

Previous sessions (with A. Nenes): cloud particle nucleation (CCN, INP, activation...)

This session focuses on processes explaining **how cloud particles grow** and eventually **form precipitation particles**. We will cover:

1. Warm cloud processes (liquid phase)
2. Cold cloud processes (ice + mixed-phase)
3. Additional cloud processes
4. Hydrodynamics
5. Drop size distribution (DSD)

Books (alphabetical order):

Lohmann, Lüönd and Marht, “An introduction to clouds...”, 2016 → LLM2016

Wallace and Hobbs, “Atmospheric Sciences...”, 2006 → WH2006

Wang, “Physics and dynamics of clouds and precipitation”, 2013 → W2013

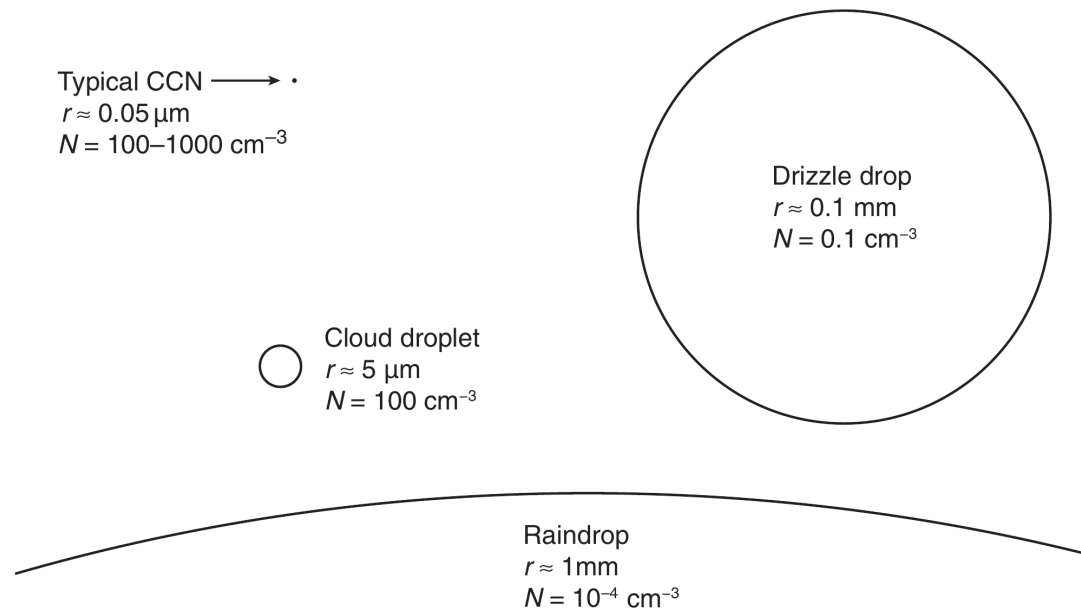
(online versions through EPFL library)

Microphysics = physical and chemical processes at the scale of individual cloud and precipitation particles (μm to cm)

Warm cloud = cloud containing only liquid droplets

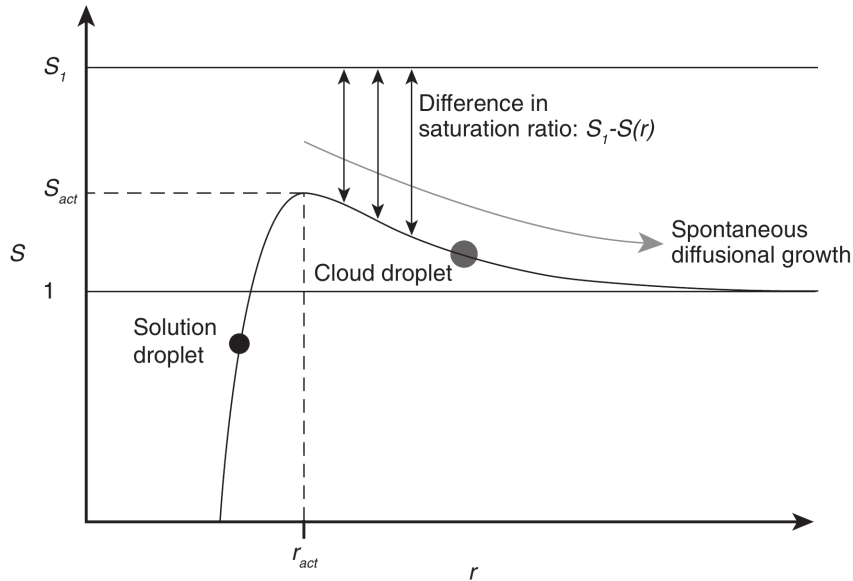
3 main processes:

- Vapor deposition (+)
- Collision-coalescence (+)
- Break-up and evaporation (-)



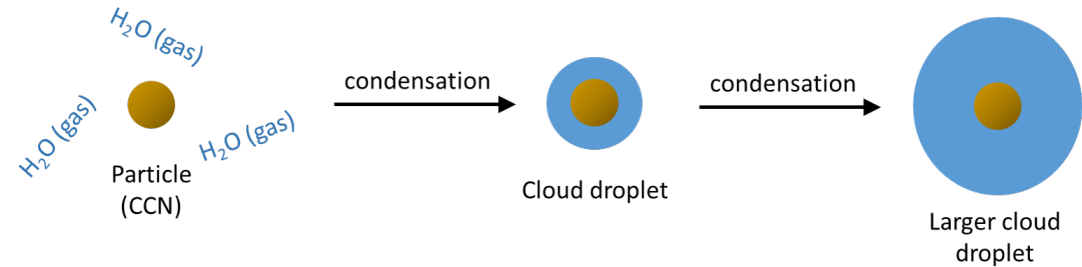
Vapor deposition

Activated droplet (Köhler's curve)



LLM2016, fig 7.2

How will this droplet grow?



<https://www.physics.uu.se/research/molcond/ongoingresearch/molecules-and-liquids/aerosols/>

Combination of 2 processes:

- Diffusion of water vapor
- Diffusion of heat

Diffusion of water vapor

Diffusion equation for isolated droplet:
(Fick's law)

$$\frac{dm_l}{dt} = 4\pi r D_v (\rho_v - \rho_{v,r})$$

m_l mass of liquid water

r droplet radius [m]

D_v water vapor diffusivity in air [$\text{m}^2 \text{s}^{-1}$]

ρ_v water vapor density – ambient [kg m^{-3}]

$\rho_{v,r}$ water vapor density – drop surface [kg m^{-3}]

Diffusion of heat / heat conduction

Latent + sensible heat:
(Fick's law)

$$\frac{dQ}{dt} = 4\pi r K (T_r - T)$$

Q heat energy transferred to droplet [J]

r droplet radius [m]

K thermal conductivity in air [$\text{J m}^{-1} \text{s}^{-1} \text{K}^{-1}$]

T_r Temperature at droplet surface [K]

T Temperature ambient air [K]

Droplet growth equation

Combining the 2 previous equations:

$$\frac{dm_l}{dt} = 4\pi r_d \frac{S - 1}{F_k^l + F_d^l} \begin{cases} F_k^l = \left(\frac{L_v}{R_v T} - 1 \right) \frac{L_v}{KT} \\ F_d^l = \frac{1}{D_v \rho_{vs}} = \frac{R_v T}{D_v e_{s,w}(T)} \end{cases} \quad \begin{aligned} K &\simeq 4.1868 \cdot 10^{-3} [5.69 + 0.017(T - T_0)] \\ D_v &\simeq 2.11 \cdot 10^{-5} \left(\frac{T}{T_0} \right)^{1.94} \left(\frac{P_0}{P} \right) \end{aligned}$$

$$r_d \frac{dr_d}{dt} = \frac{S - 1}{F_k + F_d} \begin{cases} F_k = \rho_l F_k^l \\ F_d = \rho_l F_d^l \end{cases}$$

S	saturation ratio [-]
L_v	Latent heat of vaporization [J kg ⁻¹]
R_v	gas constant of water vapor = 461.5 J kg ⁻¹ K ⁻¹
T/T_0	temperature / 273.15 [K]
ρ_{vs}	water vapor density at saturation [kg m ⁻³]
ρ_l	water density [kg m ⁻³]
$e_{s,w}$	water vapor pressure at saturation [Pa]
P/P_0	air pressure / at sea level (101325) [Pa]

Solution:

$$r_d(t) = \sqrt{r_{d_0}^2 + 2 \left(\frac{S - 1}{F_k + F_d} \right) t}$$

- For large droplets, neglecting solution and curvature effects
- **Droplet grows as \sqrt{t}**

Growth of a population of droplets

When a droplet is not isolated, we must consider interactions with ambient air and other droplets.

Developing cloud (no sedimentation, no coalescence), in which supersaturation:

- is due to ascending air – adiabatic expansion and cooling: $Q_1 \frac{dz}{dt}$ $Q_1 = \frac{1}{T} \left(\frac{\epsilon L_v g}{R_d c_p T} - \frac{g}{R_d} \right)$
- is consumed by deposition: $-Q_2 \frac{dq_l}{dt}$ $Q_2 = \rho \left(\frac{R_d T}{\epsilon e_{s,w}} + \frac{\epsilon L_v^2}{P T c_p} \right)$

So the saturation S evolves as

$$\frac{dS}{dt} = Q_1 \frac{dz}{dt} - Q_2 \frac{dq_l}{dt}$$

If updraft velocity increases, saturation increases

z altitude [m]

ϵ entrainment [-]

g acceleration due to gravity [m s^{-2}]

c_p Specific heat capacity of dry air [$\text{J kg}^{-1} \text{K}^{-1}$]

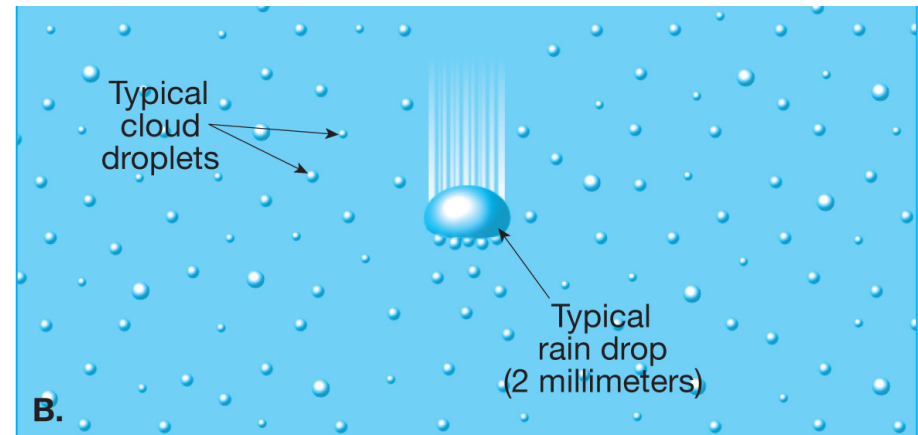
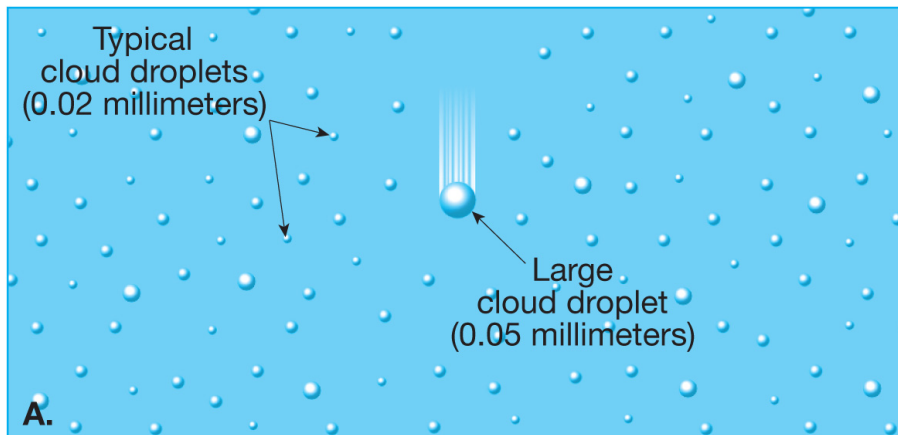
R_d Gas constant of dry air $287 \text{ J kg}^{-1} \text{K}^{-1}$

Collision-coalescence

Growth by deposition is too slow to explain formation of precipitation in cumulus clouds (~20-30 min from cloud formation to precip occurrence).

