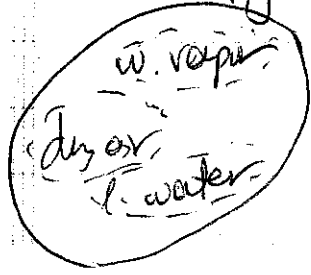
→ Enthalpy of a system containing moist air + liq. water

Q: Why enthalpy?

A: clouds are open boundary systems. H is the appropriate representation vs. U, G, \dots

panel concept.

Enthalpy: sum of all components



so: $dH = \sum dh$ (over all components) + latent heat release/loss.

Use 1 kg of air mixture: amount of (58)
 water vapor is m_w , and liquid water is m_l

$$dH = (m_d c_{pd} + m_l c_{pl} + m_v c_{pv}) dT + L_{ev} dm_v$$

\uparrow \uparrow \uparrow \uparrow
 dry air liquid vapor phase change

In the atmosphere, the mass of water vapor is much less than dry air content.

Q: How can we prove that?

A: From comparing e_s vs P . Assuming ideal gas, $\frac{e_s}{(P-e_s)} = \frac{n_w}{n_a} \ll 1$.

Similarly $m_l \ll m_d$ so:

$$dH \approx m_d c_{pd} dT + L_{ev} dm_v$$

condensation of liquid water

In intensive form (or $m_{total} = 1 \text{ kg}$):

$$dh \approx c_{pd} dT + L_{ev} dw_v$$

Now, internal energy can be written as:

$$du = (c_{vd} + w_v c_{v,v} + w_l c_{v,l}) dT + L_{ev} dw_v$$

and for a similar reason as for dh :

$$du \approx C_v dT + L_v dw_v \quad (\text{intensive})$$

Now, the first law can be written as:

$$dq = du - v dp \\ = C_v dT + L_v dw_v - v dp$$

In case where the system is adiabatic,

$dq=0$, so that: $-L_v dw_v = C_p dT - v dp$

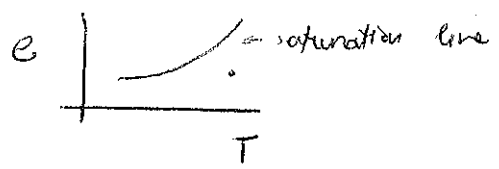
or, $dw_v = -\frac{C_p}{L_v} dT + \frac{v dp}{L_v}$

change in water vapor content is proportional to expansion and cooling in an adiabatic air mass.

Isobaric cooling (radiative fogs, stratus clouds)

$$dq = dh = c_p dT$$

If air is $RH < 100\%$ then cooling increases RH until it is $RH = 100\%$. At that point, the T of the parcel is called dew point



$T_D := e = e_s(T_D)$ definition

↑
water vapor press = saturation vapor press @ T_D .

or $w_v = w_s(T_D)$ similarly.

Same forum, define a frost point:

$T_F: e = e_{si}(T_F)$ or $w_v = w_{ssi}(T_F)$.

→ T_D can be determined from ^{EM eqns} Clausius-Clapeyron

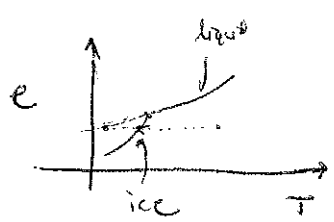
$\frac{dp}{dT} = \frac{L_{ev}}{R_v T^2}$

$\frac{d(\ln e)}{dT_D} = \frac{L_{ev}}{R_v T_D^2}$; integration from T_D to T gives:

$\ln\left(\frac{e_s}{e}\right) = -\ln H = \frac{L_{ev}}{R_v} \left(\frac{1}{T_D} - \frac{1}{T} \right)$, or:

$$H = \exp \left\{ - \frac{L_{ev}}{R_v} \left(\frac{T - T_D}{T \cdot T_D} \right) \right\}$$

so, if you know $T, H \Rightarrow T_D$
 $T, T_D \Rightarrow H$.



frost before eq. cond.

