Solution Exercise week 12 – Analysis of discharge data for hydrologic design

1 On paper

1. **Task:** Write the general equation to estimate the annual maximal discharge $Q_{est}(T)$ that corresponds to a return period T according to the frequency factor approach. Then, write the frequency factor equation for the LP3 approach.

   Solution: The frequency factor general equation is:

   $$x_T = \bar{x} + K_T\, S_x \tag{1}$$

   where x_T is the estimated value corresponding to a return period T, $\bar{x}$ is the mean of the variable and S_x the standard deviation. K_T is the frequency factor, which depends on the return period and on the specifics of the particular probability distribution. The LP3 approach is based on equation 1 but with a transformed variable $y = log_{10}(Q)$ instead of x. Thus, the discharge is estimated as:

   $$Q_{est} = 10^{\,\bar{y}+K_T\, S_y} \tag{2}$$

2. **Question:** Which terms of the equation depend on the return period? Which depend(s) on the observed discharge time series?

   Solution: From equation 2 we see that $\bar{y}$ and S_y only depend on the observed data. Then, the frequency factor K_T depends on both the return period and the data (in this case, through the skewness G_s) and so it can be expressed as $K(T, G_s)$.

3. **Task:** Estimate the discharge corresponding to a return period T=50 years, knowing that the mean and standard-deviation of the log-transformed discharge y are $\bar{y} = 0.8$ and $S_y = 0.1$ respectively, an that the skewness $G_s(y)$=-0.4. Use tables 10.4.1abcd at slide 40–42 to obtain the frequency factor $K_T(T, Gs)$.

   Solution: In this case, the equation takes the form: $Q_{est}(50) = 10^{\,\bar{y}+K_T(50,-0.4)\, S_y}$. By checking the table at slide 41 we find that $K_T(50, -0.4) = 1.834$. Thus, $Q_{est}(50) = 10^{0.8+1.834\cdot 0.1} = 9.63\ m^3/s$.

4. **Task:** Write a pseudo-code on paper to compute the estimated maximal discharges using the LP3 approach for an arbitrary number of return periods of interest.

   Solution: A possible pseudocode is reported below:

   $y \leftarrow \log 10$(discharge data)
   $\bar{y} \leftarrow$ mean(y)
   $S_y \leftarrow$ standard deviation(y)
   $G_s \leftarrow$ skewness(y)
   $T_{vec} \leftarrow$ vector of return periods of interest
   computeKTLP3(τ, G_s) $\leftarrow$ define a function that computes the frequency factor for the variable y, given a return period and a skewness
   Initialize results vector Q_{est}

for $i = 1$ to length of T_{vec} **do**
  $K_T \leftarrow$ computeKTLP3$(G_s, T_{vec}(i))$
  $log10Q_{est} \leftarrow \bar{y} + K_T \times S_y$
  $Q_{est}(i) \leftarrow 10^{log10Q_{est}}$
end for

2 Numerical application

1. **Task:** Check how the datafile `2371_Abfluss_Tagesmittel_[...].csv` is formatted. Then, open the script `compute_yearly_maxima.m` or `compute_yearly_maxima.py` and run it. It will:

   Solution: The datafile has 8 lines of header and values are separated by a semicolon (;). There are various variables (in German): the year and the flow value (m^3/s) are at columns 7 and 9. Note that the last data points are provisional (*provisorische Daten*). The data can be imported as a Matlab table with:

   ```
   S = readtable(<input_fname>,'Delimiter',';','NumHeaderLines',8).
   ```

   In the provided script, you can select a different year that will appear in the bottom plot.

2. **Task:** Now start a new script and import the yearly maxima from file `annual_maxima_LeChenit.csv`.

   Solution: As usual, you can import the data into a Matlab table using the command `readtable` (for example,
   `S = readtable('annual_maxima_LeChenit.csv','Format','%f%f','NumHeaderLines',3)`.

3. **Task:** Implement the pseudo-code you have written on paper. It is very convenient that you create a function `computeKTLP3` that takes the variables G_s and T as input, implements the frequency factor equation (see Appendix) and returns the frequency factor K_T for the LP3 distribution.

   Solution: This is an example of function that computes the frequency factor for the LP3 distribution:

```
function KT = computeKTLP3(T,Gs)

F = 1-1/T; % non-exceedance probability for return period T
z =  icdf('normal',F,0,1); % inverse cumulative frequency for a standard normal distribution, evaluated in F
b= Gs  /6; %to simplify the equation
KT = z + (z^2 - 1)*b + (1/3)*(z^3 - 6*z)*b^2 - (z^2 - 1)*b^3 + z*b^4 - (1/3)*(b)^5; % frequency factor of the LP3

end
```

4. **Task:** Test your function for a few values of T and G_s and check that they are similar to those reported in the tables at slides 40–41.

