Solution Exercise week 13 – Tracer transport and Water Transit Times

Estimating transit time metrics 1 (on paper)

1. **Task:** Write down a formula to compute the tracer concentration in streamflow (C_Q) starting from a timeseries of tracer concentration in precipitation C_J and a transit time distribution p_Q.

   Solution: The most classic convolution formula for the tracer transport problem can be written as:

   $$C_Q(t) = \int_0^{\infty} C_J(t-\tau)\, p_Q(\tau)\, d\tau \tag{1}$$

   Such an equation also works in case the transit time distribution varies over time t (in which case it is usually expressed as $p_Q(\tau, t)$).

2. **Question:** Compute the scale parameter ($1/\beta$) of a gamma distribution that has mean value $\mu = 100$ days and shape parameter $\alpha = 0.8$.

   Solution: In a gamma distribution, the mean μ can be computed directly from the parameters: $\mu = \alpha/\beta$. Thus, in this case, $\beta = \alpha/\mu = 0.008$ [1/d] and $1/\beta = 125$ [d]

3. **Question:** A soil receives about 1200 mm of precipitation per year. The soil depth Z is 2 meters, with porosity $n = 0.48$ and mean saturation $\bar{s} = 0.7$. What is the mean water transit time through this soil?

   Solution: The long-term mean transit time is given by the ratio between the (mean) water storage and the (mean) water fluxes. In this case, the storage is computed as $Z \times n \times \bar{s} = 2000 \times 0.48 \times 0.7 = 672$ mm and the water influx is 1200 mm/y. Thus the mean transit time through the soil is ≈ 0.56 years ≈ 204 days.

4. **Question:** What is the "Young Water Fraction" (F_{yw}) and how can you estimate it?

   Solution: F_{yw} was originally defined as the fraction of streamflow younger than 2–3 months. If we refer to the cumulative transit time distribution as P_Q, then $F_{yw} = P_Q(\tau \leq 2-3\,months)$. Importantly, this quantity can be computed from tracer cycles even *without* knowing the transit time distribution. If the tracer cycle amplitudes can be computed in precipitation (A_P) and in streamflow (A_Q), then $F_{yw} \approx A_Q/A_P$

Estimating transit time metrics 2 (on paper)

1. **Question:** What are ^{18}O and ^{2}H? What are their units? Why is their value negative?

   Solution: ^{18}O and ^{2}H are rare isotopes of oxygen and hydrogen in the water molecule (the abundant isotopes are ^{16}O and ^{1}H). The isotopic composition is expressed as a ratio (R) between the moles of the rare isotope and the moles of the abundant isotope. However, it is usually convenient to express it as the relative difference with respect to a standard (δ notation, in "permil" ‰). In δ notation, whenever a water sample has a smaller amount of rare isotopes with respect to the standard, the δ value will be negative. For example, if a water sample has δ^{18}O $= -10$‰, it means that it has 10‰ (or 0.1%) less rare isotopes than the standard. The typical standard used in hydrology is the 'Vienna Standard Mean Ocean Water' (VSMOW), which contains a relatively large amount of rare isotopes. Therefore, as precipitation and streamflow usually contain lower amounts of rare isotopes than the VSMOW, their δ value is usually negative.

2. **Task:** Estimate by eye the amplitude of the seasonal fluctuations of the two isotope signals. Use this approximate amplitude to estimate F_{yw} at Erlenbach.

   Solution: It is quite difficult to estimate an amplitude by eye. A rough estimate could be $A_P \approx 10$‰ and $A_Q \approx 4$‰, which leads to $F_{yw} = A_Q/A_P = 0.4$.

Computing tracer transport (on a computer)

- **Task:** Open the data files `ERL.isoPrcp` and `ERL.isoStrm` using a text editor (like Notepad++). How is missing data reported? Which columns store the timestamp and isotope information?

  Solution: The headers of the data files report all the metadata:

  - For the dataset with isotopes in precipitation, missing values are reported as `-9999`, the timestamp is simply termed `date` and the isotope compositions are indicated as `d18O_prcp` for δ^{18}O and `dDeuterium_prcp` for δ^2H;
  - For the isotopes in streamflow, missing values are reported as `NA`, the timestamp is termed `Sampling_date` and the isotope compositions are indicated as `d18O_mean` for δ^{18}O and `d2H_mean` for δ^2H;

- **Task:** Make a plot with the δ^{18}O timeseries in precipitation and in streamflow.

  Solution: See the example already provided in the template exercise.

- **Task:** Compute an approximate F_{yw} by taking the ratio between the standard deviations of the isotope signals in streamflow and precipitation. Is this similar to your previous estimate?

  Solution: In this case, the std of δ^{18}O in streamflow is 1.36 and the std of δ^{18}O in precipitation is 3.84, so $F_{yw} \approx 0.35$. This estimate is different from our previous guess, but it is broadly comparable (the fraction of young water in streamflow is still a bit more than a third of all stream water).

- **Task:** Prepare the input concentration timeseries....

  Solution: You can copy-paste the provided commands directly into Matlab/Python

- **Task:** Define a TTD for the watershed...