→ Growth by **collision-coalescence**: two droplets (same size or not) collide and stick together

Droplets must be larger than 15-20 μm , so terminal velocity (gravity~drag) becomes significant. The larger the drop(let), the larger the differential fall velocity, the more collisions.

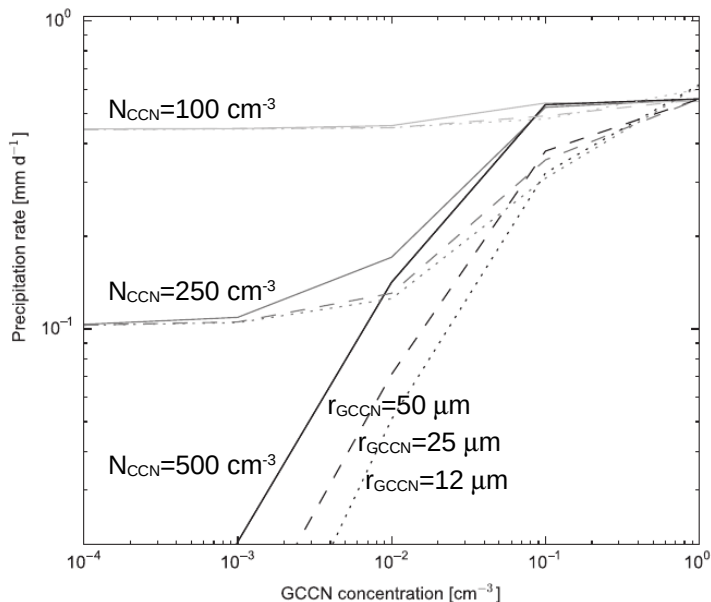


Collision – Initiation

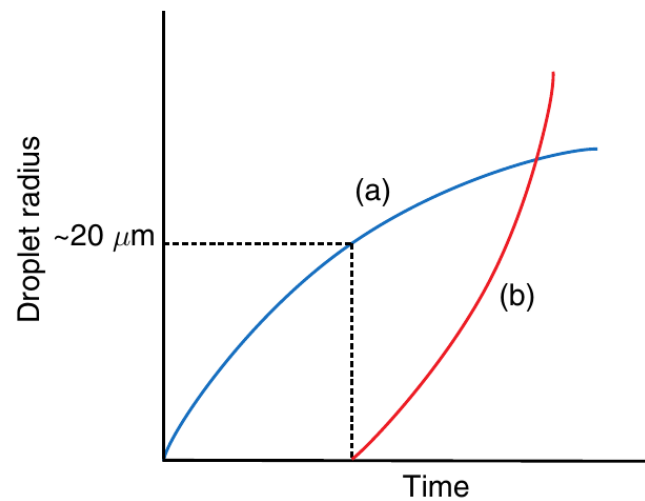
Larger droplets increase the probability of collision → we need to broaden the size distribution.

To initiate creation of larger droplets (that fall faster and hence collect smaller droplets):

- Giant CCNs → large droplets after activation (less critical when low CCN concentration)
- Turbulence → increases probability of collision



LLM2016, fig7.7



(a) growth by deposition
(b) growth by collision-coalescence

WH2006, fig6.15

Collision – Efficiency

Collision efficiency:
$$E = \frac{y^2}{(r_1 + r_2)^2}$$

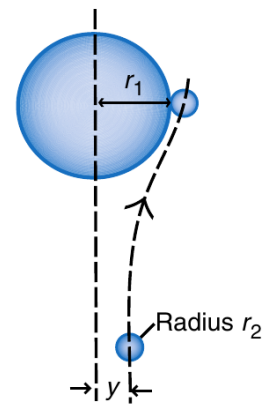


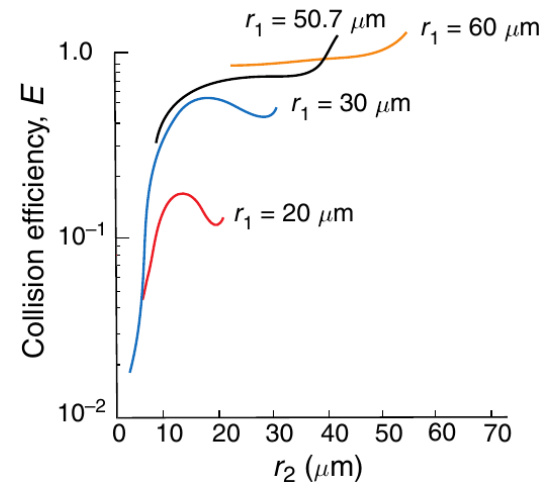
Fig. 6.19 Relative motion of a small droplet with respect to a collector drop. y is the maximum impact parameter for a droplet of radius r_2 with a collector drop of radius r_1 .

2 factors influence y and E :

WH2006, fig6.19 + 6.20

1. Air flows around droplet → larger droplets deflects more (-)
2. Inertia to cross streamlines is larger for larger droplets (+)

→ $E = f(r_1, r_2)$, E increasing with r_1 and with r_2 .
 E can be > 1 because of wake effect.



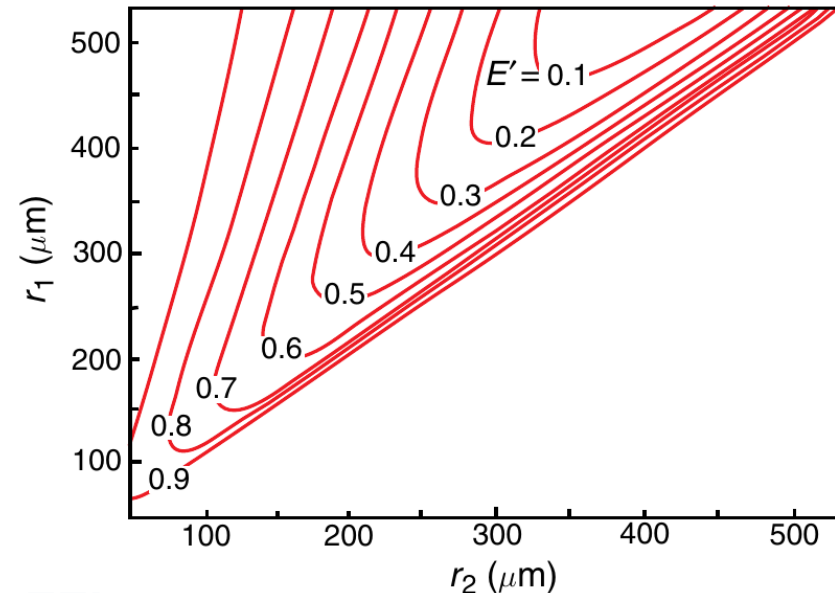
Coalescence – Efficiency

2 drops may collide but re-split after collision → we must study coalescence

Coalescence depends on relative magnitude of impact energy vs surface energy (of water)

Coalescence efficiency E' will be:

- Larger for small and large size ratios (r_2/r_1) because smaller impact energy (size / vel).
- Smaller for intermediate size ratios because higher impact energy → bouncing more likely.



WH2006, fig6.22

Collision + coalescence → **Collection efficiency $E_c = EE'$**

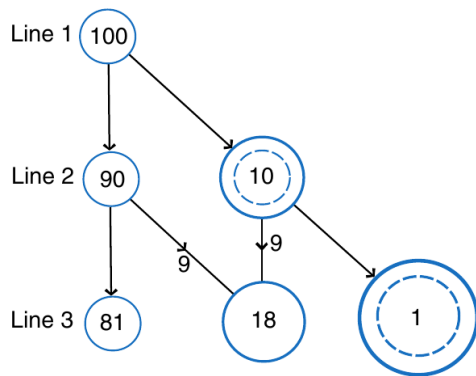
Growth model for stochastic collection

One way to (statistically) describe the process of collection (collision + coalescence)

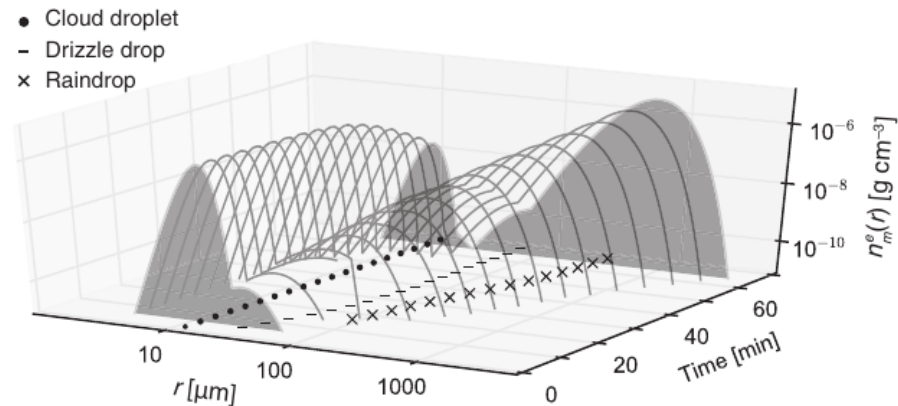
Stochastic collection equation → temporal evolution of droplet mass distribution f

$$\frac{\partial f(m_x, t)}{\partial t} = \frac{1}{2} \int_0^{m_x} f(m_x - m_y, t) f(m_y, t) K(m_x - m_y, m_y) dm_y - \int_0^\infty f(m_x, t) f(m_y, t) K(m_x, m_y) dm_y$$

K collection kernel
 m_x mass of drop x



WH2006, fig6.24



Three-dimensional illustration of the temporal evolution of a droplet size distribution (cloud droplets, drizzle and rain) in an idealized simulation over 70 minutes in steps of 200 s. The cloud class was initialized with a total number concentration $N_c = 75 \text{ cm}^{-3}$ and a cloud water content $M_l = 0.75 \text{ g m}^{-3}$, indicated by the gray shaded mass distribution at $t = 0$. Figure adapted from Sant *et al.* (2013).

Break-up

Large raindrops may break up in smaller drops because:

- Aerodynamically induced circulation within the drop may overcome surface tension force.
- **Collision with another drop** (energy transformation during collision-coalescence).

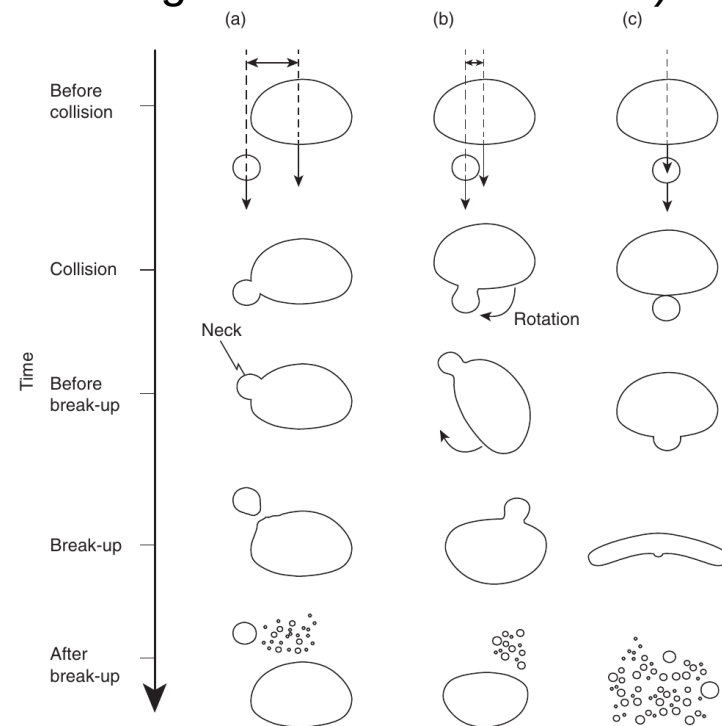
3 main types of break-up following a collision
(from theoretical and lab experiments)

1. Neck or filament BU (27%)

2. Sheet BU (55%)

3. Disk BU (18%)

Importance of the location of the point of impact...



Schematic illustration of the three common types of raindrop break-up: (a) neck or filament break-up, (b) sheet break-up and (c) disk break-up.

Evaporation

When drop(lets) are falling in unsaturated air, evaporation takes place.

