Exercise - Atmospheric processes - Dynamics

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Exercise 1: Flight levels

Flight levels are assigned to aircrafts to allow for enough vertical distance between them. Aircraft altimeters actually measure the environmental air pressure. To convert this pressure to an altitude they need (i) a reference pressure at sea level and (ii) a reference (virtual) temperature profile. For this and other reasons, the International Standard Atmosphere (ISA) was established. The ISA defines how pressure, temperature, density and viscosity varies with altitude. By using the ISA reference sea level pressure and temperature (1013.25 hPa and 15 °C, resp.), and a lapse rate of $-6.5\,°\mathrm{C\,km^{-1}}$, it is possible to determine the altitude of an aircraft above the 1013.25 hPa surface from its pressure measurement. Therefore, aircrafts fly along surfaces of constant pressure (i.e. isobars) and not constant altitude above sea level. This concept is illustrated in Fig 1. If an aircraft flies from a high pressure to a low pressure (from left to right on Fig. 1) its real altitude will decrease, despite the altimeter showing the same value.

With this freshly acquired knowledge and that of yesterday's lecture, you have just been hired by MeteoSwiss (congratulations!) as an aviation meteorologist. A typical flight level to cross the Alps is FL180. For a standard atmosphere (ISA), this corresponds to a pressure of 506 hPa or 18000 feet (5486.4 m) above the 1013.25 hPa surface. One of your duty is to provide Skyguide with the real altitude of FL180. On your first day in operations on 15 October 2020, there is a low pressure to the south of the Alps (see Fig. 2). The radiosounding in Payerne gives you a column average virtual temperature of 268.5 K and the mean sea level pressure is 1012.6 hPa.

- What will be the real altitude above sea level at which the airplane will be flying above the Alps?
- Given that the altitude of the Mt-Blanc is 4808 m a.s.l., would you advise Skyguide to assign a higher flight level to aircrafts crossing the Alps in this situation?

Figure 2 shows the geopotential height at 500 hPa, the mean sea level pressure and the thickness of the layer 1000 hPa to 500 hPa on that day. An airplane is flying the route Reykjavik to Beirut on FL180 passing above the Mont Blanc massif.

- Draw the route on Fig. 2.
- Draw a vertical cross-section showing the (approximate) evolution of the real altitude of the airplane, assuming it is flying on FL180. Add an approximate terrain height showing the Alps on your cross-section.

Notes:

- The gas constant for dry air is 287 $\mathrm{J\,kg^{-1}\,K^{-1}}$ and the average gravity at sea-level is 9.81 $\mathrm{m\,s^{-2}}$.
- MeteoSwiss provides every 12 hours a measurement of the real altitude of the FL180, which allows Skyguide to decide which flight level to attribute to fly across the Alps. When the altitude of FL180 goes below its ISA value (i.e. 18000 feet or 5486.4 m a.s.l.), Skyguide gives higher flight levels to airplanes crossing the Alps.
- The 1950 Air India Flight 245 crash in the massif du Mont Blanc is an example of what could happen if the real altitude of flight levels is not monitored around major mountain ranges.