→ Once cooling goes beyond T_D , then you have a liquid phase appearing.

→ Q: Does T decrease when you cool beyond T_D ?

A: Yes! only that $H=L$ always.

→ Q: How does T vary then?

A: Use Thermodynamics:

cooling rate → $dq = dh = c_p dT + L_v dw_v$ (4)

@ $H=L$, $w_v = w_s$ always. However, we also have appearance of a liquid water phase. We can define a liquid water mass ratio similar to w_v :

$$w_l = \frac{m_l}{m_{dry}}, \text{ so that the total}$$

amount of water, $w_t = w_v + w_l = \text{const.}$ for a closed system. Because of that,

$$dw_t = 0 = dw_v + dw_l \rightarrow dw_l = -dw_v \quad (1)$$

This simply says for liquid water that appears from condensation from gas phase.

(from previous class, $\epsilon = \frac{M_v}{M_d}$)

$$\text{Since } dw_s = + \epsilon \frac{d\epsilon_s}{P} = + \frac{\epsilon L_v \epsilon_s}{P R_v T^2} dT \quad (2)$$

↑
class's claygram

Then from (1) + (2):

$$dw_f = - dw_s = - \frac{\epsilon L_v \epsilon_s}{P R_v T^2} dT \quad (3)$$

Combining (3), (1) and (4):

$$dq = c_p dT + L_v \frac{\epsilon L_v \epsilon_s}{P R_v T^2} dT \Rightarrow$$

$$\Rightarrow dq = \left\{ c_p + \frac{L_v^2 \epsilon_s \epsilon}{P R_v T^2} \right\} dT \quad (5)$$

Temperature drop as a function of dq

$$\text{at } H < 1, \quad dq = c_p dT$$

$$H \geq 1, \quad dq = \text{from (5)}$$

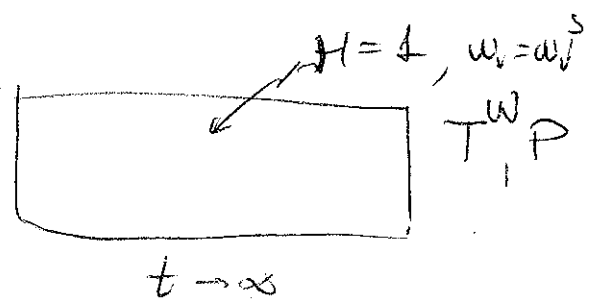
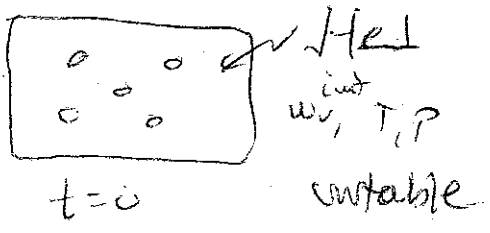
→ $dT(H < 1) > dT(H \geq 1)$ for equal dq
 Let them calculate for $dq = 1$. dT will be
 → slower for a cooling with clavel. That
 is why the T drop when fog is around is

much slower than when you are
in clear sky (at night).

→ You are putting liquid water into a system that wants to be in a single-phase state (i.e. gas, water vapor). As a result, you will have evaporation.

→ Because of evaporation, T will drop (of the air) and H will increase. The maximum T -drop that could happen would correspond to the maximum liquid water that will evaporate, i.e., until the air becomes saturated ($H=1$).

