

ENV 407 – Atmospheric Processes: From Clouds to Global Scales
Hydrostatic Equilibrium – Hypsometric Equation

1. Using the equation we derived for barometric pressure as a function of altitude calculate H for a dry atmosphere with an effective temperature of 273°K.

assume isothermal, dry atmosphere

$$\frac{dp}{dz} = -\frac{\rho g}{R_d T} \Rightarrow \ln \frac{P}{P_0} = -\frac{g z}{R_d T} \Rightarrow H = \frac{R_d T}{g} = \frac{8.314 \cdot 273}{29 \cdot 10^{-3} \cdot 9.81} \Rightarrow H = 7.98 \text{ km}$$

2. For the value of H calculated above determine the altitude where $P = 0.5 \text{ atm}$.

$$\ln \frac{P}{P_0} = -\frac{z}{H} \Rightarrow z = H \ln \frac{P_0}{P} \Rightarrow z = 7.98 \ln 2 \Rightarrow z = 5.53 \text{ km}$$

assume $P_0 = 1 \text{ atm}$

3. For the value of H calculated above what is the air pressure in atmospheres at the top of Mt Everest (8.85km)?

$$\ln \frac{P}{P_0} = -\frac{z}{H} = -\frac{8.85}{7.98} \Rightarrow P = P_0 \exp(-1.1) = P_0 \cdot 0.33 \Rightarrow P = 0.33 \text{ atm}$$

4. On Mars the atmosphere is mainly CO_2 , the temperature is 220°K and the acceleration of gravity is 3.7 m/s². What is the scale height of the Martian atmosphere? Compare the scale height to the Earth's atmosphere and explain why they scale heights are different.

$$H = \frac{R_v T}{g} = \frac{8.314 \cdot 220}{44 \cdot 10^{-3} \cdot 3.7} = 11.24 \text{ km}$$

MW of CO_2

So P drops less with altitude for Mars ($H = 11.24 \text{ km}$) compared to Earth (7.98 km). That is not likely because of the reduced gravity of Mars, because its atmosphere has "heavier" molecules (CO_2 vs N_2/O_2).