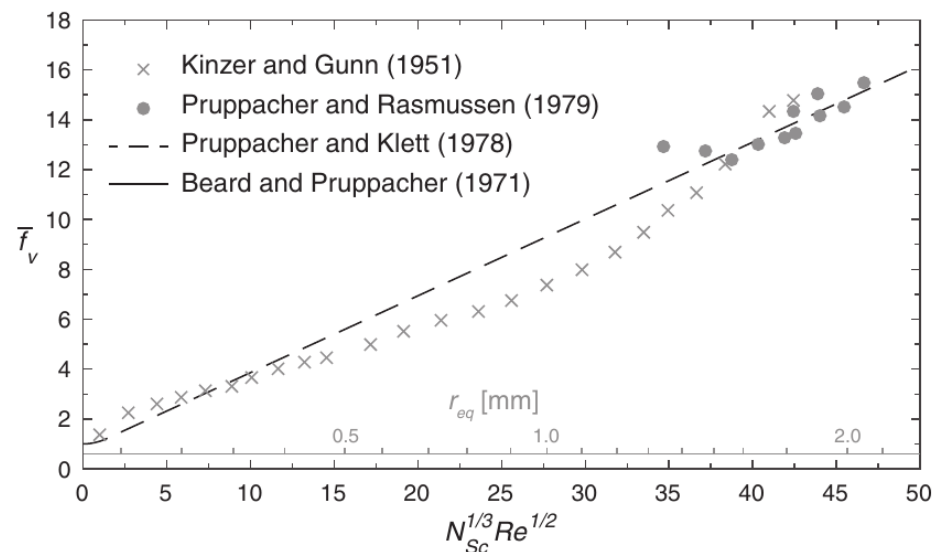
The droplet growth equation (p.5) can be used to describe this process.

Raindrops being much larger, the flow around them is turbulent, enhancing evaporation

→ **ventilation coefficient** $\bar{f}_v = \frac{dm/dt}{(dm/dt)_0}$ $(dm/dt)_0 =$ water mass flux for motionless drop

\bar{f}_v can be empirically described as function of Reynolds and Schmidt numbers

$\bar{f}_v \sim 1$ for drops $r_{eq} < 60 \mu\text{m}$
 ~ 9 $r_{eq} = 1 \text{ mm}$
 ~ 15 $r_{eq} = 2 \text{ mm}$



1. What are the main factors influencing growth by vapor deposition?
2. Explain the collision-coalescence mechanism.
3. Why cannot raindrops grow very large?

Nucleation – reminder

Homogeneous nucleation:

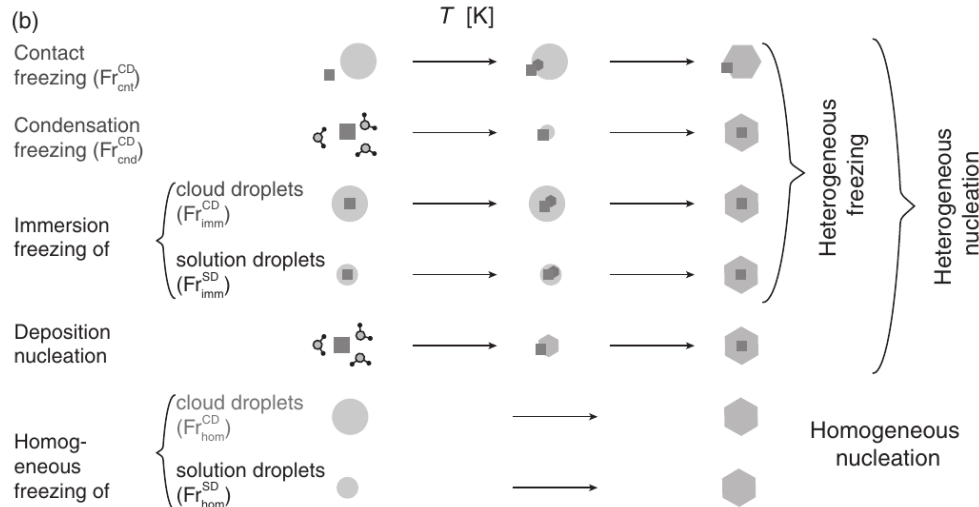
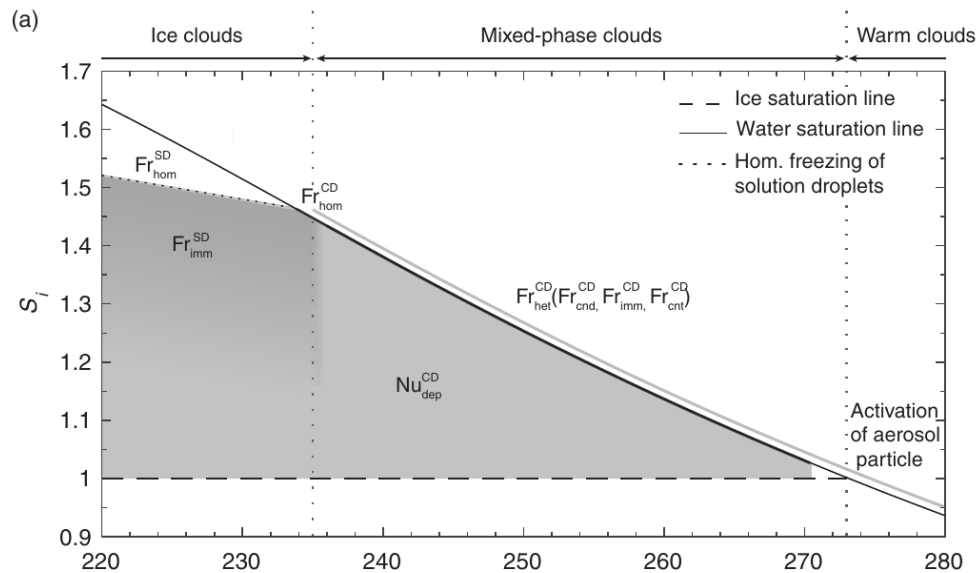
- liquid: not possible in atm conditions...
- ice phase: via freezing of liq droplets ($T < -37^\circ\text{C}$)

Heterogeneous nucleation:

On ice nucleating particles (INP), via

1. Immersion freezing
2. Condensation freezing
3. Contact freezing
4. Deposition nucleation

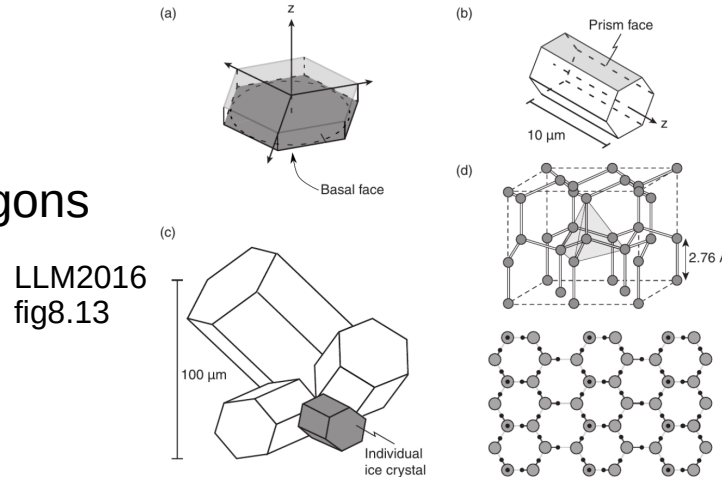
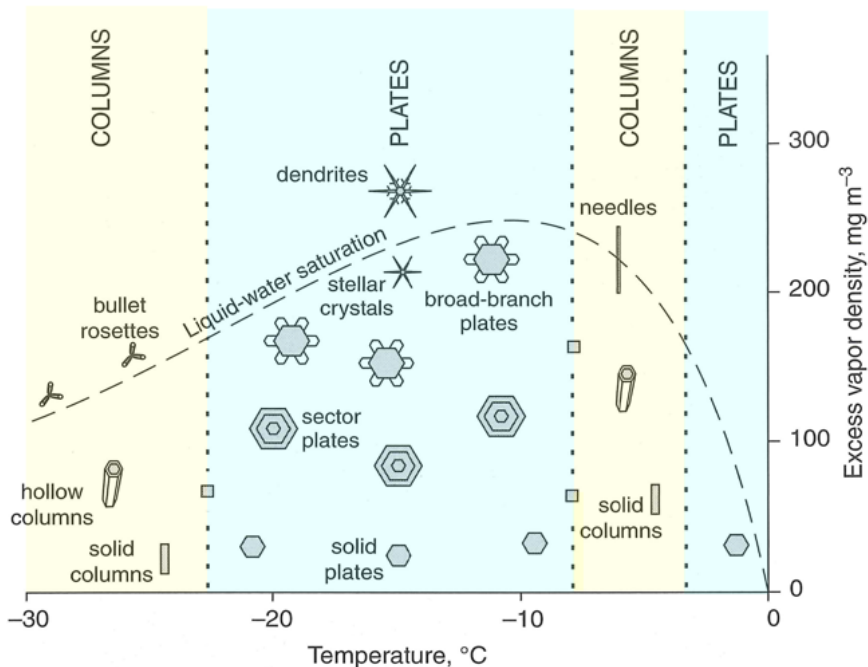
Contact freezing and deposition nucleation are 2nd order in mixed-phase clouds.



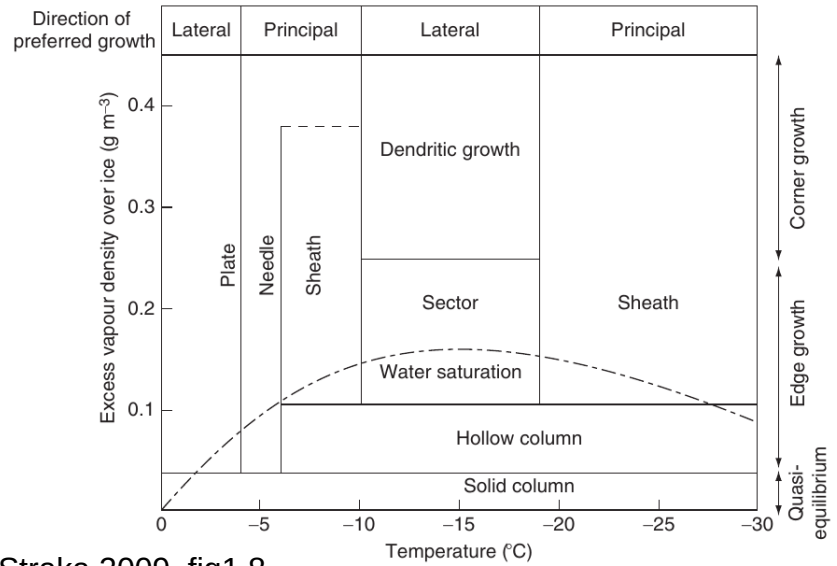
Ice crystal habits

Water molecules form H bonds → tetrahedral arrangement
 → ice lattice = stack of hexagons

Habit depends on temp. and saturation (liq. or ice)