   Solution: You can check that $K_T(T = 50, G_s = -0.4) = 1.8348$, which is very similar to the value provided in the table at slide 41.

5. **Task:** Run your code over a dense range of return periods so you can obtain a smooth function $Q_{est}(T)$, and then plot it. If you want, you can also plot the measured discharge Q against the empirical return period as computed from the Weibull plotting position formula (just like you did in assignment 1 for precipitation data).

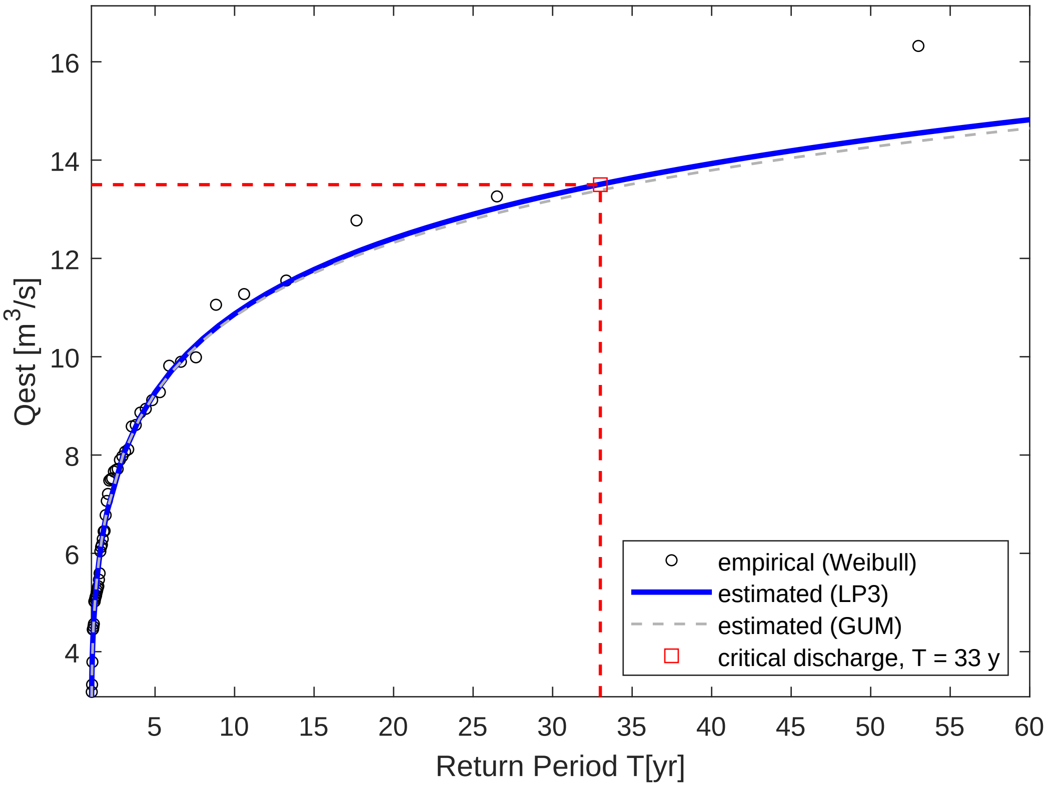

Figure 1: Estimated discharge as a function of return period

Solution: Figure1 shows in blue the discharge Q_{est} computed for return periods in [0.1,60] (note that you cannot compute the frequency factor for $T = 1$). The plot also shows the empirical return period of the discharge data according to the Weibull plotting position approach. As you can see, there was a large event (in 2018) whose return period is estimated to be >100 years.

6. **Question:** What is the probability that the bridge will be flooded during the 5-year mandate of the mayor (*maire* in french)?

   Solution: The return period corresponding to a discharge of 13.5 m^3/s is approximately 33 years (see Figure1). The probability that the bridge will be flooded during the president's mandate is therefore : $\text{P(flood in 5yr)} = 1 - \text{P(no flood in 5yr)} = 1 - (1 - 1/33)^5 = 14.3\%$.

7. **Optional Task:** Use the Gumbel distribution, instead of the LP3 distribution, for the estimation of return periods and use it to provide another estimate of the flooding probability of the bridge during 5 years.

8. **Solution:** Using the method of moments, the parameters of the Gumbel distribution are $\alpha = 0.48$ and $u = 6.13$. The distribution is plotted in Figure 1 and it is visually very close to the LP3 estimate. According to the Gumbel estimate, the critical return period for the event $Q \geq 13.5$ is 35 years and so the flooding probability over 5 years is 13.5%.