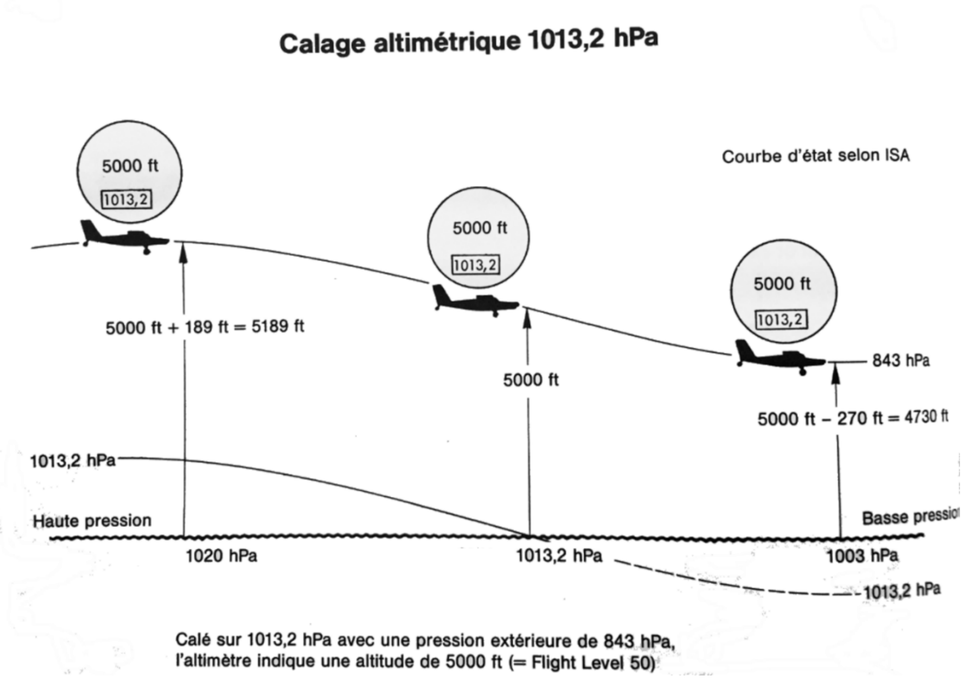

Figure 1: evolution of the cruising altitude of an aircraft flying on FL050 (5000 ft) going from a high pressure (left) to a low pressure (right). Source : K.H. Hack Météorologie pour aviateurs, éditeur Aéroclub de Suisse

Solution

Recall that the hypsometric equation states that the thickness between two pressure levels is directly proportional to the average virtual temperature between these two levels, we can compute the real altitude of FL180 on that day with:

$$\Delta Z = \frac{R_d\,T_v}{g_0}\,\ln\left(\frac{p_1}{p_2}\right)$$

Given the values in the exercise, this gives:

$$\Delta Z = \frac{287 \times 268.5}{9.81}\,\ln\left(\frac{1012.6}{506}\right) = 5449\,\mathrm{m}$$

Since the pressure of 1012.6 hPa in Payerne represents mean sea level pressure (i.e. it has been reduced to sea level considering the temperature measurement in Payerne with the barometric