→ We can calculate these changes. Assume we have $w_i, H < 1$ and also that we have $P = \text{const}$, $T = T_{\text{initial}}$ and an amount of liquid water, just enough to completely evaporate (even if we had more, it wouldn't matter, as it would not evaporate more). Assume $P = \text{const}$, i.e.:



Assume no heat sources $dq = 0$, so that 1st law is for 1kg of that air!

$$dq = dh - v dp \Rightarrow dh = 0 \quad (\text{isentropic process})$$

so $dh = c_p dT - L_e dw_v = c_p dT + L_e dw_s$

by integrating from w_v to $w_v^{final} = (w_v + w_e)$
initial amount final amount of liquid water

we get:

$$c_p \int_T^{T_w} dT = -L_e \int_{w_v}^{w_v^{final} = w_v + w_e} dw_s \Rightarrow$$

$$\Rightarrow c_p (T_w - T) = -L_e [w_v^{final}(T_w) - w_v]$$

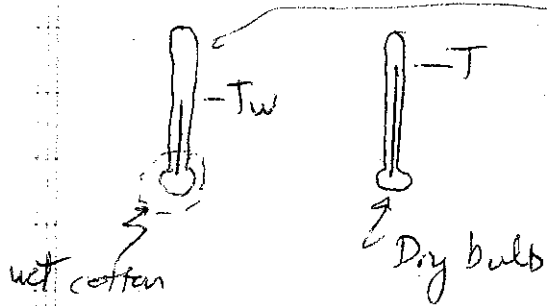
Now, since w_v^{final} = saturation w_s @ T_w , we can write:

$$T_w = T - \frac{L_e}{c_p} \{ w_s(T_w) - w_v \} \quad \text{①}$$

T_w is called "wet-bulb" temperature above is a non-linear eqn w.r.t T_w , as w_s depends on T_w .

→ Now, if we know T_w, T , we can determine w_v from ①.
↳ and $ff = \frac{w_v}{w_s(T)}$

T_w can be easily determined by a wet-bulb thermometer, which is just a thermometer whose bulb is kept continuously wet



The wet bulb evaporates water continuously, until the air around it becomes saturated. Since heat is taken away from its surroundings until $H=1$

in the vicinity of the thermometer, that means when the wet thermometer stabilizes, it will give an indication that corresponds to T_w

So, to measure H , all you need is 2 thermometers and some patience, W_s is calculated from (1).

→ If ice is evaporating, you can also have an ice bulb T_I , T_I defined similarly as:

$$T_I = T - \frac{L_{iv}}{c_p} \{ w_{si}(T_I) - w_w \}$$

Q: is $T_I >$ or $<$ than T_w ?

$$T_I = T + \frac{L_{iv}}{c_p} w_w - \frac{L_{iv}}{c_p} w_{si}(T_I)$$

$$T_w = T + \frac{L_w w_v}{c_p} - \frac{L_w w_s(T_w)}{c_p}$$

So:

$$T_i - T_w = \frac{1}{c_p} \left\{ (L_{iv} - L_{iw}) w_v - (L_{iv} w_s(T_w) - L_{iw} w_s(T_w)) \right\}$$

→ Saturation by Adiabatic, (isobaric mixing

Mix 2 air masses, so that total water is conserved; assuming adiabatic ($dq=0$) and ($dp=0$) isobaric (eg. at the tailpipe of a car exhaust on a cold morning) you can have formation of a cloud.

→ Saturation by Adiabatic Cooling from expansion

By far the most important condensation mechanism in the atmosphere.

Recall first law for an adiabatic process:

$dq = c_p dt - v dp$, from which we derived a potential temperature θ if there was no condensation and an adiabatic lapse rate $T_{dry} = \frac{g}{c_p} \approx 10 \text{ K km}^{-1}$

As our rises and expands, $T \downarrow$ and $H \uparrow$.
 when $H = 1$ (saturation level), standard form
 We can determine P, T where that
 happens as:

$$H = \frac{e}{e_s} \Rightarrow d(\ln H) = d(\ln e) - d(\ln e_s) \quad (2)$$

but

$$dP = de$$

(because as the gas expands,
all components decompress
equally, $\frac{e_1}{P_1} = \frac{e_2}{P_2}$)

