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Large-scale dynamics of the mid-latitude atmosphere: Part I

Josué Gehring – Meteorologist at MeteoSwiss

What do you see on this satellite animation?

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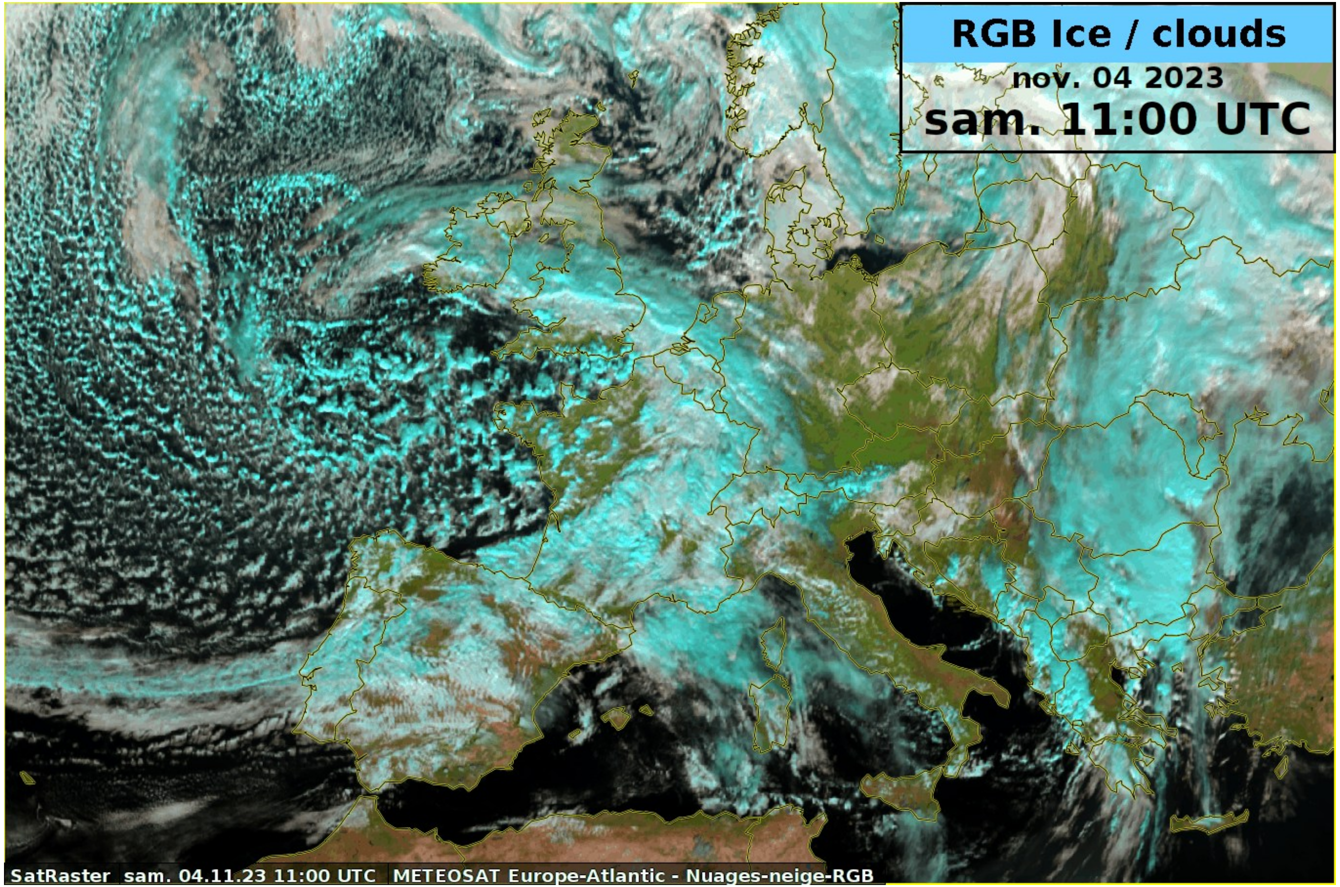
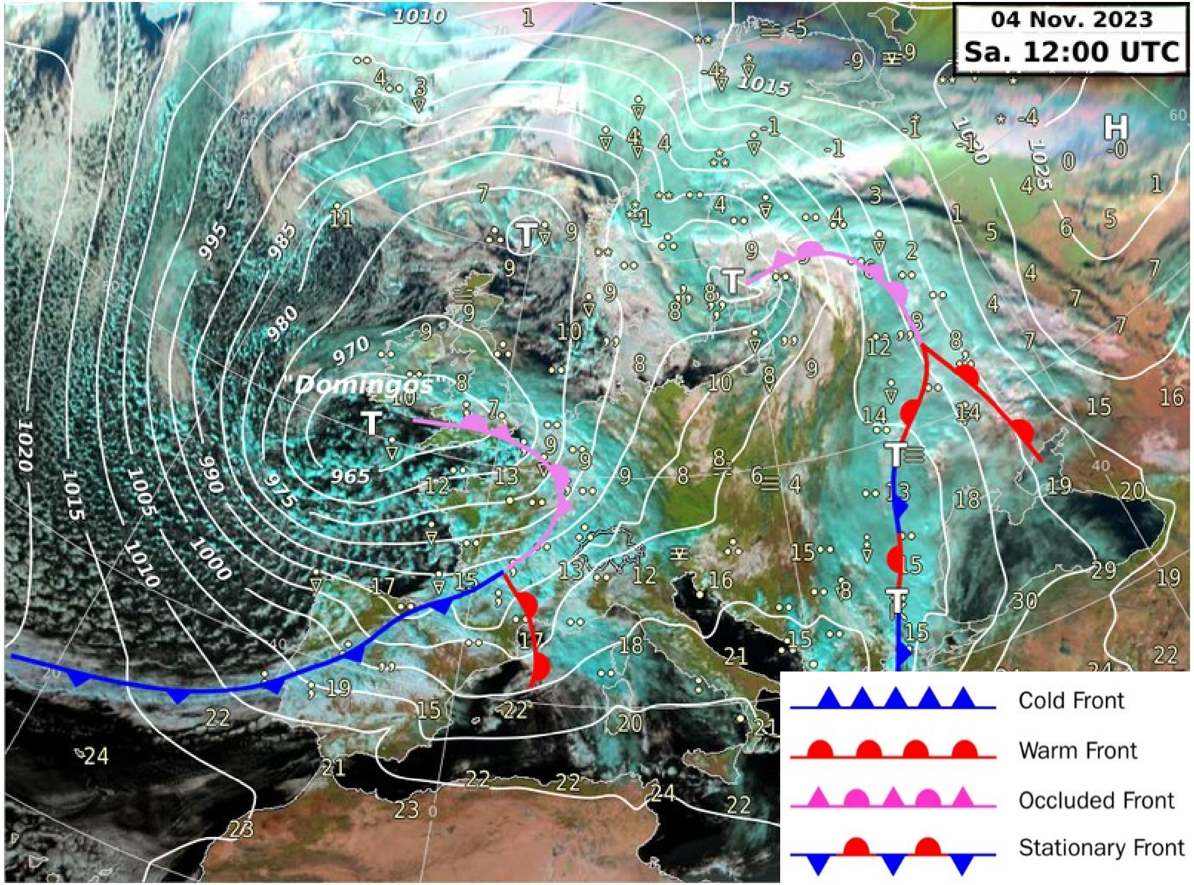


What do you see on this satellite animation?

Questions that we will investigate in this course:

- How do these systems form?
- How do they evolve?
- How are they related?
- What kind of weather are associated with them?



