ENV-407 Clausius - Clapeyron worksheet solution

1b) $$\ln\left(\frac{e_2}{e_1}\right) = \frac{1}{R_v}\int_{T_1}^{T_2} \frac{L_{lv}(T)}{T^2}\,dT$$

→ expand $L_{lv}(T) = 2.501 \times 10^6 - 2400T + 2400 \cdot 273$
$= (3.155 \times 10^6) - 2400T$

→ Plug $L_{lv}(T)$ into the integral

$$\ln\left(\frac{e_2}{e_1}\right) = \frac{1}{R_v}\int_{T_1}^{T_2} \frac{3.155\times10^6 - 2400T}{T^2}\,dT$$

$$= \frac{1}{R_v}\left[3.155\times10^6\int_{T_1}^{T_2}\frac{1}{T^2}\,dT - 2400\int_{T_1}^{T_2}\frac{1}{T}\,dT\right]$$

→ Since:

$$\int\frac{1}{T^2}\,dT = -\frac{1}{T} \quad \text{and} \quad \int\frac{1}{T}\,dT = \ln T$$

we get:

$$\ln\left(\frac{e_2}{e_1}\right) = \frac{1}{R_v}\left[3.155\times10^6\left(\frac{1}{T_1} - \frac{1}{T_2}\right) - 2400\ln\left(\frac{T_2}{T_1}\right)\right]$$

→ Plug in the values

$R_v = 461.5\ J/kg\,K$

$T_1 = 273.15K$, $e_1 = 611.2\ Pa$

$T_2 = 303.15K$

we get: $\ln\left(\frac{e_2}{e_1}\right) \simeq 1.93$ } $\frac{e_2}{e_1} = e^{1.93} \approx 6.89$

↓

$e_2 = 611.2 \times 6.89 \simeq 4215\ Pa$

$e_s(30°C) = 4.2\ kPa$

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1a) Using the integrated form:

$$e(T) = e(T_0)\, \exp\left[-\frac{L_{lv}}{R_v}\left(\frac{1}{T} - \frac{1}{T_0}\right)\right]$$

→ Now plug in numbers

Reference: $T_0 = 273.15\,K$, $e(T_0) = 611.2\,Pa$

$T = 303.15\,K$ (30°C)

$L_{lv} = 2.5 \times 10^6\,J/kg$

$R_v = 461.5\,J/kg\,K$

Exponent part → $-\frac{L_{lv}}{R_v}\left(\frac{1}{303.15} - \frac{1}{273.15}\right) \approx 1.95$

So:

$e(30°C) \approx 611.2 \times e^{1.95} \approx 611.2 \times 7.02 \approx 4290\,Pa$

$e_s(30°C) \approx 4.3\,kPa$

1c) Looking up the value of e_s from the table

$e_s(30°C) \cong 4.24\,kPa$