So substitution into the 1st law gives

$$d(\ln e) = \frac{C_p}{R_d} d(\ln T) \quad (3)$$

from Clapeyron: $d(\ln e_s) = \frac{L_{ev}}{R_v T} d(\ln T) \quad (4)$

So, substituting (3), (4) into (2):

$$d(\ln H) = \left(\frac{C_p}{R_d} - \frac{E L_{ev}}{R_d T} \right) d(\ln T)$$

So, integrating from $H = H^{init}$ to $H = 1$:

$$H = \exp \left\{ - \left(\frac{C_p}{R_d} \ln \frac{T_s}{T} \right) - \frac{E L_{ev}}{R_d} \left(\frac{1}{T_s} - \frac{1}{T} \right) \right\} \quad (4)$$

→ So, from (4), if you know H, T you can determine T_s (T at saturation).

→ The saturation pressure can be determined from ideal gas law:

$$\ln \frac{P_s}{P} = \frac{C_p}{R_d} \ln \frac{T_s}{T} \rightarrow P_s = P \left(\frac{T_s}{T} \right)^{C_p/R_d}$$

from hypsometric equation, we can write:

and
$$\left. \begin{aligned} d(\ln e) &= -\frac{g}{R_d T} dz \\ d(\ln e) &= \frac{L_{ev}}{R_d T_0^2} dT_0 \end{aligned} \right\} \rightarrow$$

$$\rightarrow \frac{dT_0}{dz} = -\frac{T_0^2 g}{\epsilon L_{ev} T} = -\frac{T_0^2 C_p}{\epsilon L_{ev} T}$$

dew point depression.

→ The height for which condensation occurs is the lifting condensation level.
determine from $T_d = \frac{T - T_s}{z_s - z} c \Rightarrow z_s = \frac{T - T_s}{\Gamma_d}$

Cost Trajectories

(1)

In a dry atmosphere, you can write

$$ds = c_p d(\ln T) - R_d d(\ln P) \quad \left(\text{or } c_p \frac{dT}{T} - R_d \frac{dP}{P} \right)$$

adiabatic process, $ds=0$

$$c_p d(\ln T) - R_d d(\ln P) = 0$$

integrating from
 $(\theta, P_0) \rightarrow (T, P)$

gas

$$\theta = T \left(\frac{P_0}{P} \right)^{R_d/c_p} \quad (1)$$

↑ potential temperature constant for
adiabatic dry motions

→ Similar for moist parcel, define an analogous

θ_e we call it equivalent potential T

1st law:

$$dq = c_p dT - \left(\frac{R_d T}{P} \right) dP + L_w dw \quad \leftarrow \text{water assumption}$$

divide by T:

$$\frac{dq}{T} = c_p \frac{dT}{T} - R_d \frac{dP}{P} + \frac{L_w}{T} dw$$

$$ds = c_p \frac{dT}{T} - R_d \frac{dP}{P} + \frac{L_w}{T} dw$$

isentropic process $ds=0$

$$0 = C_p dT - R d \ln p + \frac{L w}{T} dw_s \quad (3)$$

from (1):

$$C_p d \ln \theta = C_p d \ln T - R d \ln p \quad (2)$$

from (3), (2) we get

$$0 = C_p d \ln \theta + \frac{L w}{T} dw_s$$

Integrating the above to where all water has condensed out (i.e., $w_s \rightarrow 0$) we can then define θ_e :

$$-L w \int_{w_s}^0 d \left(\frac{w_s}{T} \right) = C_p \int_{\theta}^{\theta_e} d \ln \theta \quad \text{or:}$$

$$L w \frac{w_s}{T} = C_p \ln \frac{\theta_e}{\theta} \Rightarrow \theta_e = \theta \exp \left(\frac{L w_s}{C_p T} \right)$$

$\theta_e > \theta$, b/c $L w > 0$.