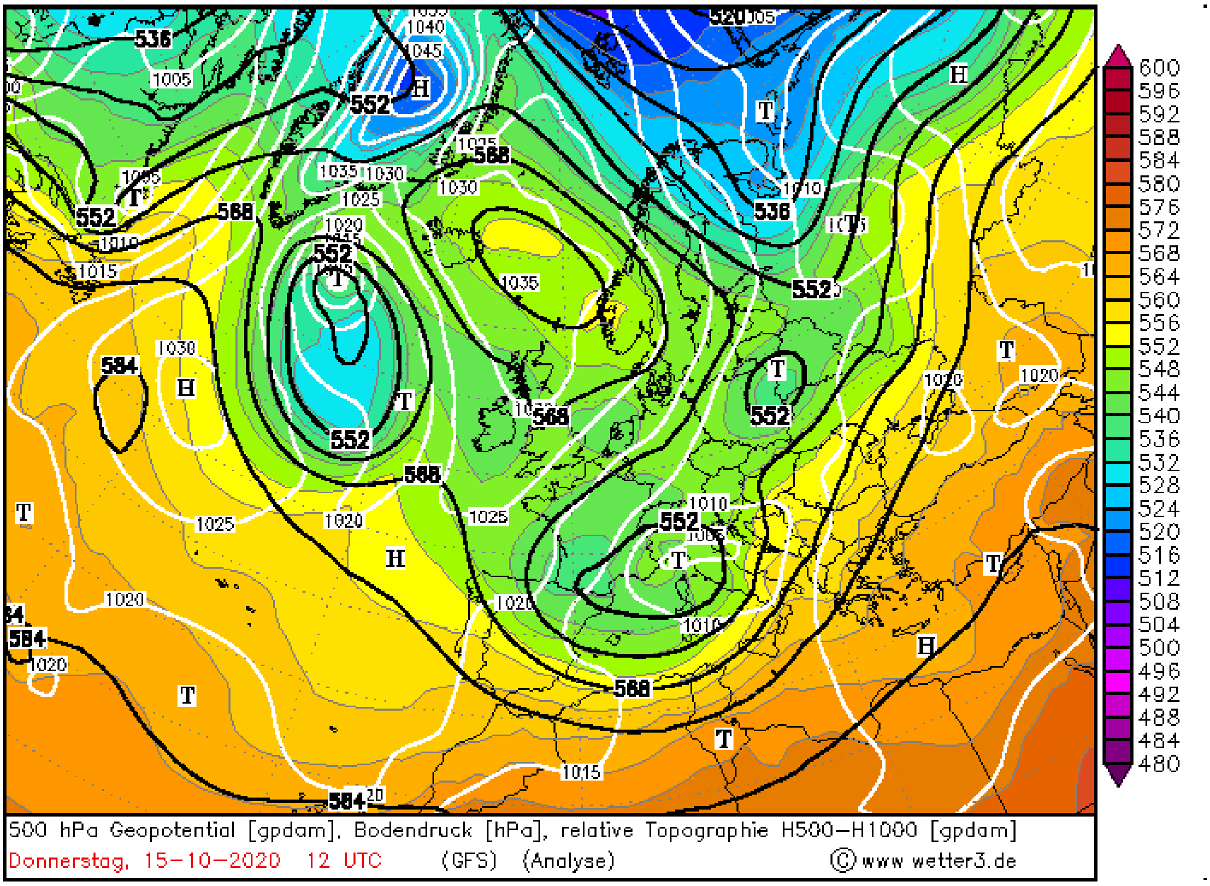

Figure 2: Geopotential height at 500 hPa (black contours, labels in decametre), mean sea level pressure (white contours, labels in hectopascal, "T" represents low pressure centres and 'H" high pressure centres), and 500 hPa to 1000 hPa thickness (colour filled in decametre) on 15 October 2022 at 12 UTC. Remember that the 500 hPa to 1000 hPa thickness is directly proportional to the average virtual temperature between these two levels. Source: GFS analysis.