Topics covered in the coming 3 lectures:

Today

- Application of momentum conservation to large-scale atmospheric flows

Next week

- Jet stream
- Weather fronts

In two weeks

- Extratropical cyclones

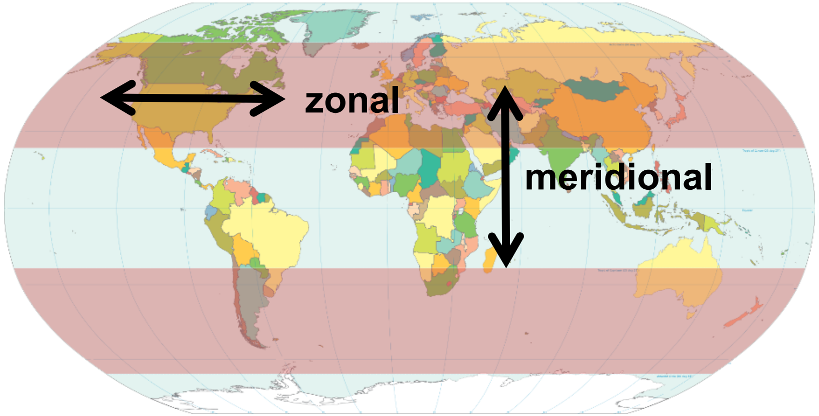
Books 

- J.E. Martin, «Mid-latitude atmospheric dynamics, a first course», 2006 → JEM
- J.M. Wallace & P.V. Hobbs, «Atmospheric science, an introduction survey», 2006 → W&H

Some basic definitions

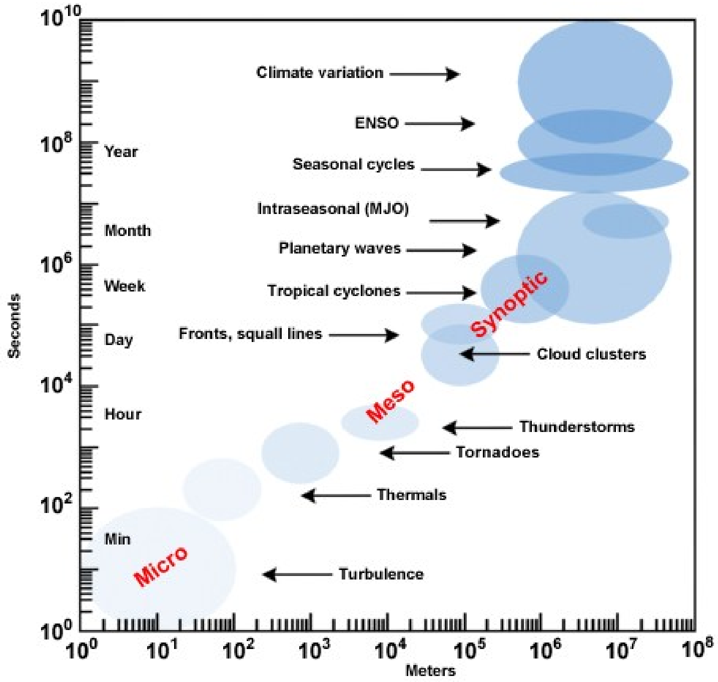
What does «**mid-latitude** large-scale atmospheric dynamics» mean?

- **Mid-latitude**: zone between the tropics (+/- 23° N) and the polar circles (+/- 66° N)



What does «mid-latitude **large-scale** atmospheric dynamics» mean?

- **Synoptic** or **large-scale**:
  - Temporal scale: **days to week**
  - Spatial scale: **1’000-10’000 km**
  - Weather systems: **cyclones** and **anticyclones**, **fronts** (to some extent)



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What is «mid-latitude large-scale **atmospheric dynamics**» ?

A bit of atmospheric science slang: despite both being physics, we distinguish between atmospheric **dynamics** and atmospheric **physics** as below

Atmospheric dynamics	Atmospheric physics
Study of the **dynamics** of **atmospheric flows** and their effect on weather	Study of other **smaller-scale physical processes** in the atmosphere: e.g. radiation, cloud and precipitation formation

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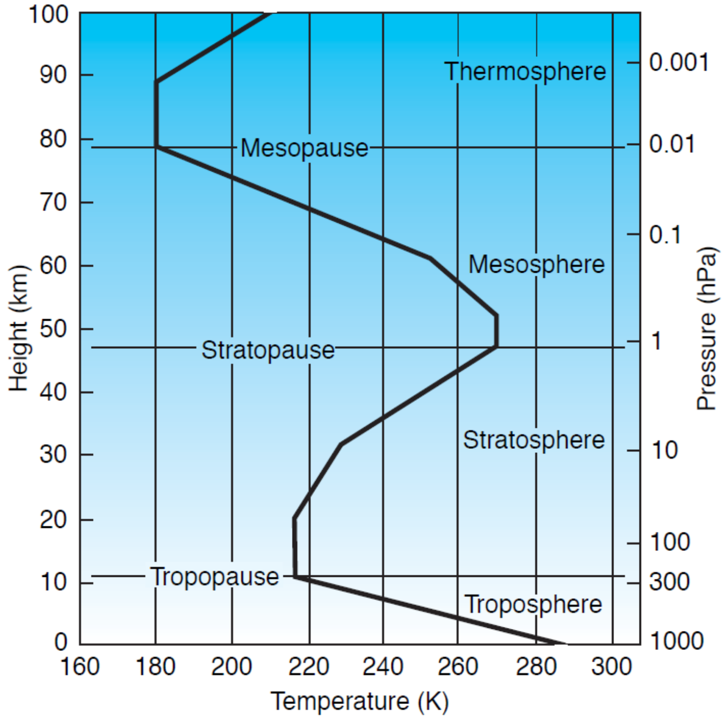
The thermal structure of the atmosphere

- Troposphere:
  - where **most weather systems** occur
  - **~85 % of the mass** of the atmosphere and virtually **all atmospheric water**
- Tropopause:
  - separates the **troposphere** from the **stratosphere,** but not totally “hermetic”
  - Defined as a sharp change in the **vertical temperature gradient** (increase in static stability)
  - **not a constant surface**, but varies significantly in space and time
    - Higher in the tropics and in summer
    - Lower in the polar regions and in winter



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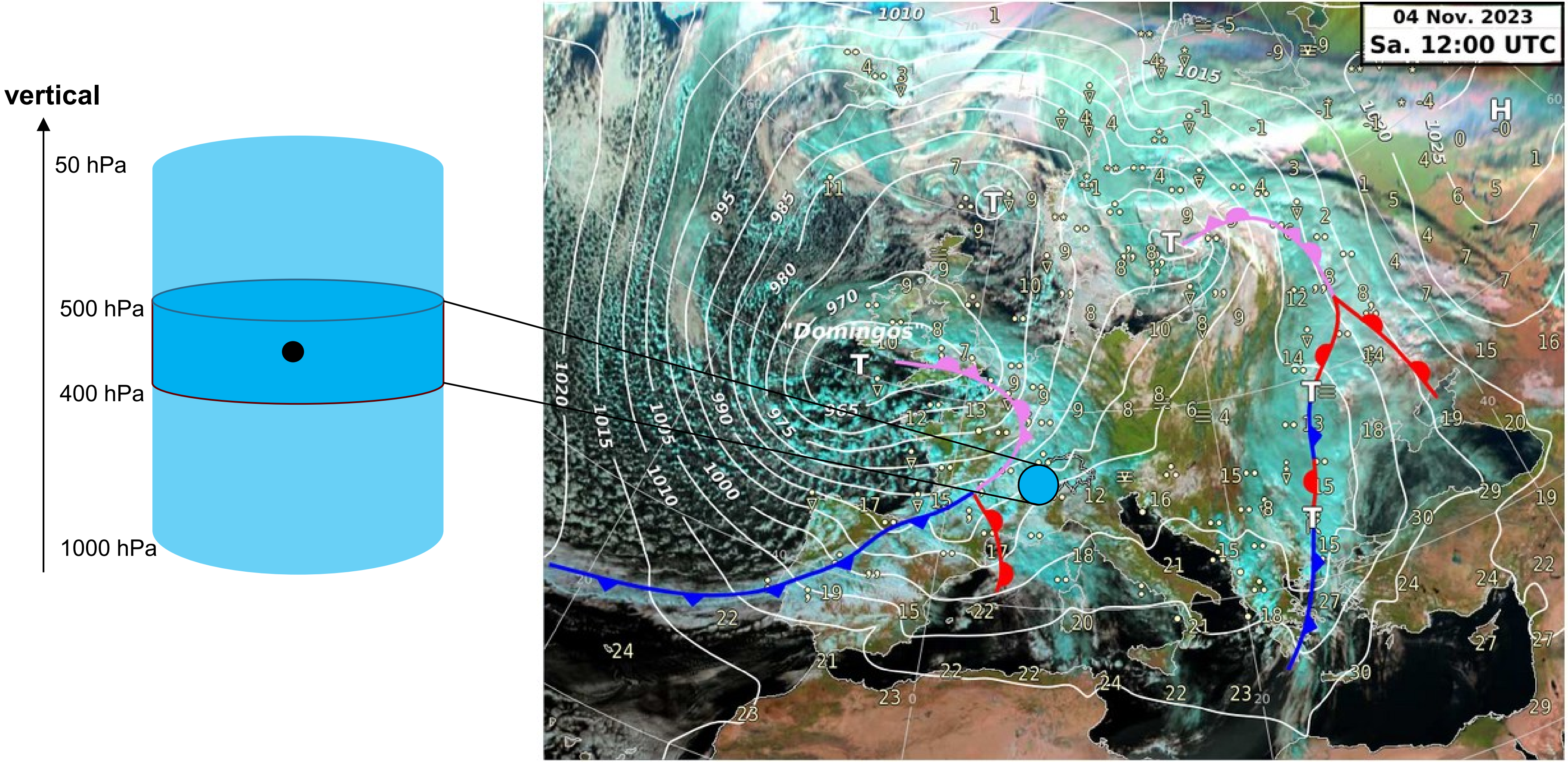
Which forces act on this air parcel?