LLM2016 fig8.13

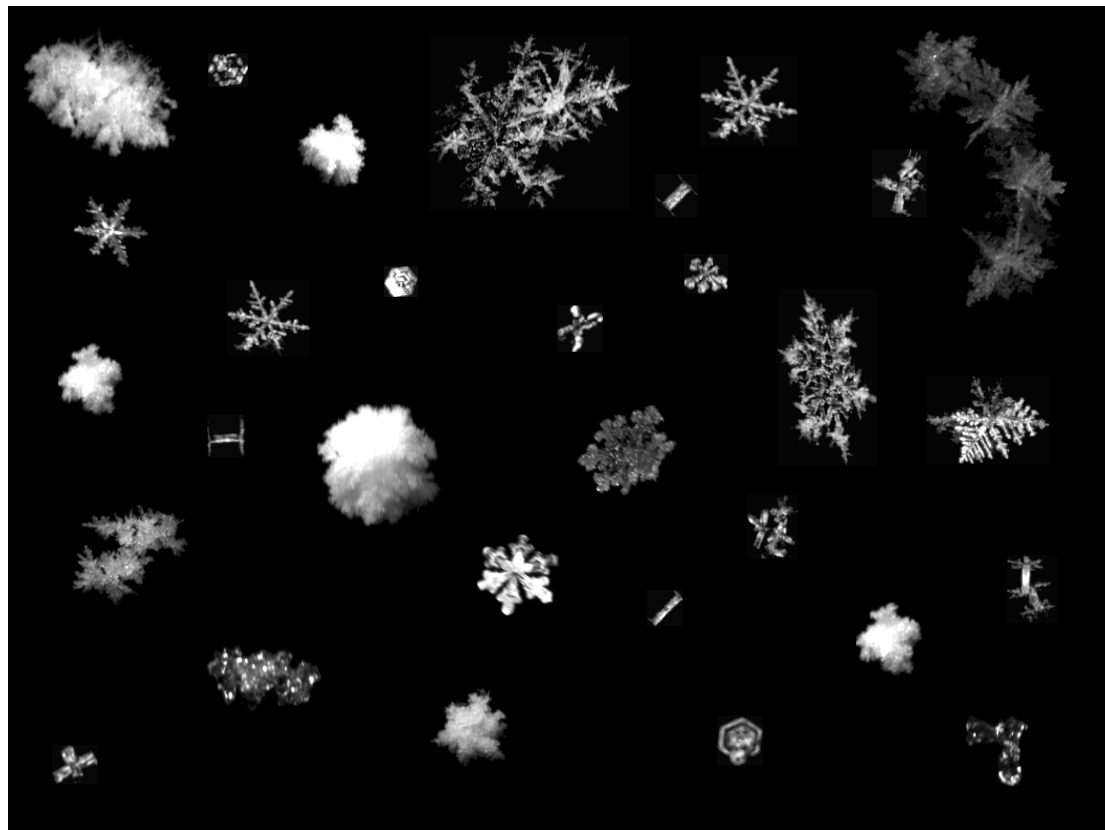
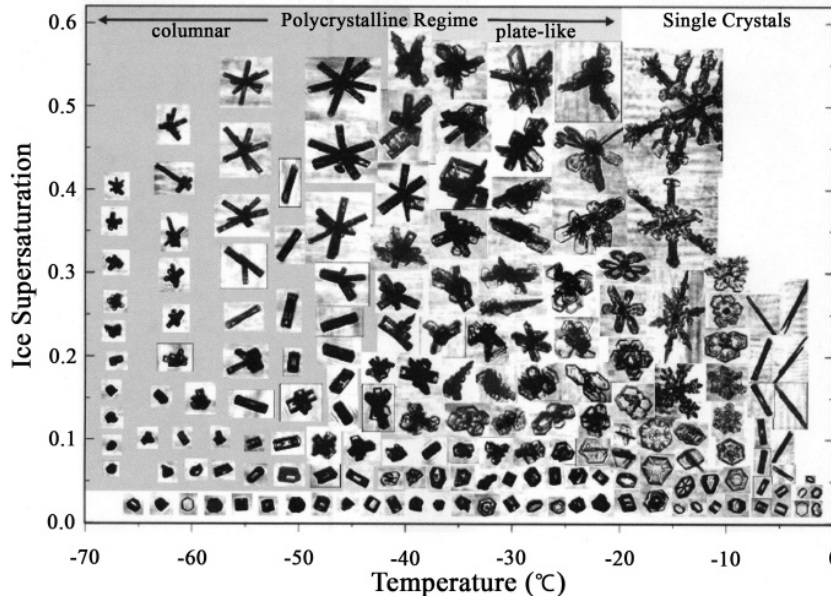
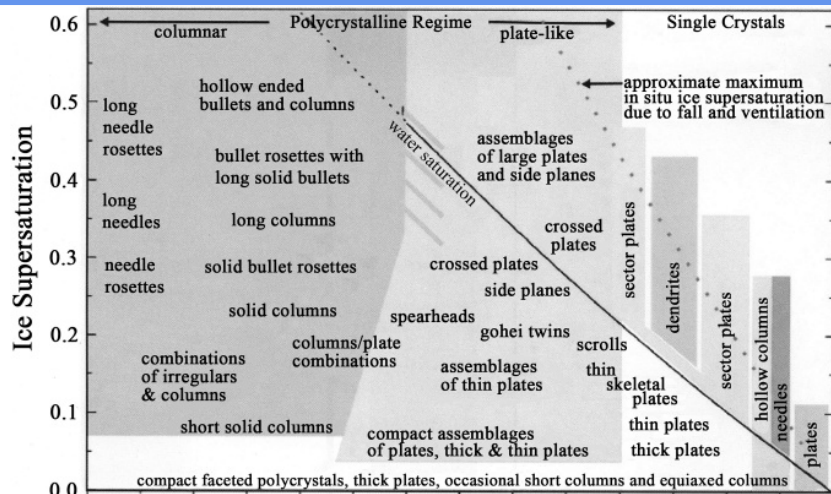


Straka 2009, fig1.8

Ice crystal habits

More complete diagram →
 Natural crystals photographed by a MASC ↓

W2013, fig9.10



Ice crystal growth

Ice embryo can grow through various processes:

1. Growth by deposition

A newly formed ice crystal will grow faster by deposition than an activated droplet because saturation wrt ice is higher than wrt liquid.

Equation similar to droplet growth by deposition:

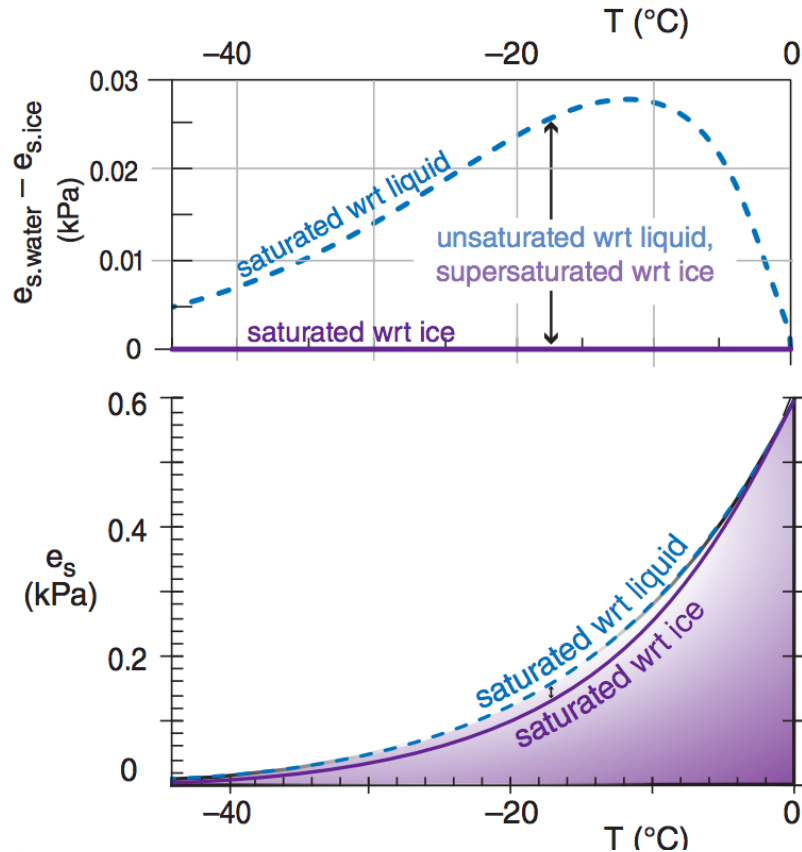
$$\frac{dm_i}{dt} = 4\alpha_m \pi C \frac{S_i - 1}{F_k^i + F_d^i} \begin{cases} F_k^i = \left(\frac{L_s}{R_v T} - 1 \right) \frac{L_s}{KT} \\ F_d^i = \frac{R_v T}{D_v e_{s,i}(T)} \end{cases}$$

α_m accommodation coef [-]
 C capacitance, f(size,shape) [m]
 L_s latent heat of sublimation [J kg⁻¹]
 S_i supersaturation / ice [-]
 $e_{s,i}$ saturation vapor pressure sat / ice [Pa]

Because water molecules from vapor will not join the crystal for every collision
 α_m in [0.2,1]

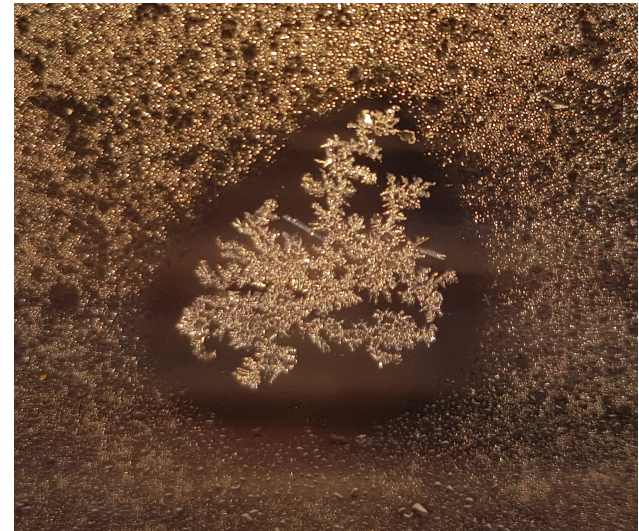
Ice crystal growth

An important factor is the difference between saturation vapor pressure wrt ice or liquid water



→ Max growth in mixed-phase clouds (MPC) $\sim -14^\circ\text{C}$

→ **Wegener-Bergeron-Findeisen process**: in MPC, ice crystals grow at the expense of supercooled liquid water (SLW) droplets.



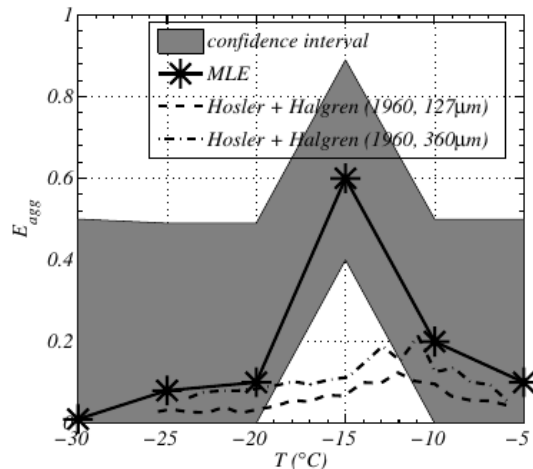
2. Growth by aggregation

Ice crystals can collide with other ice crystals and stick together → aggregation

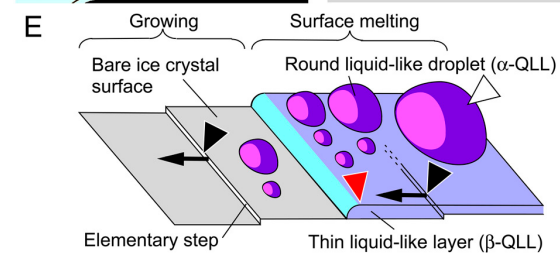
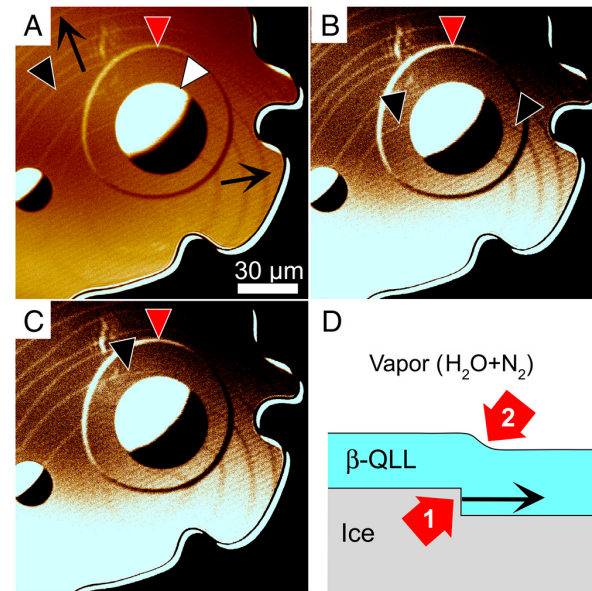
Aggregation efficiency depends on temperature and size/shape of crystals.

- Branch interlocking for dendrite like crystals ($\sim -15^\circ\text{C}$)
- Increased stickiness close to 0°C (quasi-liquid layer thick enough) → sintering

But it remains debated, poorly documented and understood...



