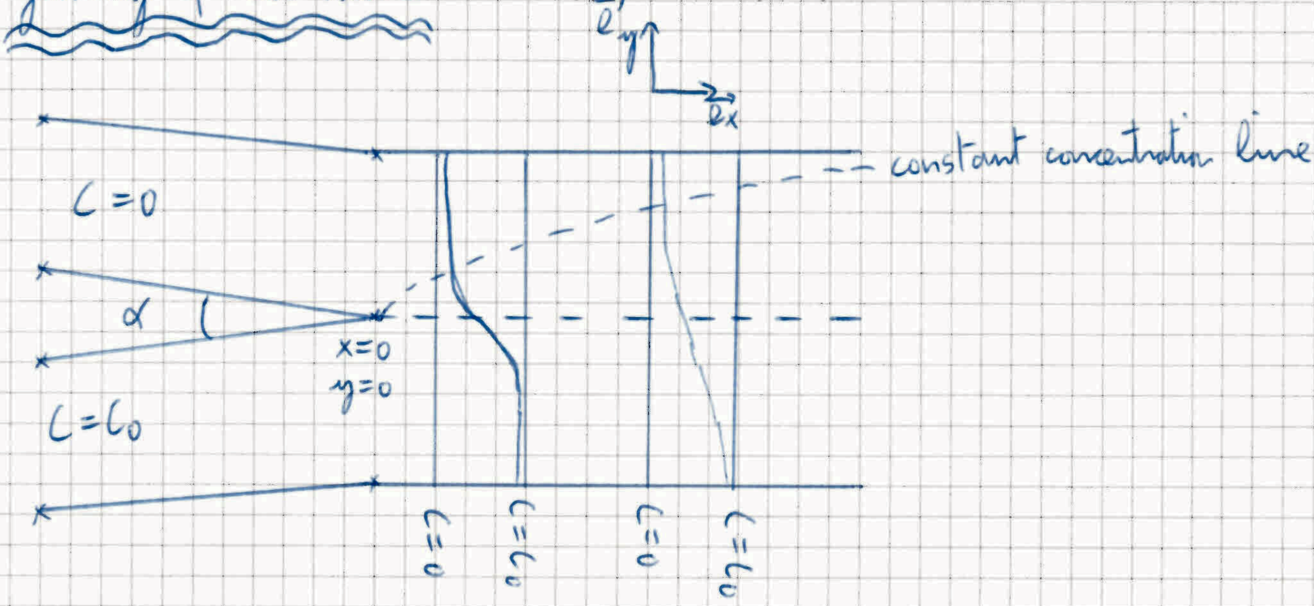
Joining of two rivers



$$\left.\begin{aligned} C(x=0, y>0) &= 0 \\ C(x=0, y<0) &= C_0 \end{aligned}\right] \quad C_0\,\Theta(-y)$$

We look for a steady state $\Rightarrow$ no I.C.

$$\left.\begin{aligned} C(x, y\to\infty) &= 0 \\ C(x, y\to-\infty) &= C_0 \end{aligned}\right] \quad \text{wide river}$$

$\alpha \ll 1$ small angle $\Rightarrow$ negligible advection in the transverse direction

ADE (2D)

$$\underbrace{\cancel{\frac{\partial C}{\partial t}}}_{\text{steady state}} + u\frac{\partial C}{\partial x} + \underbrace{\cancel{v\frac{\partial C}{\partial y}}}_{\text{no transverse advection}} = \underbrace{\cancel{D_x\frac{\partial^2 C}{\partial x^2}}}_{Pe_x = \frac{D_x}{u^2 t} \ll 1} + D_y\frac{\partial^2 C}{\partial y^2}$$

Formal proof later in the lecture

The 2D approach is valid for a shallow river

The 2D ADE reduces to

$$u \frac{\partial C}{\partial x} = D_y \frac{\partial^2 C}{\partial y^2}$$

We set $\eta = \frac{x}{u}$ and $\xi = y$

↳ Flow time from the junction

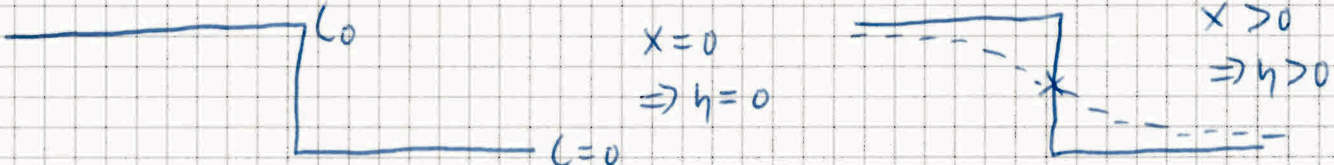
$$\frac{\partial}{\partial x} \cdot = \frac{\partial \eta}{\partial x} \frac{\partial}{\partial \eta} \cdot + \frac{\partial \xi}{\partial x} \frac{\partial}{\partial \xi} \cdot = \frac{1}{u} \frac{\partial}{\partial \eta} \cdot$$

$$\frac{\partial}{\partial y} \cdot = \frac{\partial}{\partial \xi} \cdot$$

The 2D ADE in the new coordinates becomes

$$\frac{\partial C}{\partial \eta} = D_y \frac{\partial^2 C}{\partial \xi^2} \quad (*) \quad \text{D.E. (no advection)}$$

B.C. on x gives the I.C for the D.E. (*)



This initial condition corresponds to the gate opening problem with the solution

$$C(\eta, \xi) = \frac{C_0}{2} \left[1 - \operatorname{erf}\left(\frac{\xi}{\sqrt{4D\eta}} \right) \right]$$

$$\Rightarrow C(x, y) = \frac{C_0}{2} \left[1 - \operatorname{erf}\left(\frac{y}{\sqrt{4D\frac{x}{u}}} \right) \right] \quad \text{No time (steady state)}$$