Conservation of momentum: the equation of motion

Conservation of momentum: equation of motion

JEM Sec. 3.2

$$\underbrace{\frac{d\vec{U}}{dt}}_{\text{Acceleration following the motion}} + \underbrace{2\vec{\Omega} \times \vec{U}}_{\text{Coriolis force}} - \underbrace{\Omega^2 \vec{r}}_{\text{Centripetal acceleration}} = \underbrace{-\frac{1}{\rho}\vec{\nabla} p}_{\text{PGF}} + \underbrace{\overrightarrow{g^*}}_{\text{Gravity}} + \underbrace{\vec{F}}_{\text{Frictional force}}$$

Note that:

(i) all terms are expressed as forces per unit mass, so they are actually acceleration terms

(ii) The Coriolis force and the centripetal acceleration are apparent forces resulting from the acceleration of our reference frame fixed to the earth. See JEM Sec. 2.2.

(iii) The gravitational and centripetal acceleration are both directed normal to the surface. Since there is no way for a mass on earth to distinguish them, they are often combined in the effective gravity term (JEM Sec. 2.2.1)

$$\vec{g} = \overrightarrow{g^*} + \Omega^2 \vec{r}$$

(iv) $\frac{d\vec{U}}{dt}$ is the total derivative and represents the rate of change following the motion, i.e. the vector rate of change of the velocity field (u,v,w) of a fluid particle as it moves through the fluid. See JEM Sec. 1.2.4.

$$\frac{d\vec{U}}{dt} = \frac{\partial \vec{U}}{\partial t} + \left(\vec{U} \cdot \nabla\right)\vec{U}$$

Total derivative and advection

JEM Sec. 1.2.4

$$\underbrace{\frac{dT}{dt}}_{\text{Change with time following the motion (i.e. Lagrangian)}} = \underbrace{\frac{\partial T}{\partial t}}_{\text{Change with time at a fixed location (i.e. Eulerian)}} + \underbrace{(\vec{U} \cdot \nabla)}_{\text{Advection}} T$$

Change with time following the motion (i.e. Lagrangian)

Change with time at a fixed location (i.e. Eulerian)

Advection

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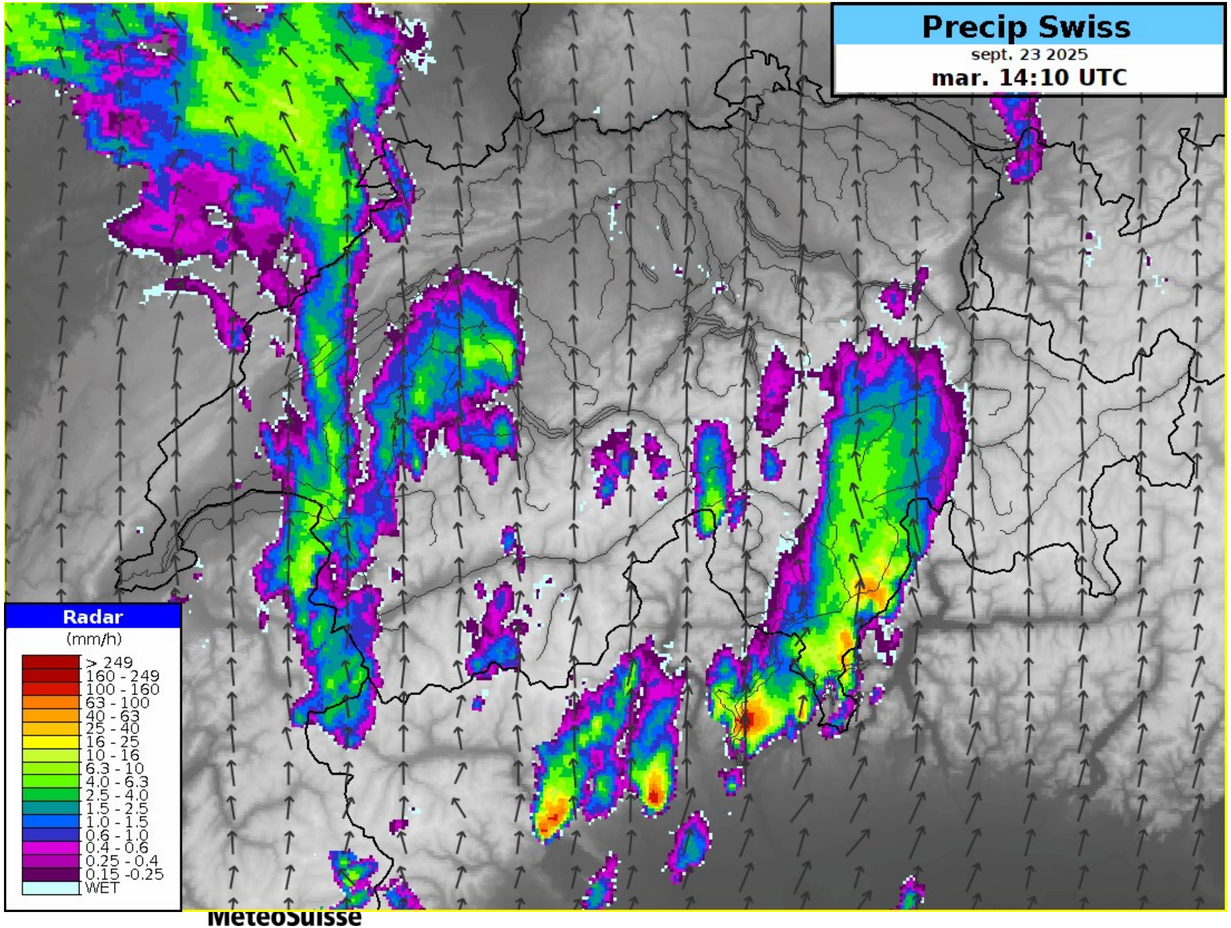
Lagrangian vs. Eulerian perspective: a concrete example

JEM Sec. 1.2.4



$$\frac{\partial R}{\partial t} = \frac{dR}{dt} - \left(\vec{U} \cdot \nabla\right) R$$

Radar-derived precipitation intensity and wind at 4000 m from the ICON-CH1-EPS model