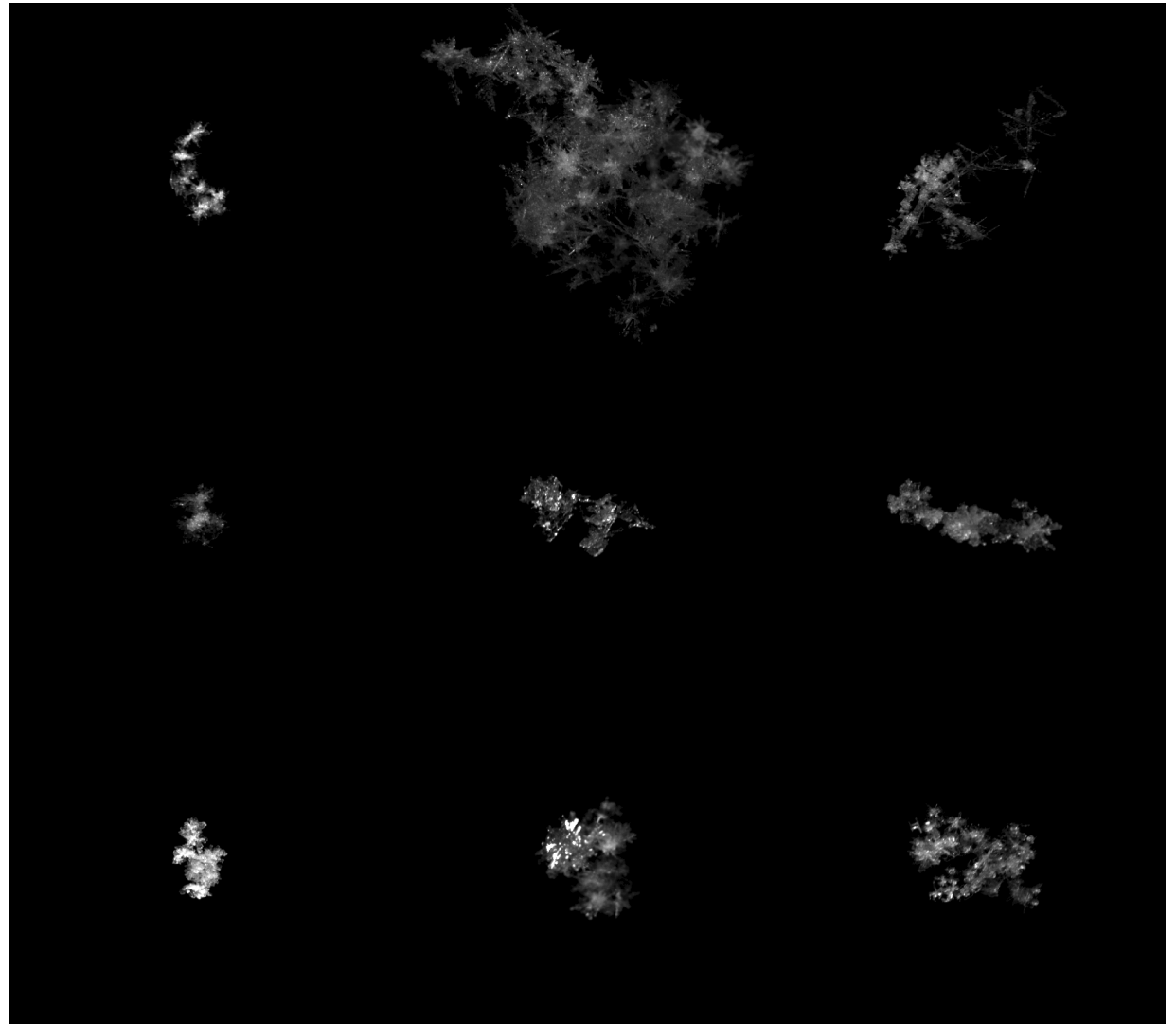
Connolly et al, ACP (2012)



Sazaki et al, PNAS (2012)

2. Growth by aggregation

Examples of aggregates
(collected at ground level)



3. Growth by riming

In MPC, ice crystals can collide with supercooled liquid water (SLW) droplets which freeze upon contact → **riming**
Sticking efficiency ~1

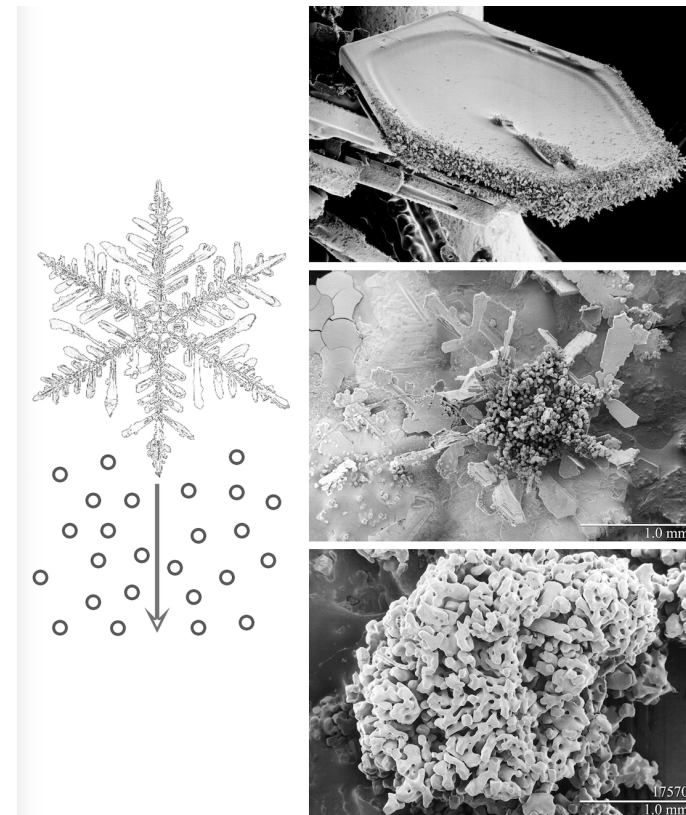
Efficient process to convert cloud water into precip water.

If initial crystal shape not discernible any more → **graupel**

Hailstone is an extreme case of riming

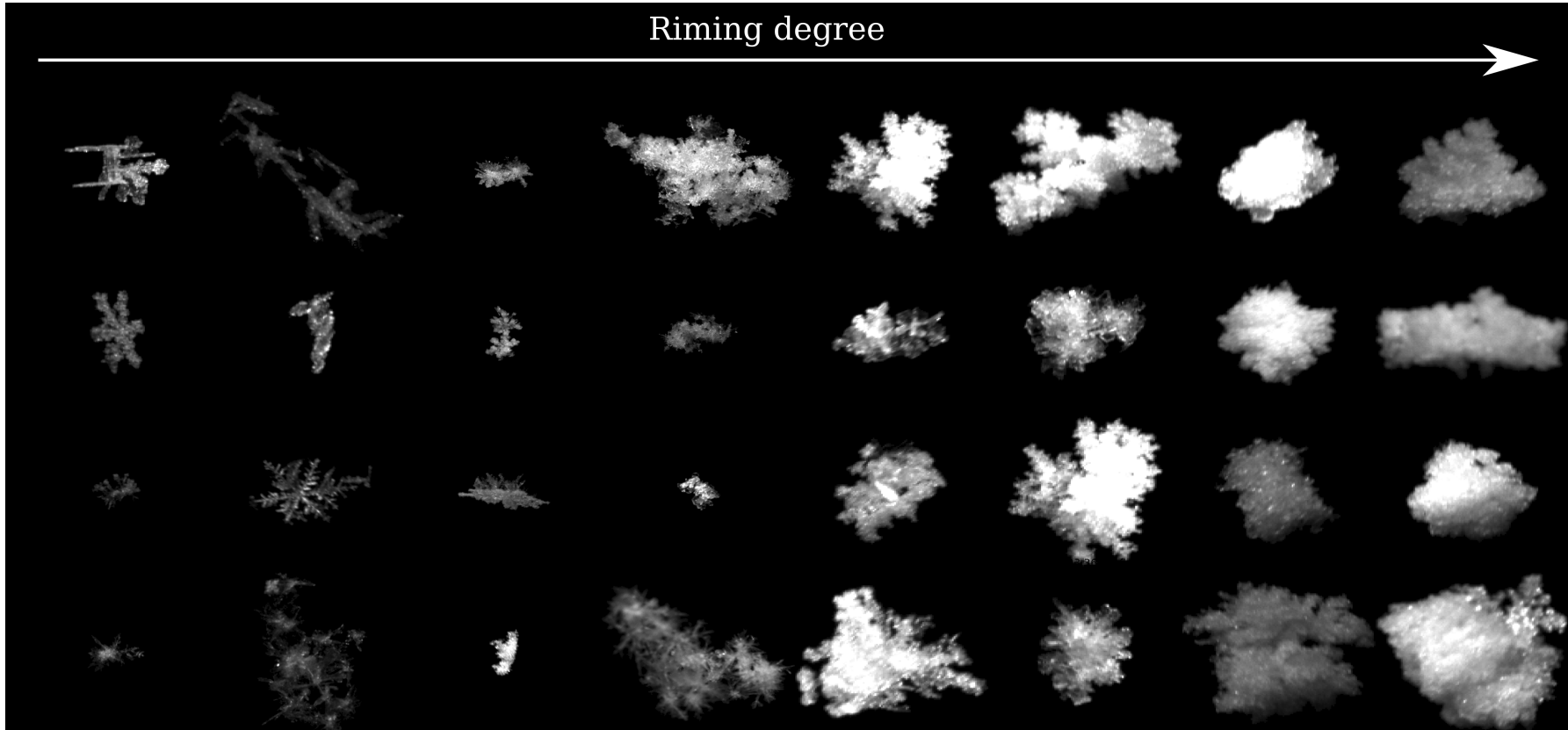


Praz et al, AMT (2017)



3. Growth by riming

Aggregates can also rime...



From MASC

3. Growth by riming

Riming not only active for cloud and precipitation particles...

