First order reactions

$$\frac{dC}{dt} = \pm K C \Rightarrow \frac{dC}{C} = \pm K dt$$

$$\ln C = \pm K t + \ln C_0$$

$$C = e^{\pm K t + \ln C_0} = C_0 e^{\pm K t}$$

For the decay $\ominus$ we have the half-life such that

$$C(t_{1/2}) = C_0/2 = C_0 e^{-K t_{1/2}}$$

$$\Rightarrow -t_{1/2} = -\frac{\ln 2}{K} \Rightarrow t_{1/2} = \frac{\ln 2}{K}$$ Independent of C_0!

For the growth $\oplus$ we can introduce similarly a doubling period $t_{1/2}$.

1D ADR equation

$$\frac{\partial C}{\partial t} + V_{adv} \frac{\partial C}{\partial x} = D \frac{\partial^2 C}{\partial x^2} \pm K C$$ $\ominus \rightarrow$ we restrict ourselves to decay

We set $C = \phi(x,t) e^{-Kt}$

We have $\frac{\partial C}{\partial t} = \frac{\partial \phi}{\partial t} e^{-Kt} - K \phi e^{-Kt}$

and $\frac{\partial^2 C}{\partial x^2} = e^{-Kt} \frac{\partial^2 \phi}{\partial x^2}$

The ADR equation becomes $\frac{\partial \phi}{\partial t} + V_{adv} \frac{\partial \phi}{\partial x} = D \frac{\partial^2 \phi}{\partial x^2}$ ADE

with solution $\phi(x,t) = \frac{M}{A\sqrt{4\pi D t}} e^{-(x - V_{adv} t)^2 / 4Dt}$ $\qquad \phi(x, t \to 0) = \frac{M}{A}\delta(x)$

$$C(x,t) = \frac{M}{A\sqrt{4\pi D t}} e^{-(x - V_{adv} t)^2 / 4Dt} e^{\pm K t}$$