The horizontal and vertical components of the equation of motion

JEM Sec. 3.2.1

Coordinate system: (x, y, z) points eastward, northward, and upward, respectively. Similarly the wind field $\vec{U} = (u, v, w)$ corresponds to westerly, southerly and upward motion. This choice of coordinate system leads to “spherical terms” (i.e. 1/a[1])

$$\frac{du}{dt} - \frac{uv\tan\phi}{a} + \frac{uw}{a} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + 2\Omega\sin\phi v - 2\Omega\cos\phi w + F_x$$

$$\frac{dv}{dt} + \frac{u^2\tan\phi}{a} + \frac{vw}{a} = -\frac{1}{\rho}\frac{\partial p}{\partial y} - 2\Omega\sin\phi u + F_y$$

$$\frac{dw}{dt} - \frac{u^2 + v^2}{a} = -\frac{1}{\rho}\frac{\partial p}{\partial z} - g + 2\Omega\cos\phi u + F_z.$$

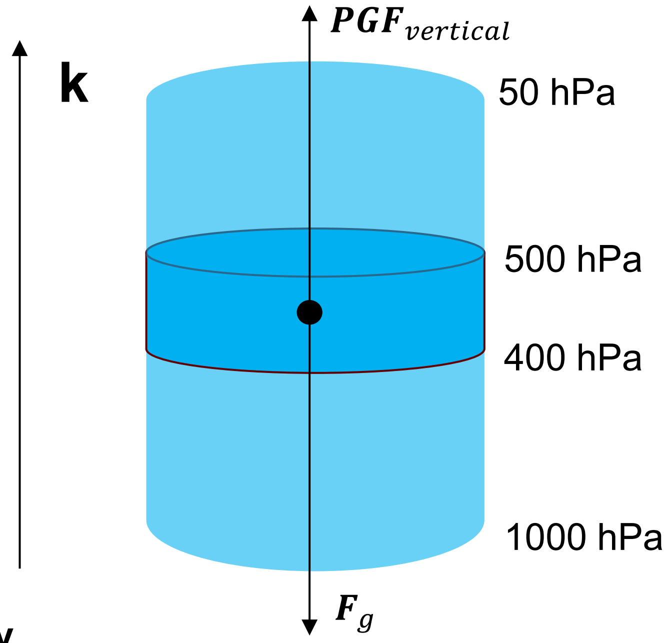
[1] a is the earth’s radius

Scale analysis of the vertical equation of motion

Scale analysis of the vertical equation of motion

$$\frac{dw}{dt} - \frac{u^2 + v^2}{a} = -\frac{1}{\rho}\frac{\partial p}{\partial z} - g + 2\Omega\cos\phi\, u + F_z.$$

	1	2	3	4	5	6
	$\frac{dw}{dt}$	$-2\Omega\cos\phi\, u$	$\frac{-(u^2+v^2)}{a}$	$-\frac{1}{\rho}\frac{\partial p}{\partial z}$	$-g$	F_z
Characteristic scales	$\frac{UW}{L}$	★ $f_0 U$	$\frac{U^2}{a}$	$\frac{p}{\rho H}$	g	$\frac{\nu W}{H^2}$
Magnitudes ($\mathrm{m\,s^{-2}}$)	10^{-7}	10^{-3}	10^{-5}	10	10	10^{-15}



→ Balance between the **vertical pressure gradient force** and **gravity**, how is it called?

★ $f = 2\Omega\sin(\phi)$ is the Coriolis parameter. f_0 is for $\phi_0 = 45°$ in the mid-latitudes, such that $f_0 = 2\Omega\sin\phi_0 = 2\Omega\cos\phi_0 \cong 10^{-4}\ \mathrm{s^{-1}}$

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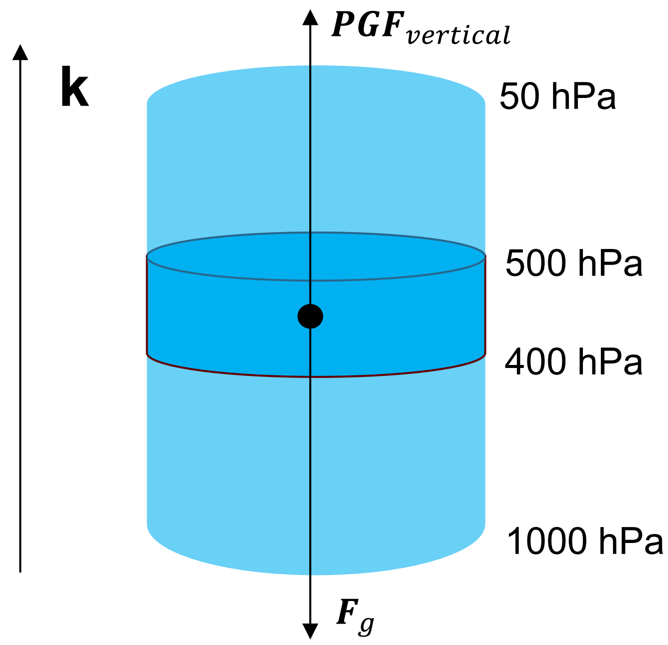
Hydrostatic balance

JEM Sec. 3.1

Upward: $\overrightarrow{PGF}_{vertical} = -\frac{1}{\rho}\frac{\partial p}{\partial z}\hat{k}$

Downward: $\overrightarrow{F_g} = -g\hat{k}$

If atmosphere at rest → $\frac{\partial p}{\partial z} = -\rho g$

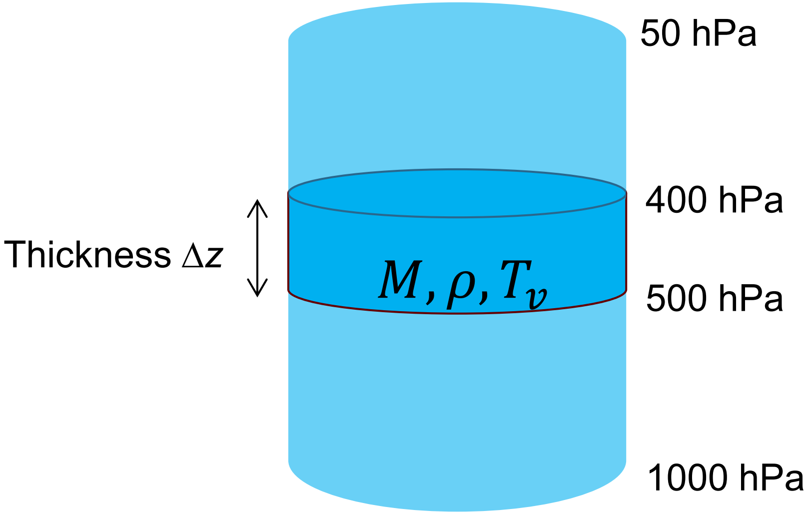


- Vertical pressure gradient force is balanced by gravity (assumes atmosphere at rest on the vertical)
- This balance is valid for nearly all conditions in the atmosphere