Potential risk for aircraft



<http://www.securiteaerienne.com/rappel-theorique-niveaux-de-givrage/>

Rime accumulation near La Dôle



https://www.juralacs-meteo.fr/Galerie_photo/La_Dole_hiver_2.html

Summary of microphysical Processes in cold clouds

Deposition

WBF process

Aggregation

Riming

Secondary ice production

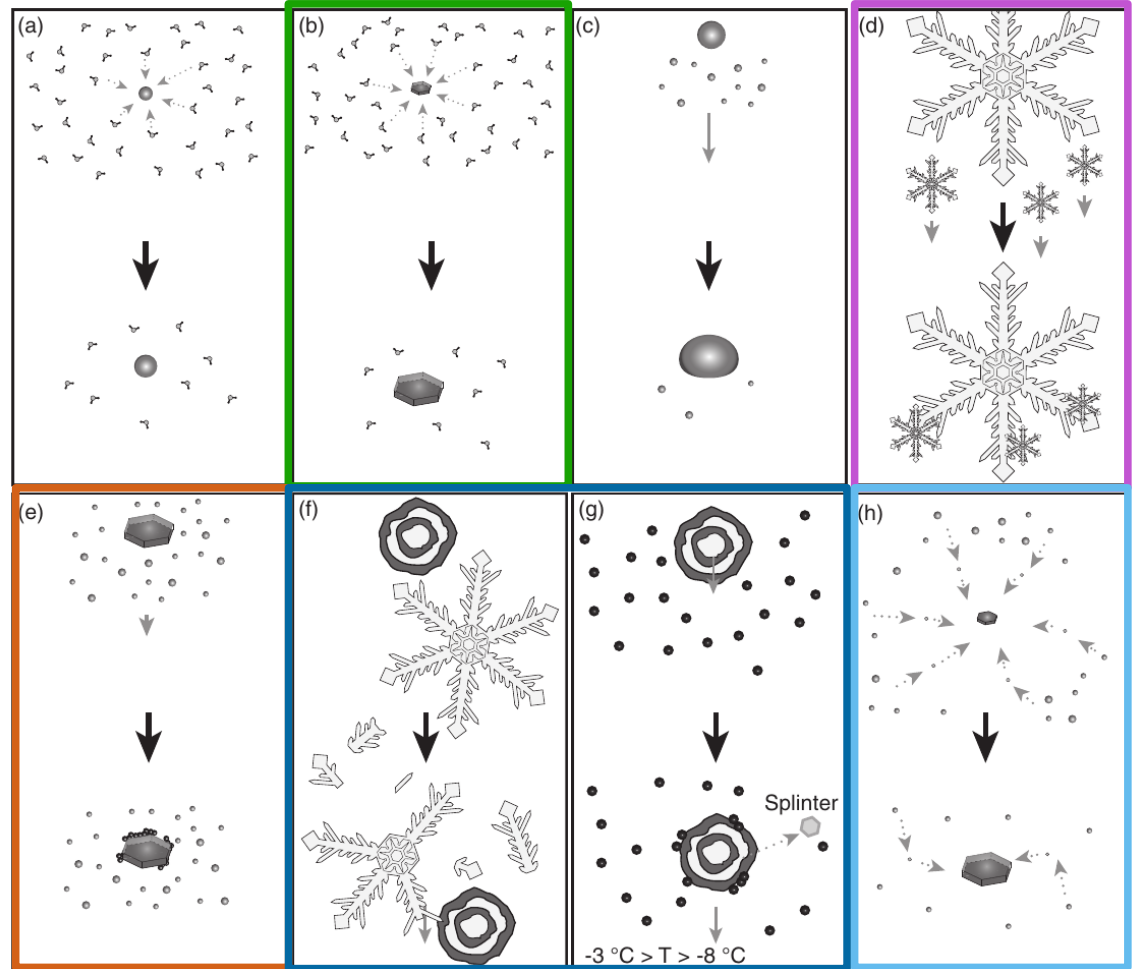


Fig. 8.18

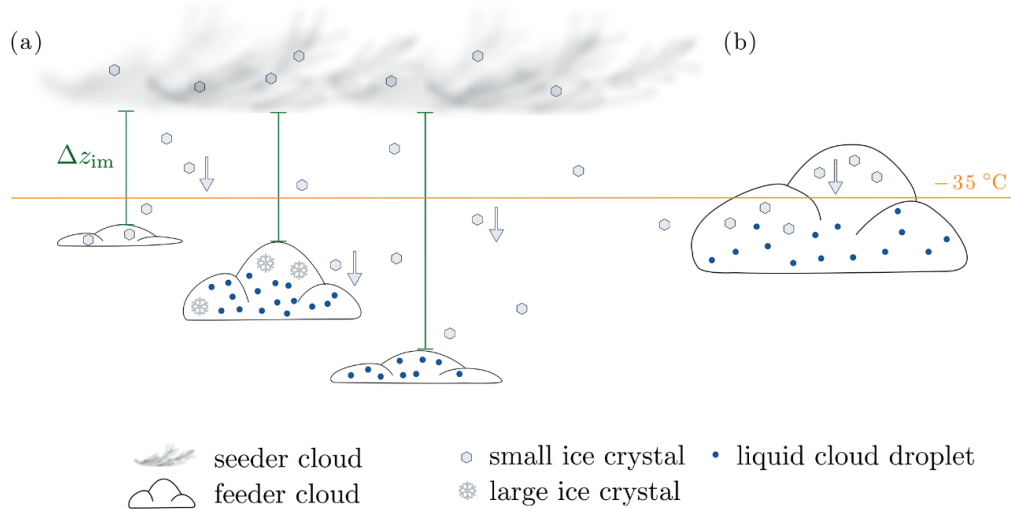
Schematic of microphysical processes in an earlier and later stage: (a) condensational growth, (b) depositional growth, (c) collision-coalescence, (d) aggregation, (e) riming, (f, g) ice multiplication, (h) Wegener-Bergeron-Findeisen process. The processes shown in (c), (d) and (e) are also referred to as growth by accretion.

1. What is the Wegener-Bergeron-Findeisen process?
2. At which temperatures are there peaks in aggregation efficiency?
3. What is riming?
4. What is secondary ice production?

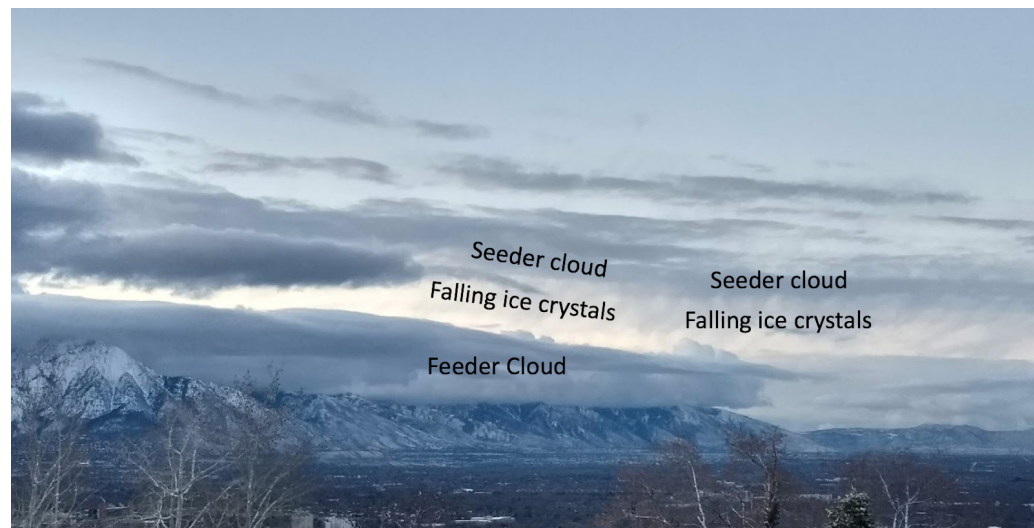
Seeder-feeder mechanism

Falling particles from upper cloud (seeder) grow by collecting particles in lower cloud (feeder)
 For both liquid and ice cloud particles.

Frequent in orographic context



Proske et al. ACP, 2021



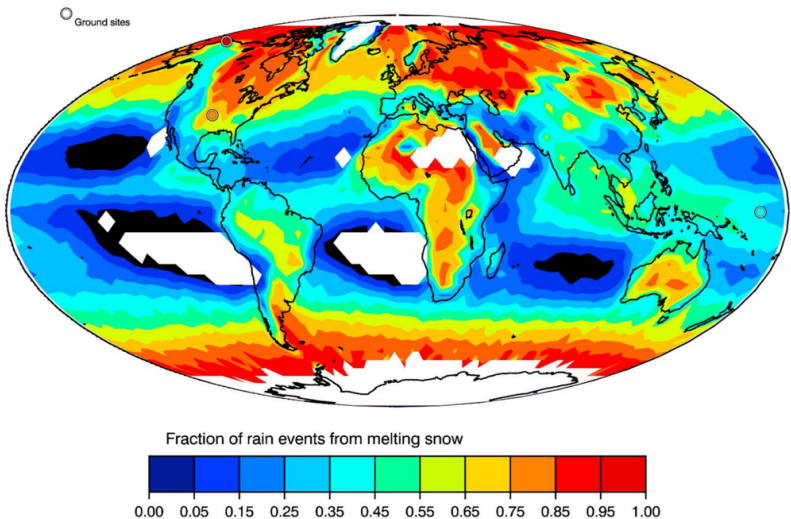
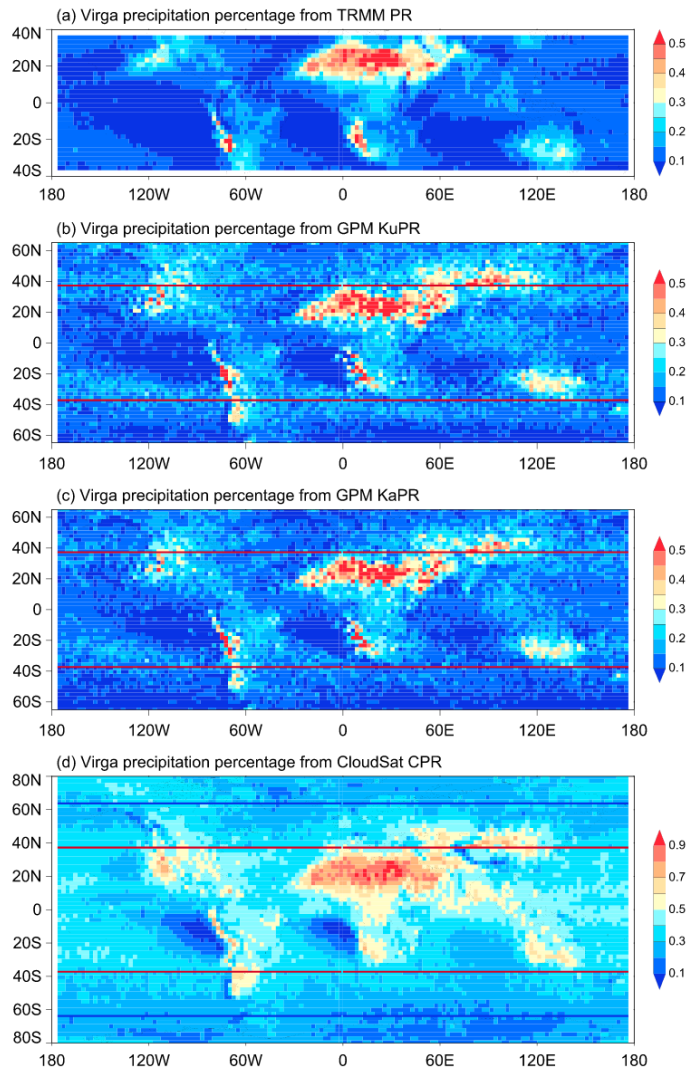
<https://wasatchweatherweenies.blogspot.com>

Over CH, ~30% of cloud situations...

Melting and evaporation/sublimation

Wang, GRL, 2018

Evaporation/sublimation: virga are frequent on a global scale

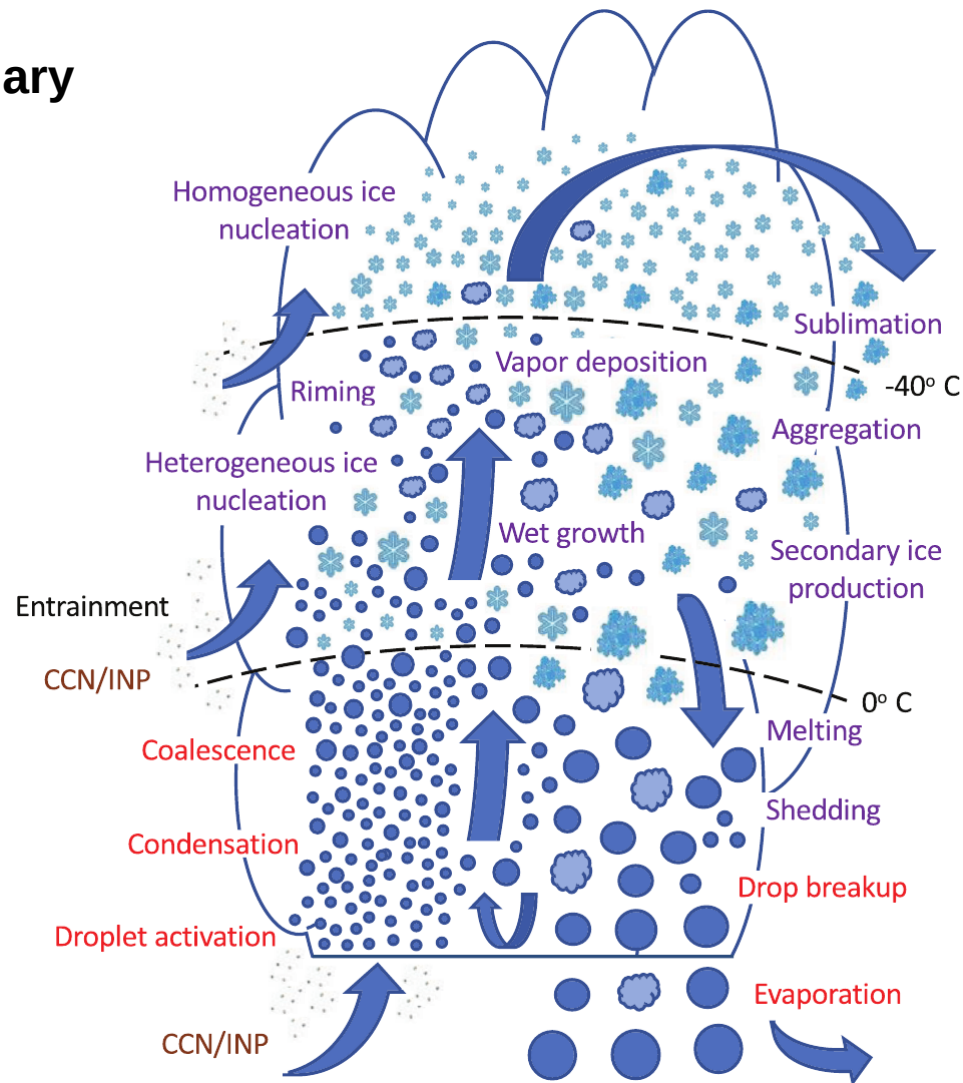


Field, GRL, 2015

In Antarctica, sublimation of ~30% precip in coastal areas (Grazioli et al, 2017)

Melting: raindrops often comes from snowflakes aloft

Overall summary



Knowledge gaps remain about detailed understanding of microphysical processes, in particular in the ice phase...