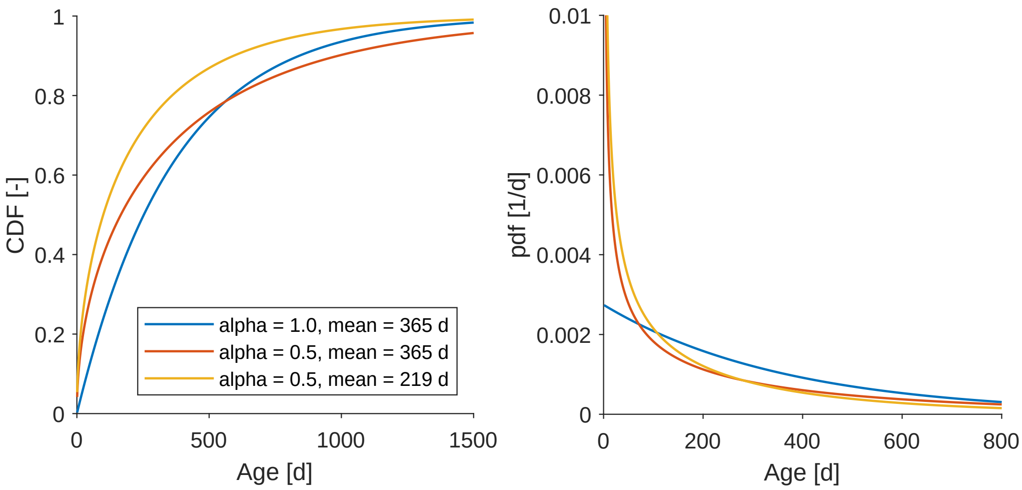
  Solution: You can copy-paste the provided commands directly into Matlab/Python. Note that it is practical to choose a mean value for the distribution and then compute the scale (or inverse scale) parameter based on this mean value. Also note that the cutoff N is here a function of the mean value, such that it is automatically larger/smaller when the mean is larger/smaller.

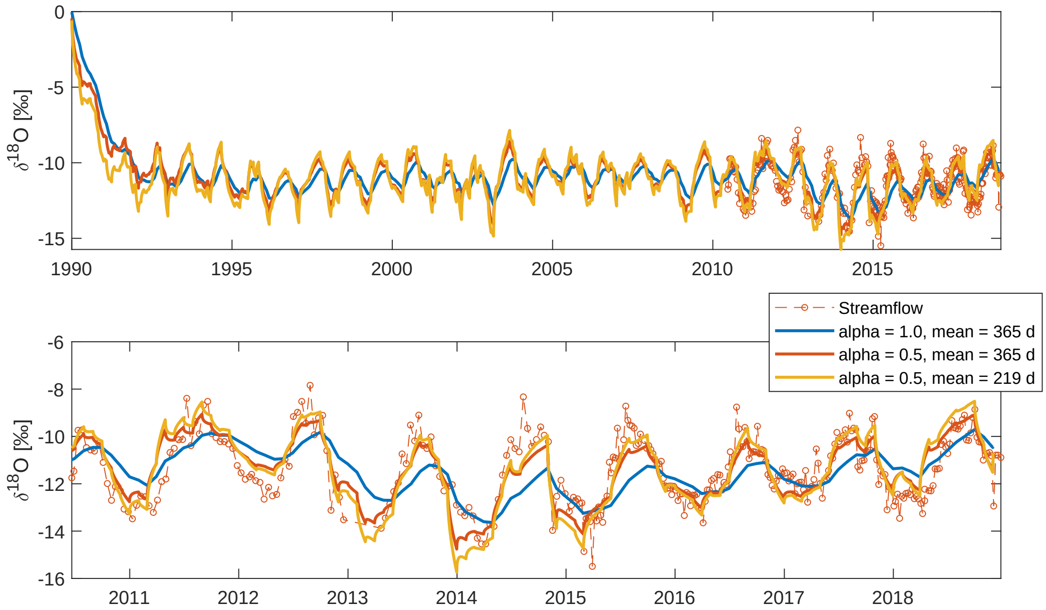
- **Task:** Implement the convolution... Make a plot with your output simulation (continuous line) and the output measurements (dots).

  Solution: You can use a standard convolution approach of the type :
  `C_Q(i+j-1) = C_Q(i+j-1) + C_J(i)*TTD(j)*dt`, for `i=1:length(C_J)` and `j=1:length(TTD)`. To crop the output to the right length you can use `C_Q = C_Q(1:length(C_J))`.

- **Question:** How does your TTD change when you increase/decrease the parameters α and the mean value? And how does the output change?

  Solution: The gamma distribution's shape can change considerably depending of the value of the shape parameter α. When $\alpha > 1$, the distribution is bell-shaped and larger values of α move the peak rightwards. When $\alpha \leq 1$, the distribution decreases monotonically, and the decrease is more marked for smaller values of α. The scale parameter simply changes the scale of the distribution and not its shape. Larger values of $1/\beta$ imply larger values in the distribution mean. In general, both larger values of $1/\beta$ and larger values of α lead to larger mean values, which results in more damped and shifted output. You can see examples in the figures below, where the blue curve has higher shape and scale parameter values and the corresponding tracer output is more damped and delayed than the others.





- **Task:** Modify the parameters α and β until you find a good visual match between the simulated C_{out} and the measured C_Q.

  Solution: There are many parameter combinations that result in similar good fits to the measurements. A simulation with $\alpha = 0.5$ and mean value $\mu = 300$ days is already a good one. But many others are similarly good and this leads to some uncertainty in the estimation of the 'true' distribution parameters based on these tracer data. This issue occurs frequently and it is called 'equifinality'.

- **Question:** Now you can use this 'good' TTD to compute the $F_{ywleadto}$. Is it similar to your previous estimates?

  Solution: With the parameters indicated above, one can compute the values of the cumulative gamma distribution that correspond to $x = 2 - 3$ months (55–91 days). The Matlab `cdf` function can be used as `cdf('gamma',x,shape_parameter,scale_parameter)`. The Python `cdf` function can be used as `gamma.cdf(x, a=par_alpha, scale=par_scale)`. The result in this case is $F_{yw} = 0.33 - 0.42$, which includes all our previous estimates.