Implications of the hydrostatic balance: the hypsometric equation

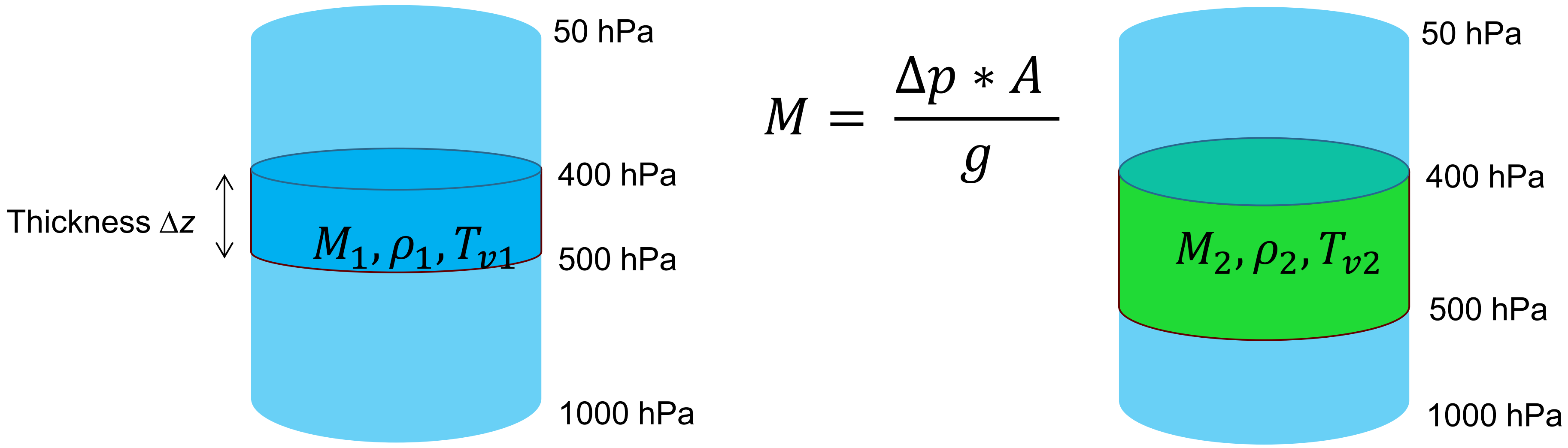
Hypsometric equation

JEM Sec. 3.1.1



$$M = \frac{\Delta p * A}{g} = \frac{1 * 10^4 [Pa] * 1[m^2]}{9.81\ [ms^{-2}]} = 1019\ [kg]$$

Hypsometric equation



$$M = \frac{\Delta p * A}{g}$$

- Since $M_1 = M_2$ and $V_1 < V_2$ → $\rho_1 > \rho_2$
- Ideal gas law ($p = \rho R_d T_v$) → $T_{v1} < T_{v2}$
- **Thickness** between two isobaric surfaces is related to the **column average virtual temperature** with larger (smaller) thickness associated with a warmer (colder) air mass

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Hypsometric equation

Combining the hydrostatic equation ($\frac{\partial p}{\partial z} = -\rho g$) and ideal gas law ($p = \rho R_d T_v$) yields: $\frac{\partial p}{\partial z} = -\frac{pg}{R_d T_v}$, which after integration yields:

$$\frac{R_d \overline{T_v}}{g} \ln\left(\frac{p_1}{p_2}\right) = \Delta z$$

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The concept of geopotential

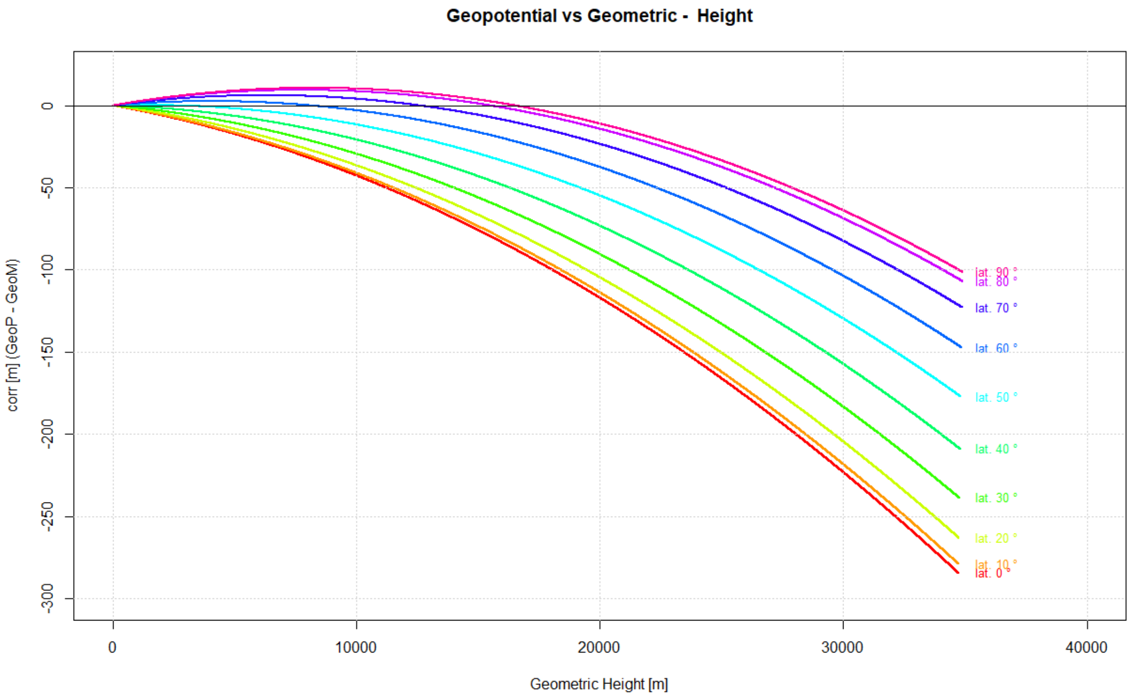
- Geopotential: **work** required to **raise a unit mass** a distance dz above the ground →

$$d\Phi = g dz \; [\mathrm{m^2 s^{-2}}] \mathrm{or} \; [\mathrm{J \; kg^{-1}}]$$
$$\rightarrow \phi(z) = \int_0^z g \, dz$$

- **Geopotential height**:

$$\mathrm{Z} = \frac{\Phi}{g_0}$$

where g_0 is the global average gravity at sea level (9.81 $\mathrm{m \, s^{-2}}$) → **geometric height** (z) and **geopotential height** (Z) are just about **equal** in the troposphere (since $g \approx g_0$)



The concept of geopotential

- **Hydrostatic equation** ($\frac{\partial p}{\partial z} = -\rho g$) becomes

$$\partial p = -\rho \partial \Phi \text{ or } \partial p = -\rho g_0 \partial Z$$

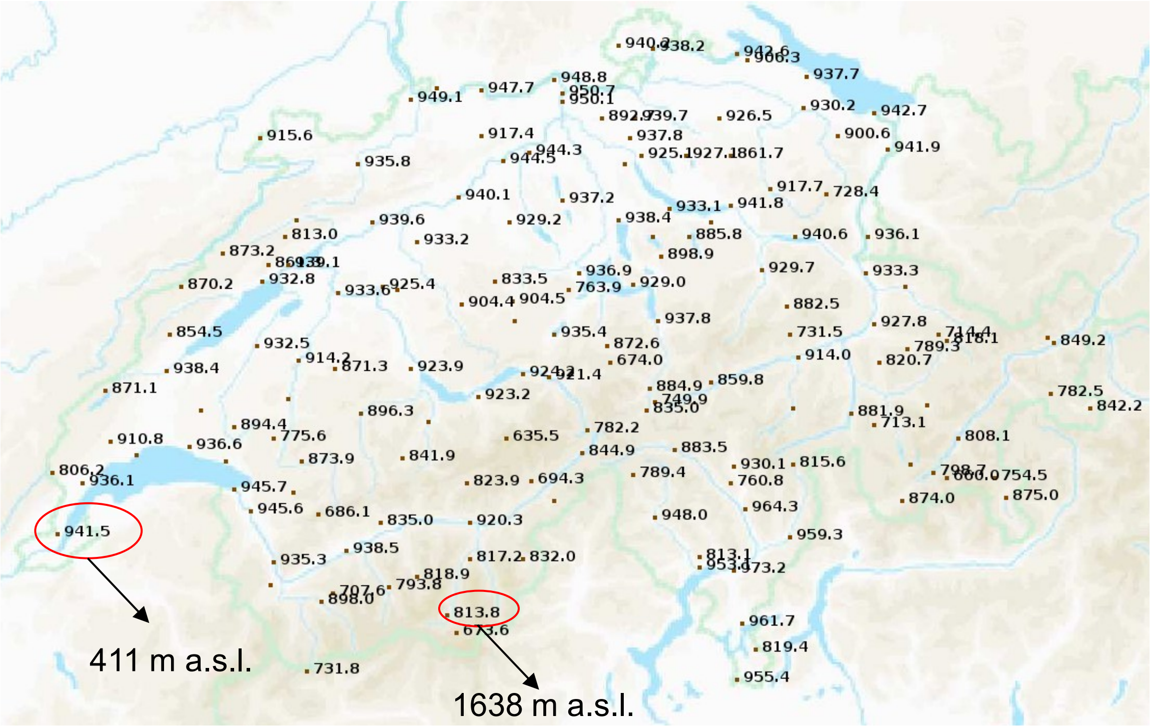
- **Hypsometric equation** becomes:

$$\frac{R_d \overline{T_v}}{g_0} \ln\left(\frac{p_1}{p_2}\right) = \Delta Z$$

→ *Geopotential height difference between two isobars is a function of both the average virtual temperature and the pressure of the lower isobaric level (p_1).*