Morrison et al, JAMES, 2020

Fall velocity of cloud droplets and raindrops

Terminal velocity v_t results from balance between gravity and drag forces

Drops < 1.5 mm are roughly spherical $\rightarrow v_t$ depends on size and flow regime (Re)

$$\begin{array}{llll}
 v_t = k_1 r^2 & k_1 \simeq 1.2 \times 10^6 & [\text{cm}^{-1}\text{s}^{-1}] & r < 30\mu\text{m} \\
 v_t = k_2 r & k_2 \simeq 8000 & [\text{s}^{-1}] & 30 \leq r < 600\mu\text{m} \\
 v_t = k_3 \sqrt{r} & k_3 \simeq 2010 & [\text{cm}^{-0.5}\text{s}^{-1}] & 600 \leq r < 1500\mu\text{m}
 \end{array}$$

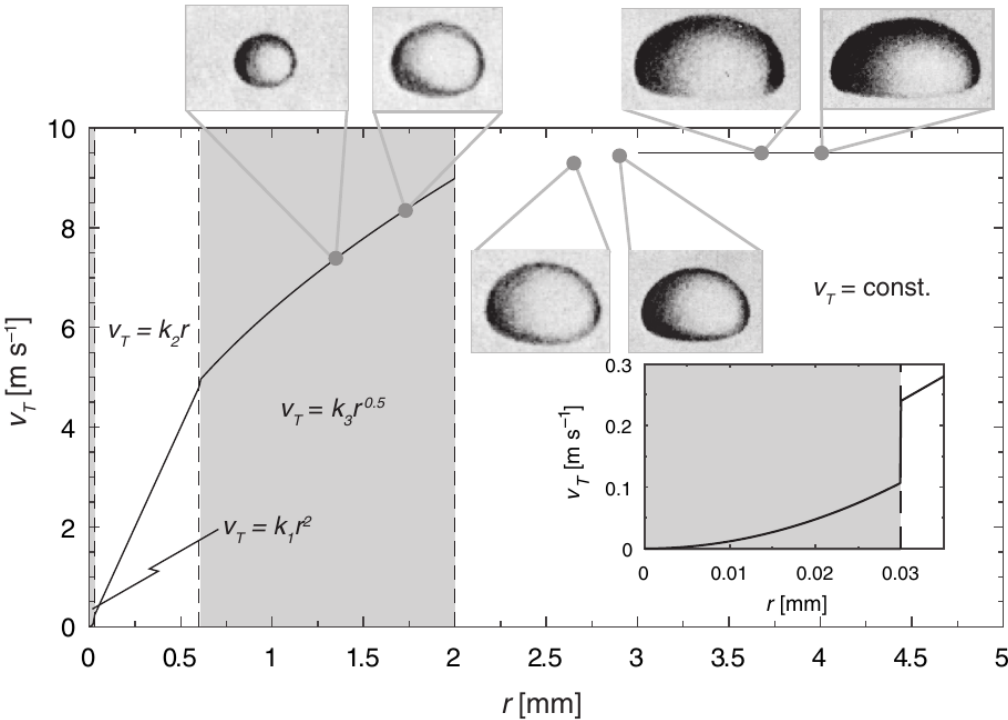
Raindrops have more complex fall velocities because additional factors come into play (non-sphericity, deformation, break-up...) \rightarrow empirical fit to lab data.

Ex: Beard, JAS, 1976 ([link](#))

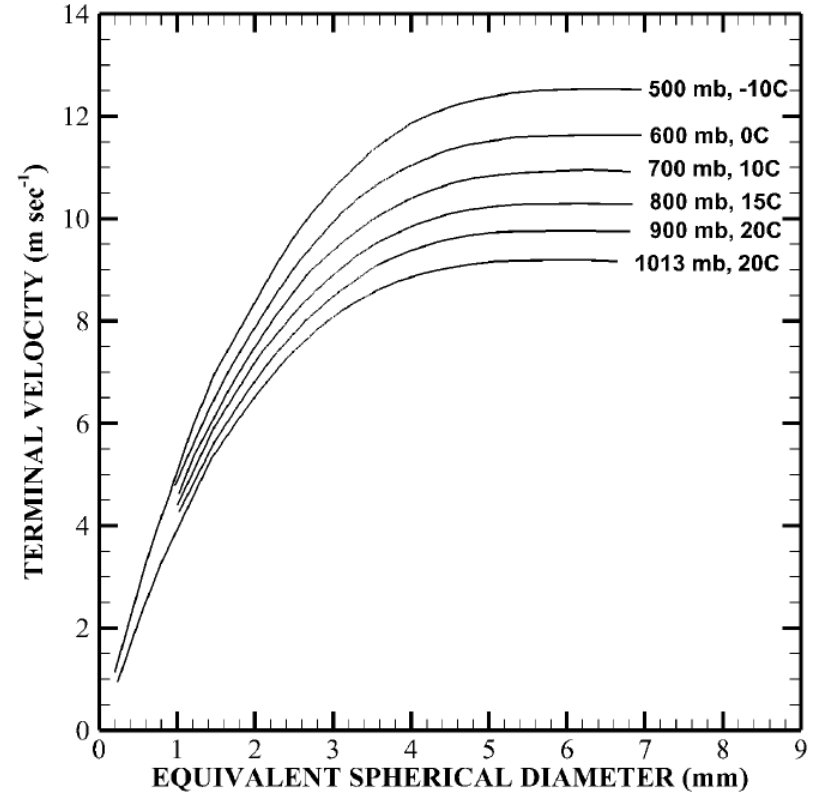
Fall velocity of cloud droplets and raindrops

Droplets

Raindrops



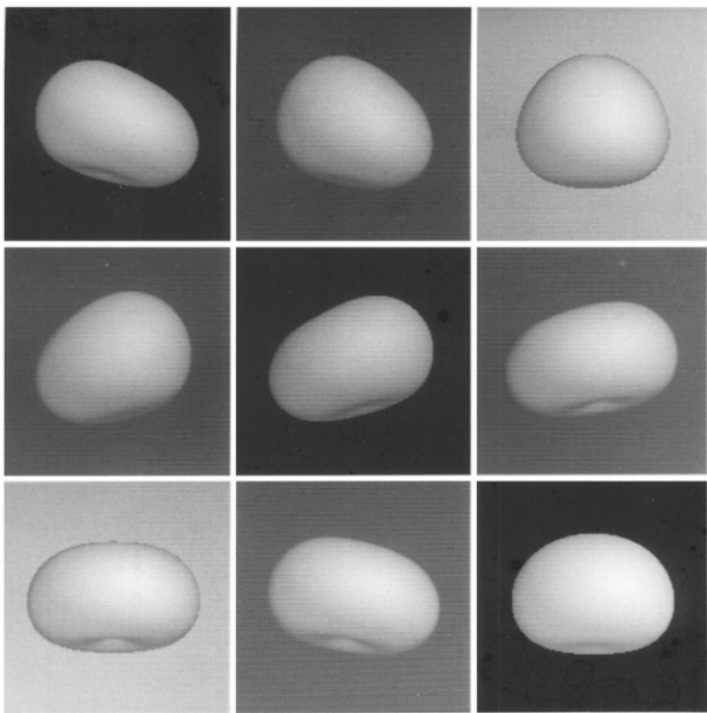
LLM2016, Fig7.10



W2013, fig8.25
from Beard 1976

Raindrop shape

Oscillations



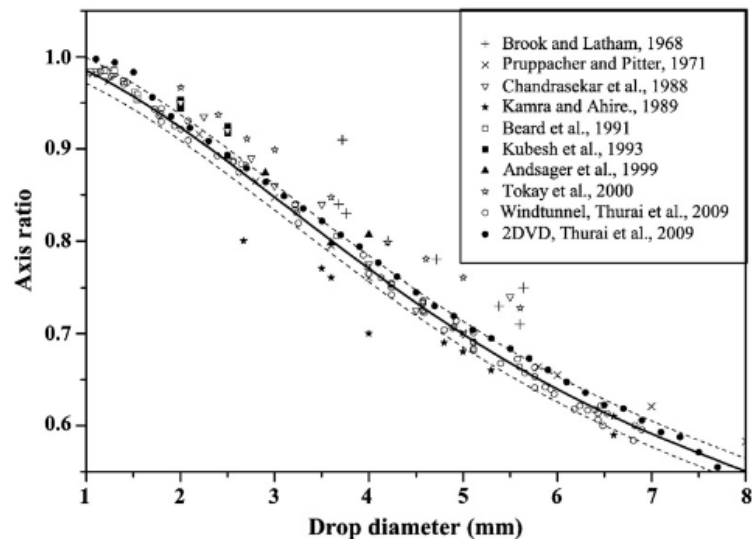
Beard et al, AR, 2010

Raindrops

Large raindrops are not spherical



Beard et al, AR, 2010



Szakall et al, 2010

Fall velocity of ice crystals

More complicated than drops because variety of shapes, sizes etc...
Empirical estimation of velocity-size/mass relationships.

Ex: Heymsfield and Westbrook, JAS, 2010

$$A_r = A / [(\pi/4)D^2] \quad \begin{array}{l} A \text{ projected area [m}^2\text{]} \\ D \text{ part. max dim [m]} \end{array}$$

$$X^* = \frac{\rho_a 8mg}{\eta^2 \pi A_r^{0.5}}$$

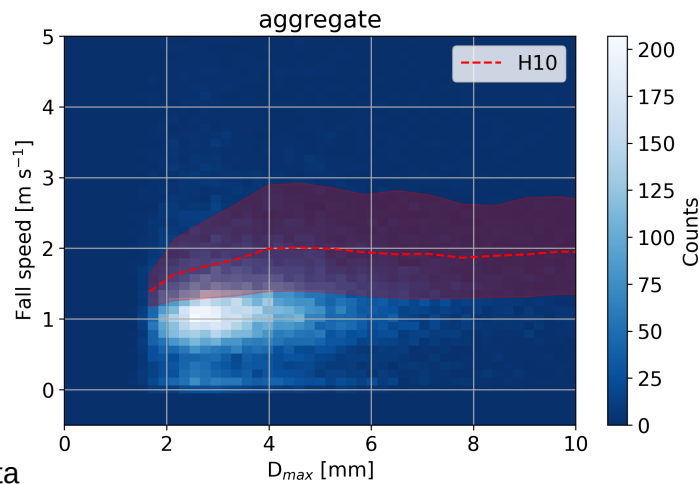
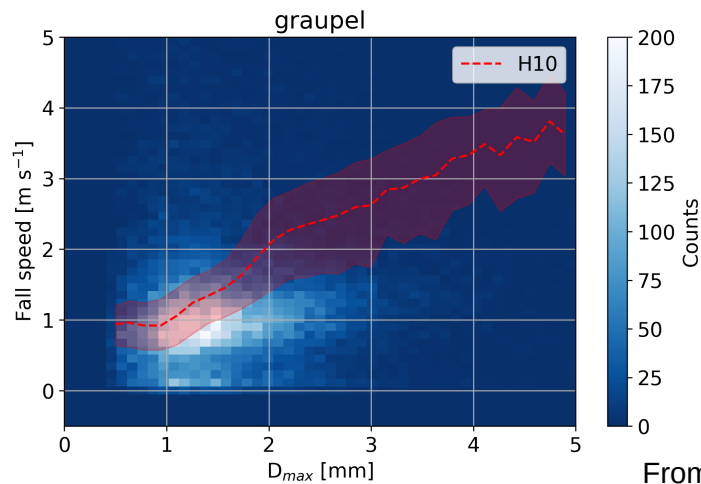
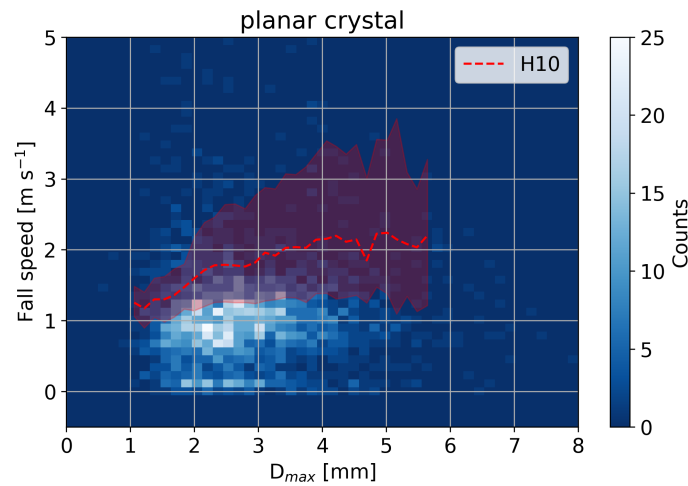
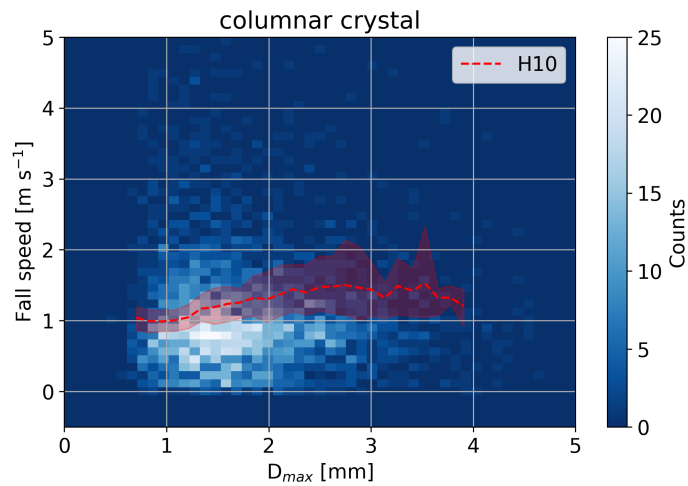
ρ_a	air density	[kg m ⁻³]
η	air dyn. visco.	[kg m ⁻¹ s ⁻¹]
m	part. mass	[kg]
g	gravity	[m s ⁻²]