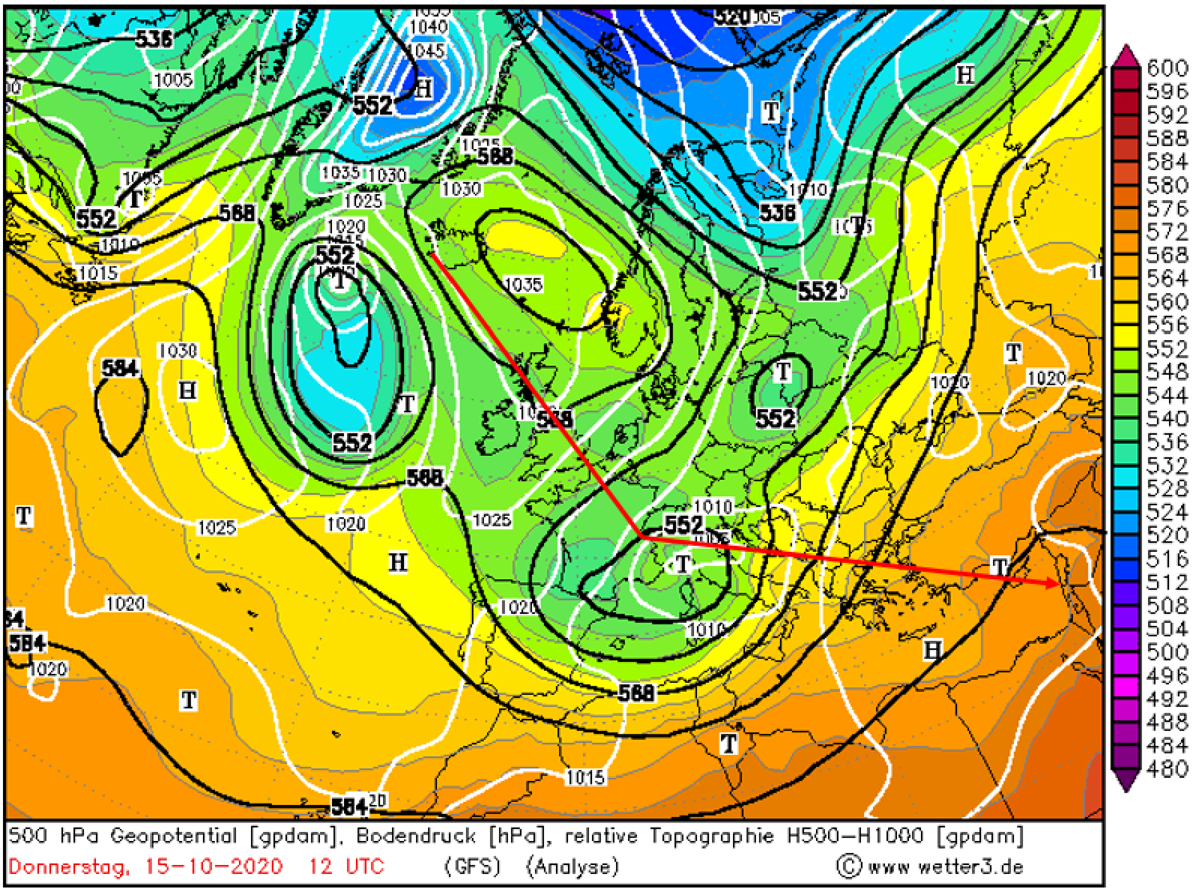

Figure 3: Same as Fig. 2, but with the flight track

formula), the thickness computed above corresponds to the geopotential height in metres above sea level (m a.s.l., remember it is almost equal to the real altitude). Since 5449 m a.s.l. is just below the ISA altitude of FL180 (5486.4 m), Skyguide will assign a higher flight level to aircrafts crossing the Alps in this situation.

The route of the aircraft flying from Reykjavik to Beirut passing over the Alps is shown in Fig. 3. The approximate evolution of the altitude of FL180 for this route is shown in Fig. 4. As the aircraft flies from a region of high geopotential heights above Iceland to lower geopotential heights above the Alps, the real altitude of FL180 decreases and passes quiet close to the highest Alpine peaks. As it heads more eastwards towards Beirut, it flies to higher geopotential heights and hence the real altitude of FL180 increases. Note how the 500 hPa geopotential height is both a function of the 500 to 1000 hPa thickness and mean sea level pressure: the surface low pressure is centred over northern Italy, while the lowest thickness (i.e. lowest mean virtual temperature) is centred over south-western France. Therefore the minimum 500 hPa geopotential height (i.e. the 552 dam contour) extends from northern Italy to south-western France, as it is a function of both the thickness and the mean sea level pressure, as stated by the hypsometric equation.

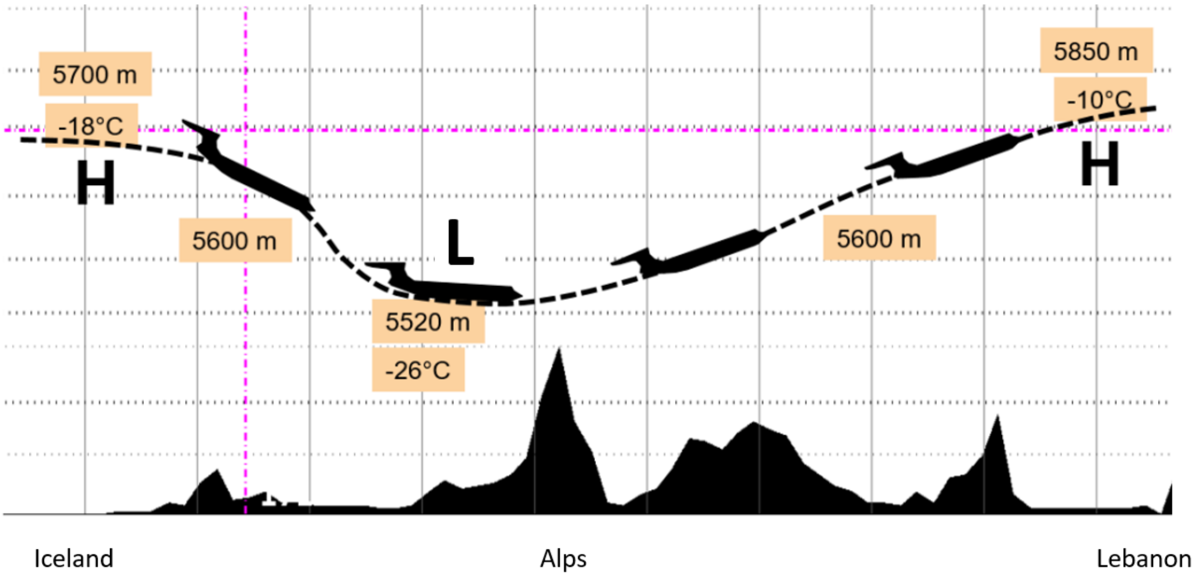

Figure 4: Evolution of the approximate altitude of FL180 along the route shown on Fig. 3 for 15 October 2020 at 12 UTC. "H" represents high geopotential heights and "L" low geopotential heights. The numbers show the altitude of the 500 hPa geopotential height and hence are a bit higher than FL180 which corresponds to 506 hPa. The temperature is given for your information, but you were not expected to find it. The topography along the route is shown at the bottom. It does not matter if your track and cross-section do not exactly correspond, to this solution. It should however be qualitatively correct.