Application of the hypsometric equation: the altimeter equation

Pressure measured at station height (hPa) on 05 Nov 2023 at 16 UTC



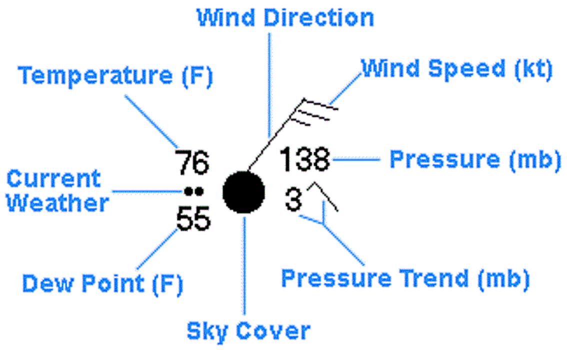
- How to compare pressure measurements from stations at different heights?
- Rearranging the hypsometric equation

$$\frac{R_d \overline{T_v}}{g} \ln\left(\frac{p_1}{p_2}\right) = \Delta z$$

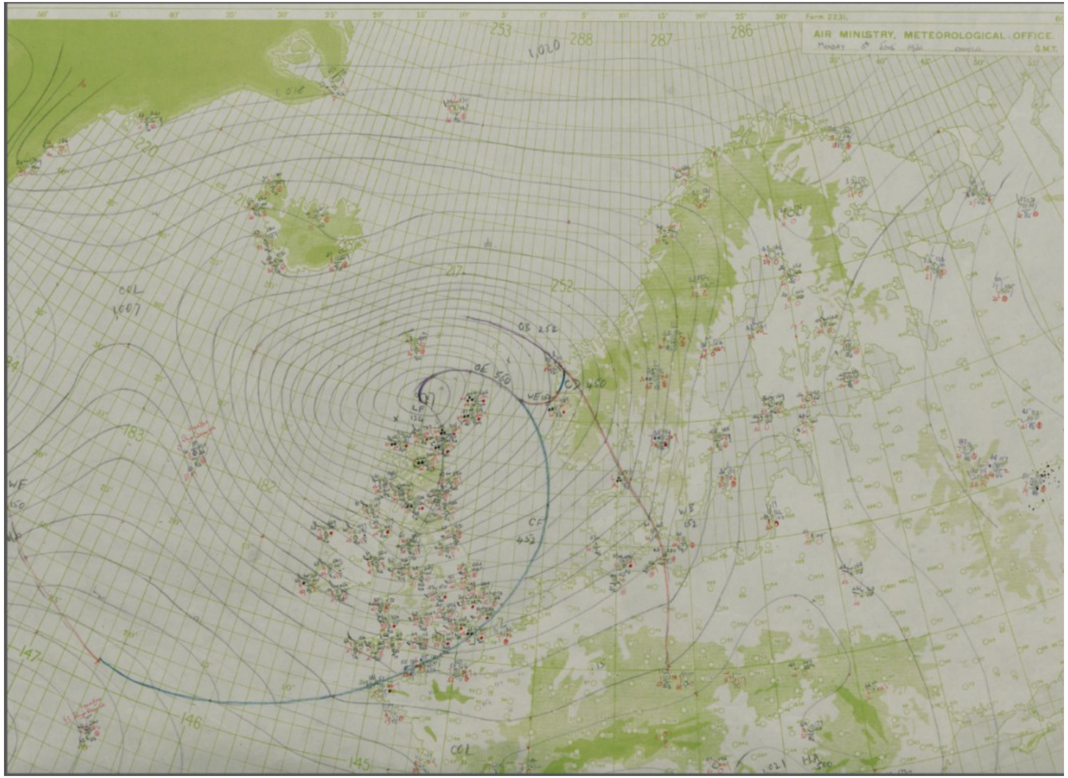
yields the altimeter equation:

$$p_1 = p_2 e^{\frac{g z_2}{R_d \overline{T_v}}}$$

Application of the hypsometric equation : the altimeter equation



Link to MeteoSwiss blog (in French) on how to read synoptic maps



Synoptic map 05 June 1944, just before « D-Day » (Normandy landings). Source: Met Office archives (UK)

Hydrostatic balance: take home messages

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Hydrostatic balance: take home messages

- A scale analysis of the vertical component of the equation of motion tells us that the atmosphere is in *hydrostatic balance*: the **vertical pressure gradient force** is balanced by **gravity**
- By combining the hydrostatic balance with the ideal gas law, we derived the *hypsometric equation*, which tells us that the **height** between two pressure levels is **proportional to the mean *virtual temperature*** in that layer
- A rearrangement of the hypsometric equation allows to **compare pressure measurements** at different altitudes by reducing them to a reference level

Scale analysis of the horizontal equation of motion

Scale analysis of the horizontal equation of motion

$$\frac{du}{dt} - \frac{uv\tan\phi}{a} + \frac{uw}{a} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + 2\Omega\sin\phi v - 2\Omega\cos\phi w + F_x$$

$$\frac{dv}{dt} + \frac{u^2\tan\phi}{a} + \frac{vw}{a} = -\frac{1}{\rho}\frac{\partial p}{\partial y} - 2\Omega\sin\phi u + F_y$$

Scale analysis of the horizontal equation of motion

	1	2	3	4	5	6	7
x equation	$\frac{du}{dt}$	$-2\Omega \sin\phi v$	$2\Omega \cos\phi w$	$\frac{uw}{a}$	$-\frac{uv \tan\phi}{a}$	$-\frac{1}{\rho}\frac{\partial p}{\partial x}$	F_x
y equation	$\frac{dv}{dt}$	$2\Omega \sin\phi u$		$\frac{uv}{a}$	$\frac{u^2 \tan\phi}{a}$	$-\frac{1}{\rho}\frac{\partial p}{\partial y}$	F_y
Scales	$\frac{U^2}{L}$	$f_0 U$	$f_0 W$	$\frac{UW}{a}$	$\frac{U^2}{a}$	$\frac{\delta p}{\rho L}$	$\frac{\nu U}{H^2}$
Magnitude ($\mathrm{m\,s^{-2}}$)	10^{-4}	10^{-3}	10^{-6}	10^{-8}	10^{-5}	10^{-3}	10^{-12}

→ Balance between the **horizontal pressure gradient and Coriolis** forces, how is it called?