$$Re = \frac{\delta_0^2}{4} \left[\left(1 + \frac{4\sqrt{X^*}}{\delta_0^2 \sqrt{C_0}} \right)^{0.5} - 1 \right]^2 \quad \begin{array}{l} \delta_0 = 8.0 \\ C_0 = 0.35 \end{array}$$

→ **Terminal velocity** $v_t = \frac{\eta Re}{\rho_a D}$ [m s⁻¹]

Fall velocity of ice crystals

Not perfect...



From MASC data

Fall velocity of hail stones

Hail is a rare phenomenon, so limited observations.
But many wind tunnel and simulation experiments...

Ex: Heymsfield et al., JAS, 2018

Terminal velocity

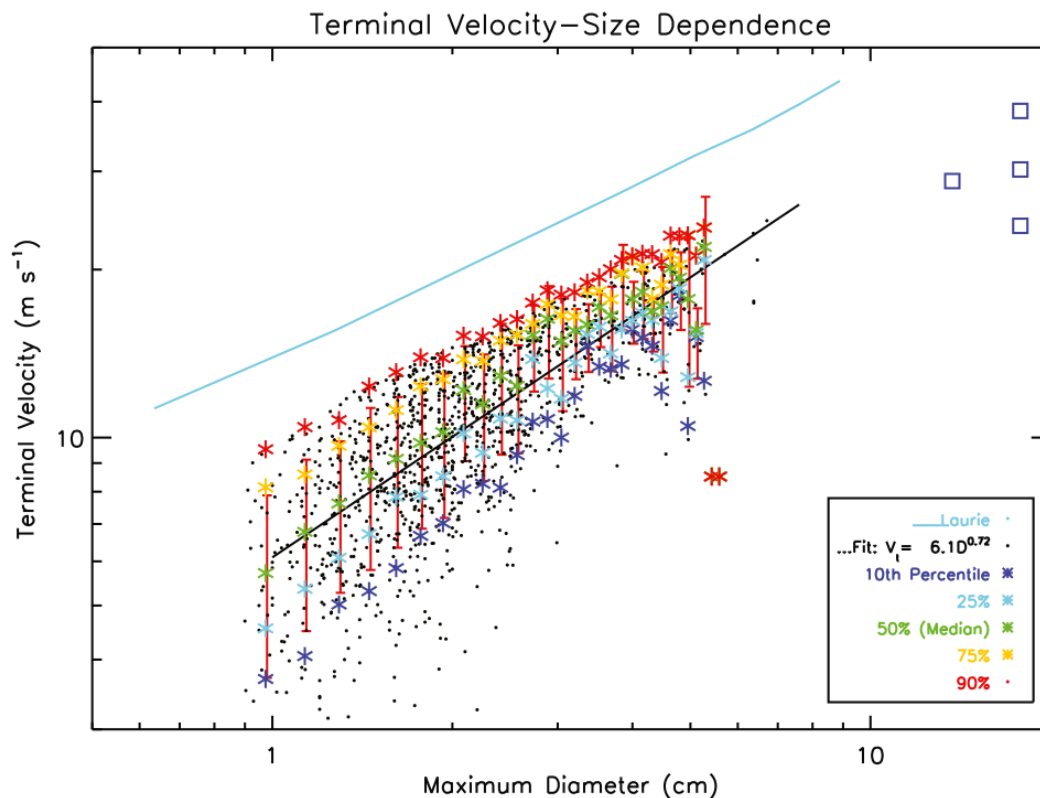
$$v_t = 6.1D^{0.72} \quad [\text{m s}^{-1}]$$

Orders of magnitude:

D ~ 5 cm v ~ 20 ms⁻¹

D ~ 10 cm v ~ 30 ms⁻¹

D ~ 20 cm v ~ 40 ms⁻¹



Size distribution of drop(let)s and ice crystals

In a given volume of atmosphere, there are many cloud/precip particles, with their respective individual properties (position, size, shape, mass, fall velocity...)

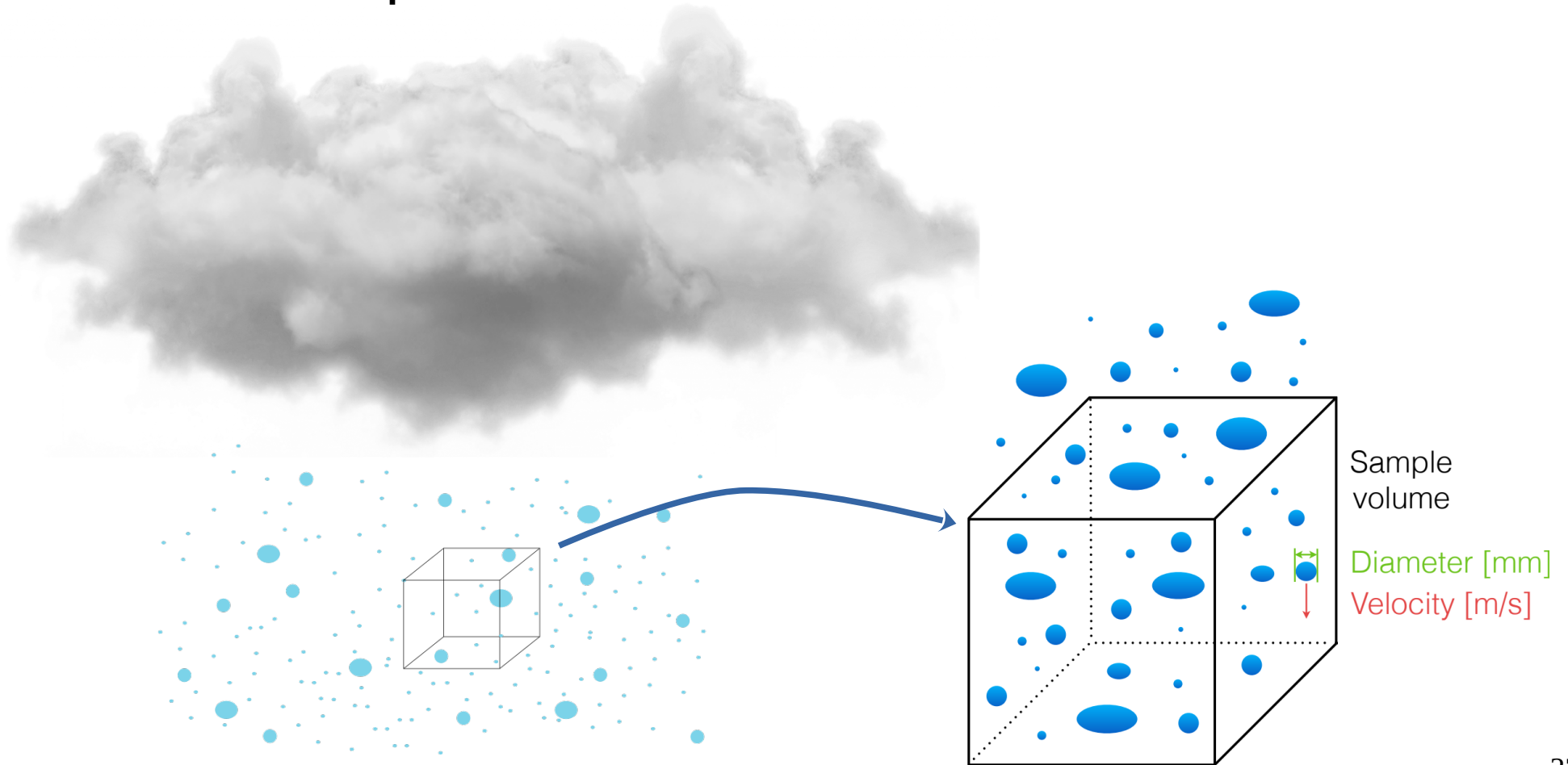
Too many particles (cloud $\sim 10^7 - 10^8 \text{ m}^{-3}$, precip $\sim 10^2 - 10^3 \text{ m}^{-3}$) to follow each of them.

- Statistical summary of those properties.
- For liquid drops: mass, shape, velocity can be accurately retrieved from size.
- **(rain)drop size distribution** (similar for ice particles, but link size / mass etc less evident)

DSD or PSD is a fundamental piece of information about cloud/precip!

It reflects the microphysical processes controlling the formation and evolution of cloud/precip particles aloft.

Size distribution of drops



Size distribution of drops and ice crystals

Frequently denoted $N(D)$, defined as drop concentration N_t x probability density of size f

$$N(D) = N_t f(D) \quad D = \text{equivolume diameter [mm]}$$

Concentration of drops with size in $[D, D+dD] = N(D)dD$

Mean drop size

$$\bar{D} = \int_{D_0}^{D_{max}} f(D) D dD = \frac{1}{N_t} \int_{D_0}^{D_{max}} N(D) D dD$$

Moment order n

$$M_n = \int_{D_0}^{D_{max}} N(D) D^n dD$$

Rain rate

$$R = \int_{D_0}^{D_{max}} N(D) D^3 v(D) dD \quad v = \text{fall velocity [m s}^{-1}\text{]}$$

Parametric models of the size distribution of drops and ice crystals

From observation, DSD/PSD represented as

Exponential distribution (Marshall & Palmer, 1948)
1 parameter: characteristic size D_0

$$N(D) = \frac{N_t}{D_0} \exp(-D/D_0)$$

Gamma distribution (Ulbrich, 1983)
2 parameters: D_0 and μ

$$N(D) = N_t \frac{D^{\mu-1}}{\Gamma(\mu) D_0^\mu} \exp(-D/D_0)$$

Generalized-Gamma distribution (ex : Lee et al, 2004)
3 parameters: D_0 , μ and c

$$N(D) = N_t \frac{c}{D_0 \Gamma(\mu)} \left(\frac{D}{D_0} \right)^{c\mu-1} \exp(-(D/D_0)^c)$$

1. What is the seeder-feeder process?
2. What is the shape of raindrops?
3. What is the typical terminal velocity of a raindrop of $20\mu\text{m}$ and 2mm -equivolume diameter at sea level and 20°C ?
4. Why is the DSD an important piece of information?

Cloud and precipitation processes

1. Warm cloud processes
 - growth by vapor deposition (saturation / liquid water)
 - growth by collision-coalescence (coll-coal efficiencies)
2. Cold cloud processes
 - growth by vapor deposition (saturation / ice)
 - growth by aggregation (coll-coal for ice crystals)
 - growth by riming (collision with SLW droplets)
3. Additional processes
 - Secondary ice production (ice-SLW or ice-ice)
 - Seeder-feeder (upper cloud seeds lower cloud)
4. Hydrodynamics
 - Fall velocity of drops (size, air temp)
 - Fall velocity of ice particles (size, shape, density...)
5. Particle size distribution
 - Definition of PSD/DSD + moments
 - Parametric models for PSD/DSD