Exercise 2: geostrophic and thermal wind balance

1. From the geopotential height field of Fig. 5, estimate the geostrophic wind at 500 hPa above Madrid (note that the distance between Madrid and the north of Portugal is about 400 km).
2. From the thickness field of Fig. 5, draw qualitatively the geostrophic wind at 1000 and 500 hPa above Madrid.
3. Is there a significant temperature advection (in the 1000 to 500 hPa column) over Madrid? If not, try to identify a region on the map where you expect warm air advection. Draw again qualitatively the geostrophic wind at 1000 and 500 hPa over that region.

Solution

The geostrophic wind on an isobaric surface in natural coordinates is given by:

$$V_g = -\frac{1}{f}\frac{\partial \Phi}{\partial n}. \tag{2.1}$$

Since by definition $\partial\Phi = g_0\,\partial Z$, we can rewrite Eq. 2.1 in terms of geopotential height Z with:

$$V_g = -\frac{g_0}{f}\frac{\partial Z}{\partial n}.$$

Hence, we can compute the geostrophic wind at 500 hPa over Madrid based on the data shown on Fig. 5. Since the geopotential gradient between Madrid and the north of Portugal is more or

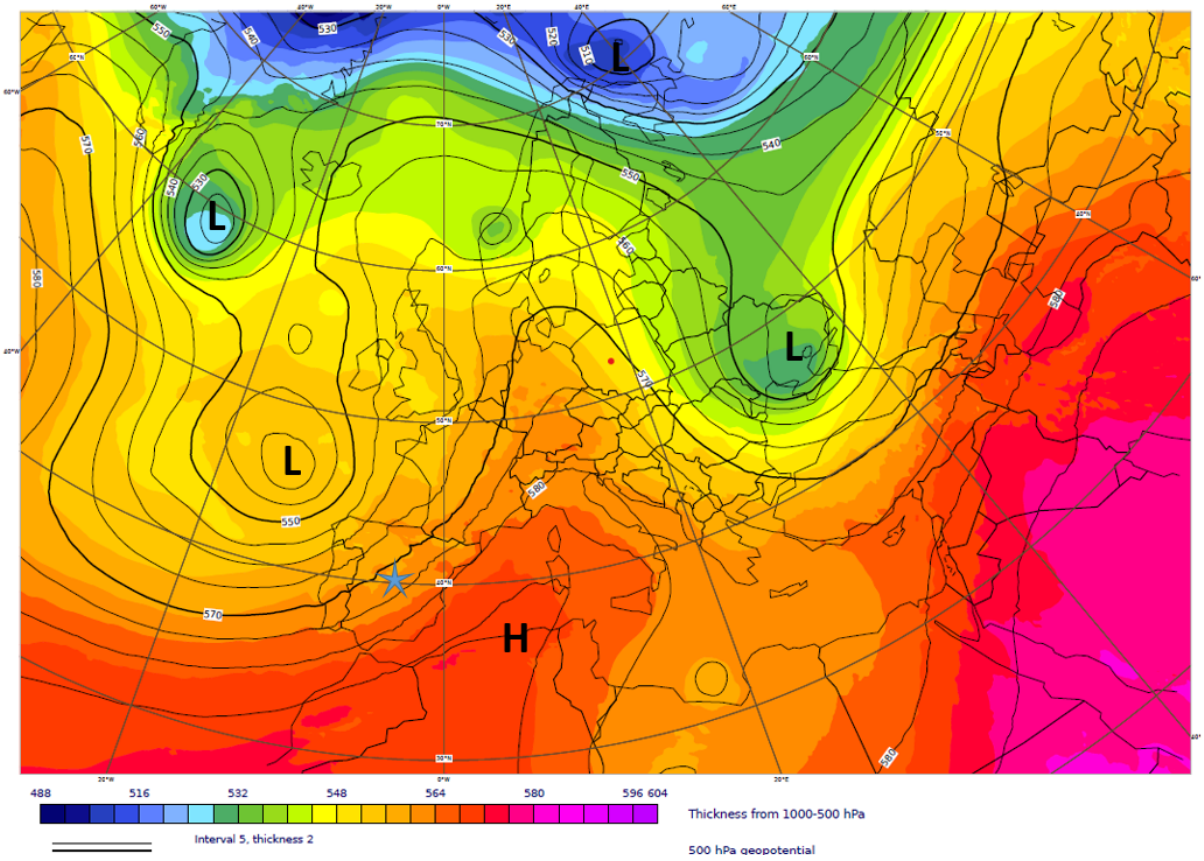

Figure 5: Geopotential height at 500 hPa (black contours, labels in decametre) and thickness between 500 and 1000 hPa (colour filled) on 20 October 2022 at 12 UTC. The approximate location of Madrid is shown with the blue star. The centres of low and high geopotential heights are shown with a "L" and "H", respectively. Source: analysis of the Integrated Forecast System (IFS) model

less constant, we can estimate the gradient over Madrid by taking the "bulk" gradient between Madrid and the north of Portugal with:

$$\frac{\Delta Z_{Madrid-NPortugal}}{Distance_{Madrid-NPortugal}} = \frac{120\ \mathrm{m}}{4 \times 10^5\ \mathrm{m}}$$

Given that $f = 2\,\Omega \sin\phi$ and that the latitude of Madrid from Fig. 5 is about 40 $^\circ$, we have:

$$f = 2 \times \frac{2\pi}{24 \times 3600\ \mathrm{s\,d^{-1}}} \times \sin(40^\circ) = 9.3 \times 10^{-5}\ \mathrm{s^{-1}}$$

and we can compute the geostrophic wind at 500 hPa above Madrid with:

$$V_g = \frac{9.81\ \mathrm{m\,s^{-2}}}{9.3 \times 10^{-5}\ \mathrm{s^{-1}}} \times \frac{120\ \mathrm{m}}{4 \times 10^5\ \mathrm{m}} = 31.25\ \mathrm{m\,s^{-1}}.$$

Since the thermal wind vector is parallel to the isotherms leaving lower temperature to its left in the northern hemisphere and that the 500 hPa geostrophic wind is also parallel to the isotherms in this case, there must be an increasing south-westerly component of the geostrophic wind with height between the 1000 hPa and 500 hPa surface. Hence the vertical wind profile must be south-westerly winds increasing in speed from the 1000 hPa to 500 hPa. Note that if the magnitude of the thermal wind vector is greater than the geostrophic wind at 500 hPa, the geostrophic wind at 1000 hPa will be in the opposite direction (i.e. north-easterly, recall that the thermal wind vector is the difference between the wind at some upper-level with the one at some lower-level). Figure 6 shows the radiosounding above Madrid. The wind profile indeed shows a south-westerly wind increasing in speed from the surface to 500 hPa. Remember that the thermal wind balance assumes geostrophic and hydrostatic balance and hence the real wind profile will deviate a bit from our inference (in particular note the "mid-level" jet around 700 hPa, which is a consequence of the ageostrophic circulation associated with the jet streak). The wind barb at the lowest level is also more southerly due to friction close to the ground, which will reduce the speed and hence the Coriolis force, thus the wind will be slightly deviated towards the low geopotential heights. Also, note that the measured wind speed at 500 hPa is about 60 knots, which is about $30\,\mathrm{m\,s^{-1}}$, a value rather close to our approximation computed above. The difference between the geostrophic wind we computed and the measured wind speed are due to (i) our approximation of the geopotential height gradient based on a simple map reading and (ii) departure from geostrophy due mainly to the flow acceleration component of the equation of motion. Since the geostrophic wind usually shows an error of about 10 to 20 % compared to the actual wind, we can be rather satisfied with our simple approximation.