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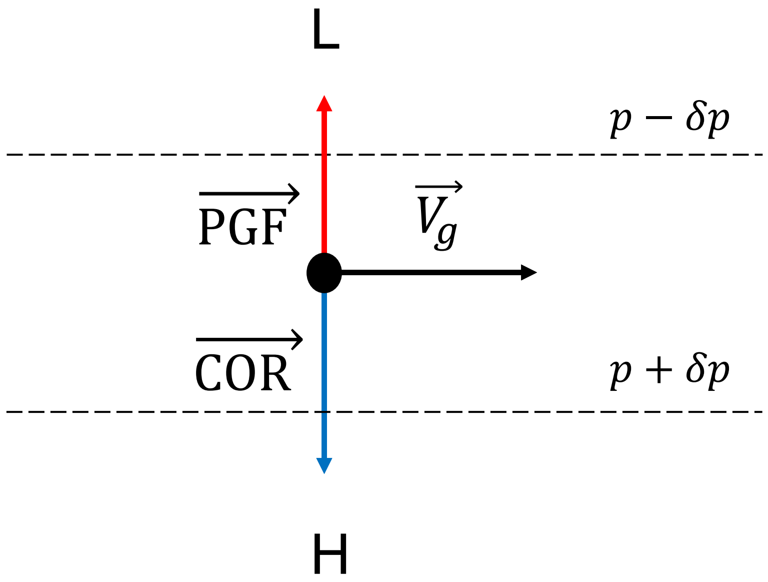
Geostrophic balance



$$\overrightarrow{V_g} = \frac{1}{\rho f}\vec{k} \times \nabla p$$

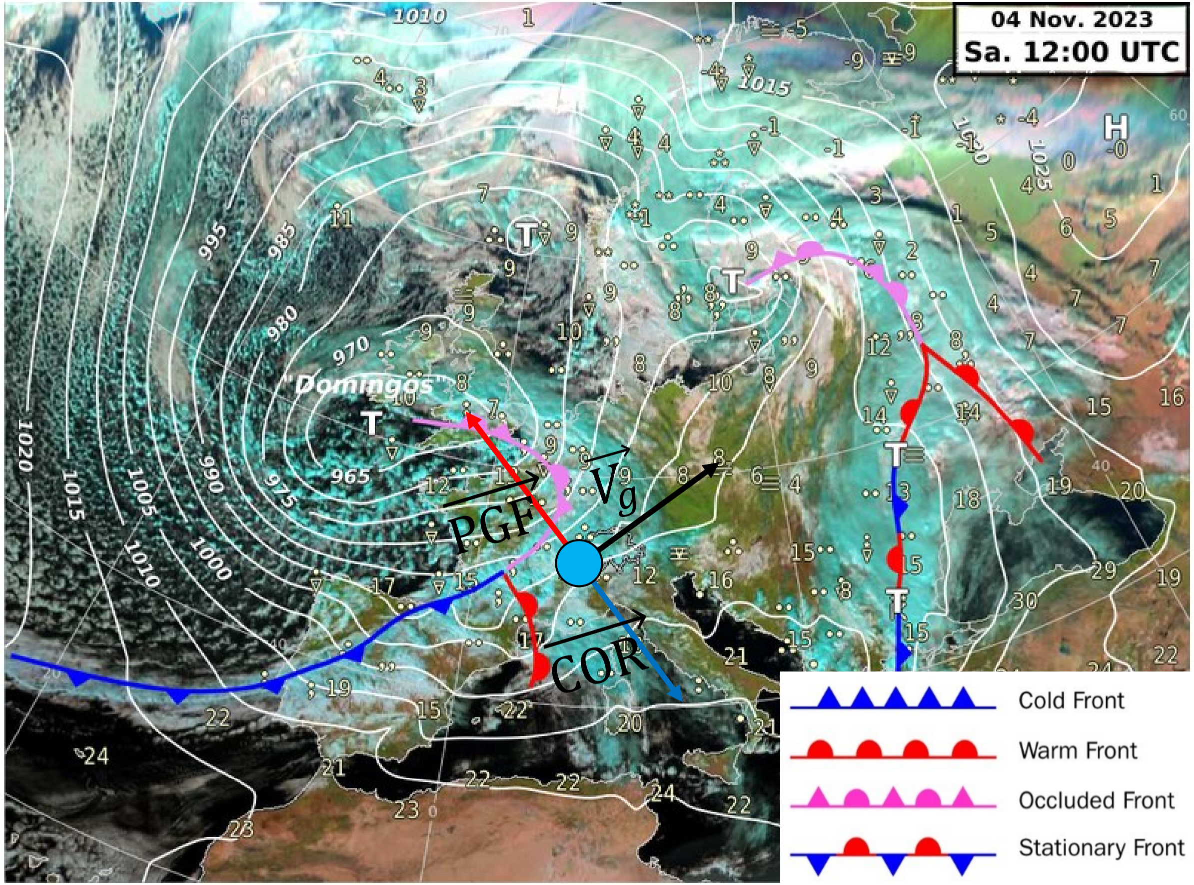
Valid if $\frac{d\vec{U}}{dt}$ is small, which is not the case in:

i) along-flow acceleration

ii) flow curvatures

Geostrophic balance

Pressure distribution (white contours, labels in hPa) during storm Domingos. T = low pressure systems



Pressure as a vertical coordinate

JEM Sec. 4.1

- On an isobaric surface, the pressure gradient force becomes

$$PGF_p = -\nabla_p \Phi,$$

where ∇_p is the horizontal nabla operator on an isobaric surface, i.e. $\nabla_p = \frac{\partial}{\partial x}\hat{\imath} + \frac{\partial}{\partial y}\hat{\jmath}$ (1).

- The geostrophic wind is hence:

$$\overrightarrow{V_g} = \frac{1}{f}\vec{k} \times \nabla_p \Phi$$

→ The **density $\boldsymbol{\rho}$ disappeared**, this is one of the main motivation to use pressure as a vertical coordinate, since air density is hard to measure accurately

- The **vertical wind velocity** in isobaric coordinates is: $\omega = \frac{dp}{dt}$ and since pressure is decreasing with height, **$\boldsymbol{\omega}$ is negative for upward motions**.
- In isobaric coordinates, the **geostrophic wind is non-divergent** (1), i.e. $\nabla \cdot \overrightarrow{V_g} = 0$

(1) The full derivation of isobaric coordinates can be found in JEM 4.1

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Equation of motion: take home messages

- From a **complicated set of equations** (i.e. the equation of motion), we conducted a **simple scale analysis**, which allowed to us to identify **two dominating forces** both in the vertical and horizontal components
  - In the vertical, the **pressure gradient force** is balanced by **gravity** (**hydrostatic balance**)
  - In the horizontal, the **pressure gradient force** is balanced by the **Coriolis force** (**geostrophic balance**)

→ *To a first order, the mid-latitude atmosphere on Earth is in hydrostatic and geostrophic balance.*

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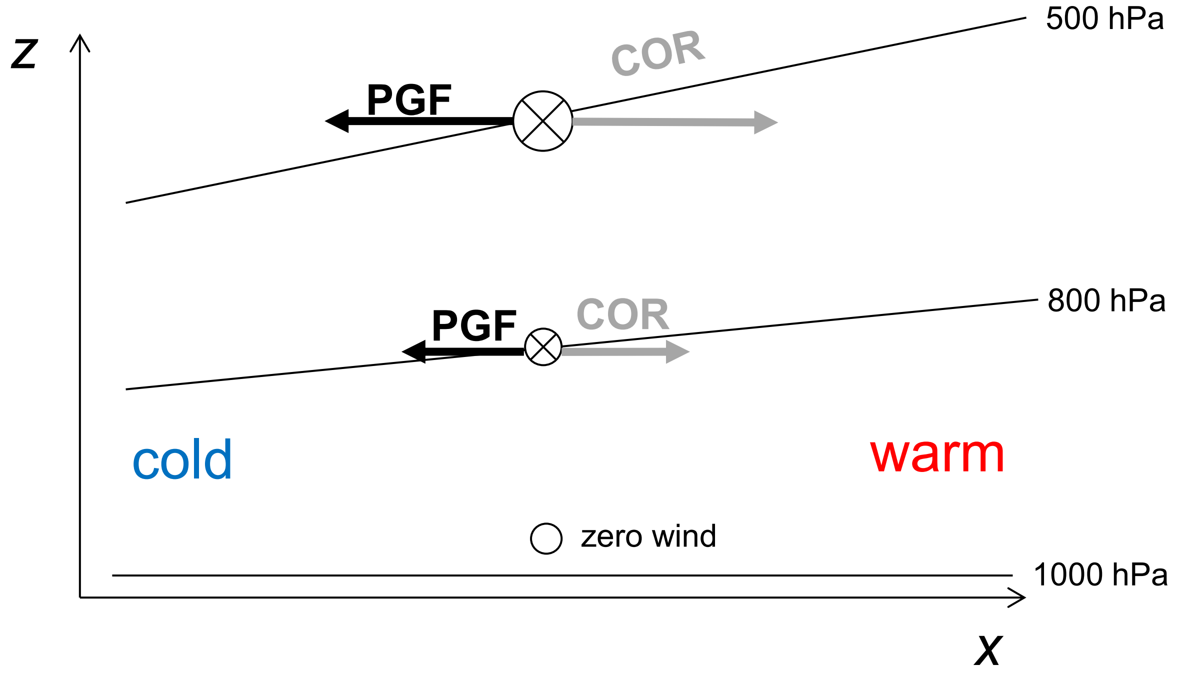
Combining the geostrophic and hypsometric equation: the thermal wind relationship

