

Formula sheet – Part A. Nenes

$R^* = 8.3144 \text{ J/}^\circ\text{K mol}$, $1 \text{ bar} = 1 \text{ atm} = 10^5 \text{ Pa}$, $1 \text{ atm} = 1.013 \times 10^5 \text{ Pa}$, $1 \text{ Pa} = 1 \text{ N m}^{-2}$,
 $1 \text{ N} = 1 \text{ J/m}$, Molar mass of Air = $29 \times 10^{-3} \text{ kg mol}^{-1}$, $L_{lv} = 2.47 \times 10^6 \text{ J kg}^{-1}$

$pv = RT$, R is the specific gas constant = R^*/M , where M is the molar mass of the gas.

For an ideal gas: $du = c_v dT$, $dh = c_p dT$, $c_p - c_v = R$

Combined 1st+2nd law: $du = dq - pdv = Tds - pdv$

Gibbs Free Energy $dg = -sdT + vdP$

Clausius-Clapeyron Equation: $\frac{de_s}{dT} = \frac{L_{lv}e_s}{RT^2}$

Wet bulb temperature $T_w = T - \frac{L_{lv}}{c_p} (w_v^s(T_w) - w_v)$, where $w_v^s(T_w)$ is the saturation water

vapor mixing ratio (at temperature T_w), and w_v is the water vapor mixing ratio.

Dew point: temperature for which $w_v^*(T_d) = w_v$

Humidity variable relationships: $w_v = 0.62 e/P$ $w_v^* = 0.62 e_s/P$ $H = e/e_s$

Saturation vapor pressure of water at room temperature: 30mbar.

Enthalpy for parcel: $dH = (m_d c_{pd} + m_v c_{pv} + m_l c_l) dT + L_{lv} dm_v$, where the subscripts d, v, and l refer to dry air, water vapor and liquid water, respectively

Hydrostatic equation: $g = -\frac{1}{\rho} \frac{\partial p}{\partial z}$, where ρ is density.

Kelvin Equation: $\frac{e_s^{curved}}{e_s^{flat}} = \exp\left(\frac{4M_w \sigma}{RT \rho_w D_p}\right)$, where σ is the surface tension of water

($=0.072 \text{ J m}^{-2}$), ρ_w is the density of water ($=1000 \text{ kg m}^{-3}$), M_w is the molar mass of water and D_p is the droplet diameter.

Idealized Raoult's Law: $P = P^{\text{sat}}(T) \chi_w$

Lognormal distribution: $n(Dp) = \frac{dN}{d \log Dp} = \frac{N}{\sqrt{2\pi \ln \sigma_g}} \exp\left\{-\frac{\ln^2(D_p/D_{pg})}{2 \ln^2 \sigma_g}\right\}$

Formula sheet – Part J. Gehring

Equation of motion: $\frac{d\vec{U}}{dt} + 2\vec{\Omega} \times \vec{U} - \Omega^2 \vec{r} = -\frac{1}{\rho} \vec{\nabla} p + \vec{g}^* + \vec{F}$

Total derivative: $\frac{d\vec{U}}{dt} = \frac{\partial \vec{U}}{\partial t} + (\vec{U} \cdot \nabla) \vec{U}$

Geostrophic balance: $\vec{V}_g = \frac{1}{\rho f} \hat{k} \times \nabla p$, where $f = 2\Omega \sin \varphi$ and φ is the latitude (positive in the Northern Hemisphere and negative in the Southern Hemisphere).

Hypsometric equation: $\frac{R_d \bar{T}_v}{g} \ln \left(\frac{p_1}{p_2} \right) = \Delta z$, where R_d is the gas constant for dry air, \bar{T}_v is the column-averaged virtual temperature.

Geopotential: $d\Phi = g dz$, where Φ is the geopotential. The geopotential height is defined as: $Z = \frac{\Phi}{g_0}$

The altimeter equation: $p_1 = p_2 e^{\frac{g z_2}{R_d \bar{T}_v}}$

The thermal wind: $\frac{\partial \vec{V}_g}{\partial p} = -\frac{R}{f p} \hat{k} \times \nabla T$. The thermal wind vector is defined as: $\vec{V}_T = -\frac{\partial \vec{V}_g}{\partial p}$

The ageostrophic wind: $\vec{V}_{ag} = \vec{V} - \vec{V}_g$.

Simplified form of the equation of the motion (retaining the three highest order terms):

$$\frac{d\vec{V}}{dt} = -f \hat{k} \times \vec{V}_{ag}, \text{ isolating the ageostrophic wind yields: } \frac{\hat{k}}{f} \times \frac{d\vec{V}}{dt} = \vec{V}_{ag}$$

Continuity equation: $\nabla_p \cdot \vec{V}_h = -\frac{\partial \omega}{\partial p}$

The pressure tendency equation: $\frac{\partial p_s}{\partial t} \approx -\int_0^{p_s} (\nabla \cdot \vec{V}) dp$

Formula sheet – Part A. Berne

Richardson number (bulk):

$$Ri_b = \frac{N_{BV}^2 (\Delta z)^2}{(\Delta U)^2 + (\Delta V)^2}$$

Brunt-Vaisala frequency:

$$N_{BV} = \sqrt{\frac{|g|}{T_v} \left(\frac{\Delta T_v}{\Delta z} + \Gamma_{d/s} \right)} \sim \sqrt{\frac{|g|}{\theta} \frac{\Delta \theta}{\Delta z}}$$

Dimensionless mountain height:

$$H = \frac{1}{Fr} = \frac{N_{BV} h}{U}$$