Since the isohypses (i.e. lines of constant geopotential height) are parallel to the isopleths of thickness or, stated more simply, the geostrophic wind is parallel to the isotherms over Madrid, there is no temperature advection. Indeed, the temperature advection in the 500 to 1000 hPa column is given by:

$$-\overline{\overrightarrow{V_g}} \cdot \nabla\overline{T}, \tag{2.2}$$

where $\overline{\overrightarrow{V_g}}$ and $\overline{T}$ are the column-averaged geostrophic wind and temperature, respectively. Since $\overline{\overrightarrow{V_g}}$ is perpendicular to $\nabla\overline{T}$, the dot product is zero and there is no temperature advection in that column. In this case, we can say that the front extending from southern Spain to Denmark (as seen by the strong thickness gradient) is stationary. In order for the front to start moving, we would need to have a component of the column averaged geostrophic wind to be crossing the isopleths of thickness.

In order to have warm advection in the 500 to 1000 hPa column, we need the geostrophic wind at some level be crossing the isotherms or isopleths of thickness from higher to lower values. In simpler terms, we need the wind to go from a warmer region to a colder region. On Fig. 5, we see that over the North Sea, the isohpyses are crossing the isopleths of thickness from higher to lower values. Hence the temperature advection represented by Eq. 2.2 is positive. From the thermal wind balance, the wind will be veering (i.e. turning clockwise) with height over the North Sea. The wind profile shown by the radiosounding at Schleswig in Denmark in Fig. 7 shows that the wind is indeed veering with height, consistent with warm air advection in the northern hemisphere. We therefore have a warm front over the North Sea and it is moving north-easterly.