Thermal wind

JEM Sec. 4.3



→ A **horizontal temperature gradient** implies a **vertical shear of the geostrophic wind**

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Thermal wind

$$\frac{\partial \vec{V}_g}{\partial p} = -\frac{R}{fp}\hat{k} \times \nabla T$$

- The vertical shear of the geostrophic wind is known as the **thermal wind**
- The **thermal wind vector** is simply the difference between the geostrophic wind at some upper level with the one at some lower level. It is hence defined as:

$$\vec{V}_T = -\frac{\partial \vec{V}_g}{\partial p}$$

- The **thermal wind vector** is **parallel to the isotherms** or to isopleths of thickness(1) leaving lower temperature to its left in the northern hemisphere (2)

(1) Recall that the thickness between two pressure levels is related to the average (virtual) temperature of that column (hypsometric equation).

(2) The Coriolis parameter $f = 2\,\Omega \sin\varphi$ is negative in the southern hemisphere, since the latitude φ is negative. So the opposite is true in the southern hemisphere.

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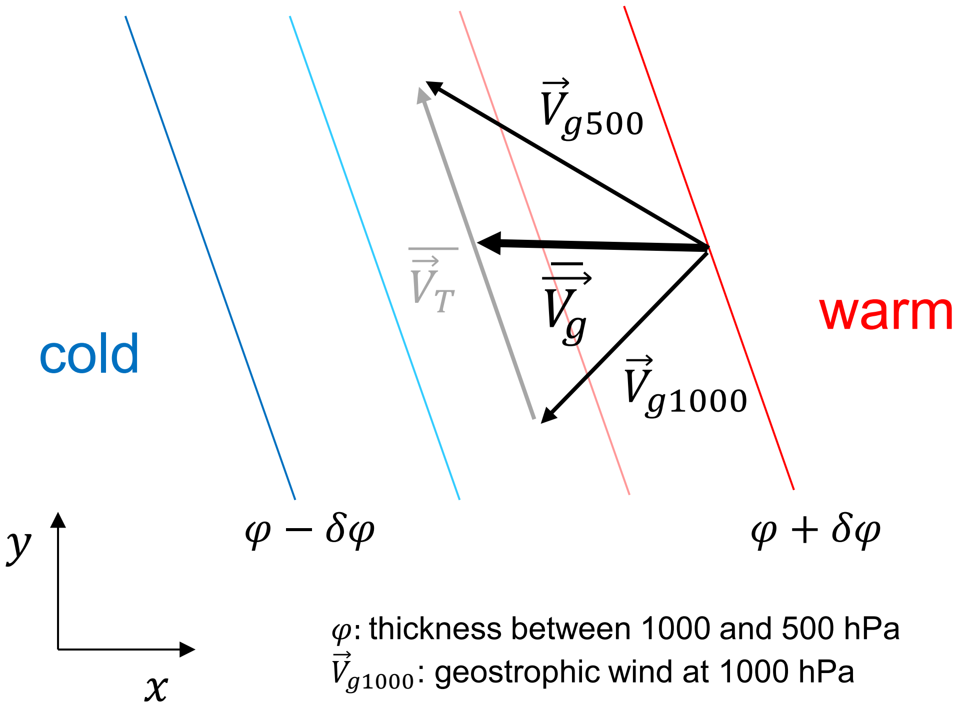
Implications of the thermal wind: temperature advection



φ: thickness between 1000 and 500 hPa
$\vec{V}_{g1000}$: geostrophic wind at 1000 hPa

$$\frac{\partial \vec{V}_g}{\partial p} = -\frac{R}{fp}\hat{k} \times \nabla T$$

Temperature advection given by:

$$-\overline{\vec{V}_g} \cdot \nabla \overline{T}$$

In the northern hemisphere:

- **Veering** (turning clockwise) winds with height is associated with **warm air advection**
- **Backing** (turning anticlockwise) winds with height is associated with **cold air advection**

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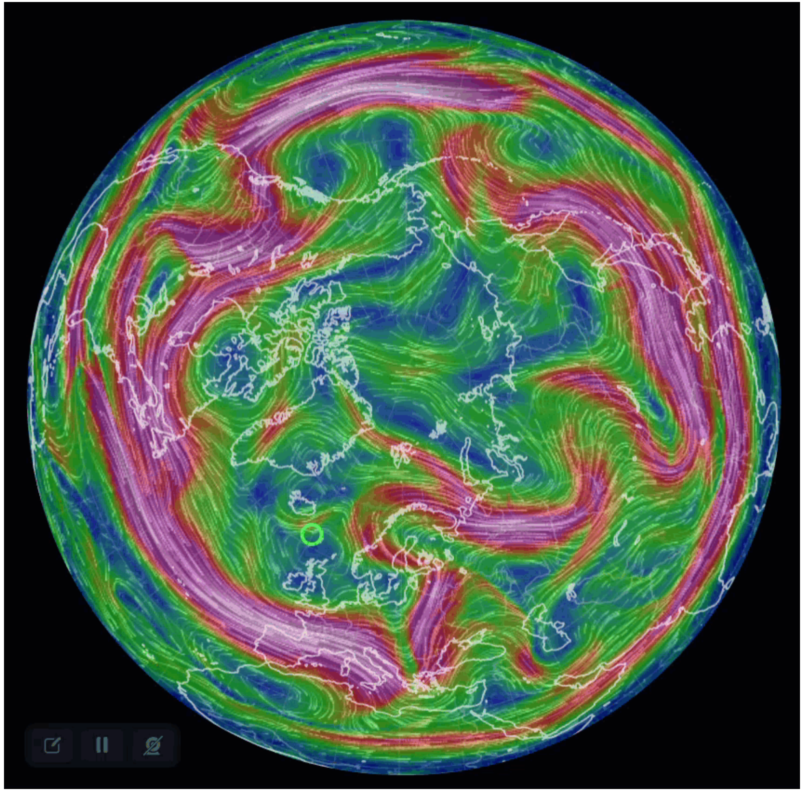
Implications of the thermal wind: the jet stream

Why we have a **jet stream**?

- **Differential solar heating** between the poles and the equator implies a **meridional temperature gradient**
- The thermal wind balance demands that this gradient be related to an **increasing westerly component** of the wind with height

→ Hence the **maximum westerly winds** just below the tropopause[1]

→ Hence **weather systems are moving from west to east**



Wind at 250 hPa on 04 November 2023. Source: earth.nullschool.net

[1] At the tropopause and above, the meridional temperature gradient decreases and so does the thermal wind.

Thermal wind balance: take home messages

- Since warmer air masses are thicker than colder ones, geopotential height gradients vary with height in the presence of a temperature gradient → **geostrophic wind will vary with height**
- If isohypses are not parallel to isopleths of thickness, it implies a **temperature advection** which can be determined simply by the **distribution of the geostrophic wind with height**
- The presence of a **meridional temperature gradient** implies that the westerly component of the geostrophic wind must be increasing with height → **jet stream** at the tropopause and **weather systems are moving eastward**