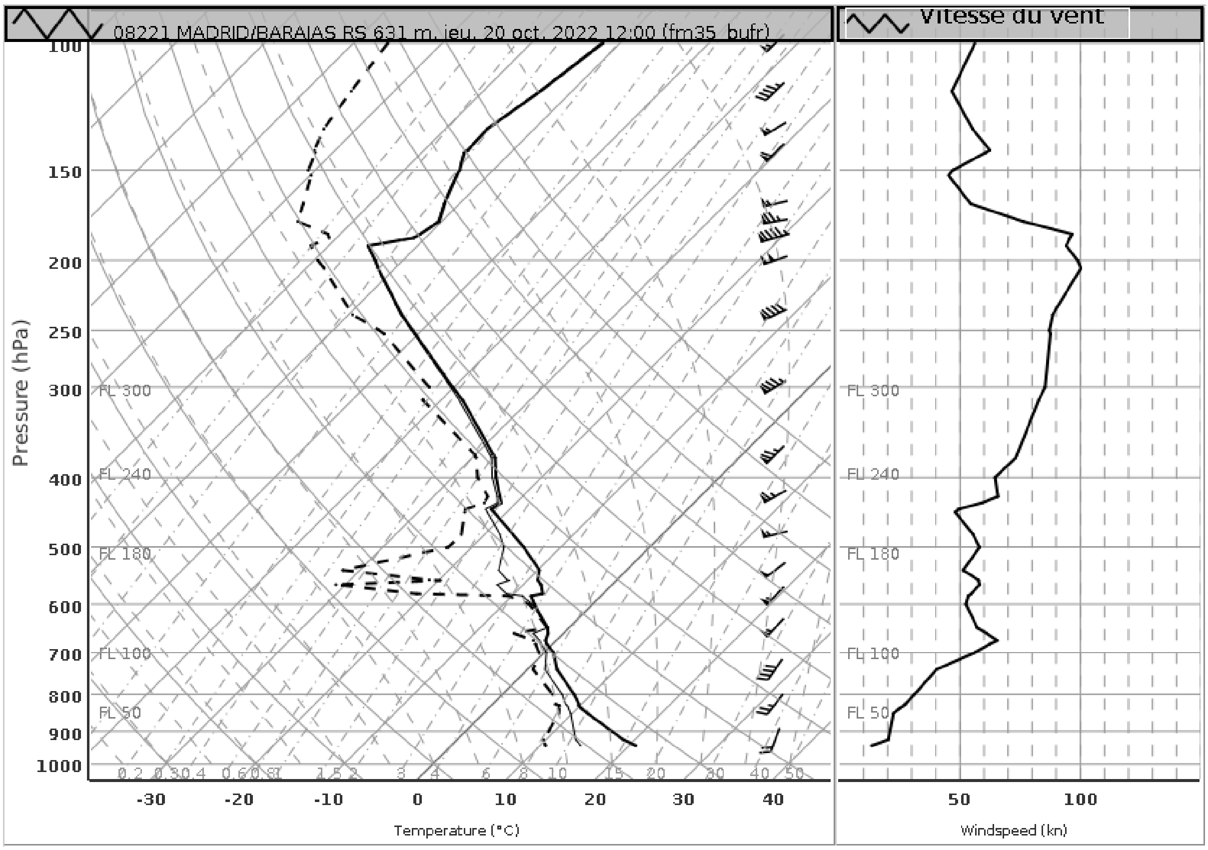

Figure 6: Radiosounding over Madrid on 20 October 2022 at 12 UTC. The wind profile is shown by the wind barbs and the speed is represented in the right panel.

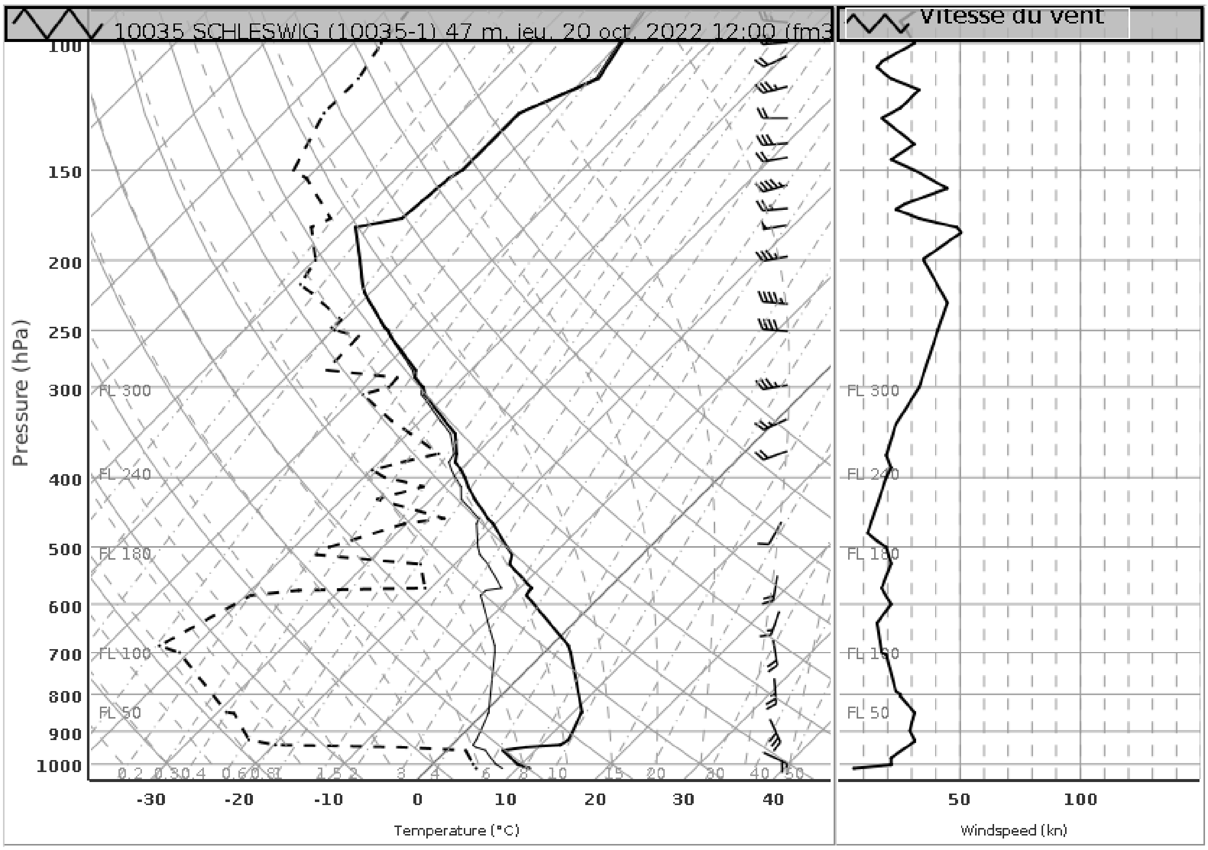

Figure 7: Radiosounding over Schleswig (in Denmark) on 20 October 2022 at 12 UTC. The wind profile is shown by the wind barbs and the speed is represented in the right panel.