Environmental transport phenomena:
Lecture I

Benoît Crouzy
(benoit.crouzy@meteoswiss.ch)

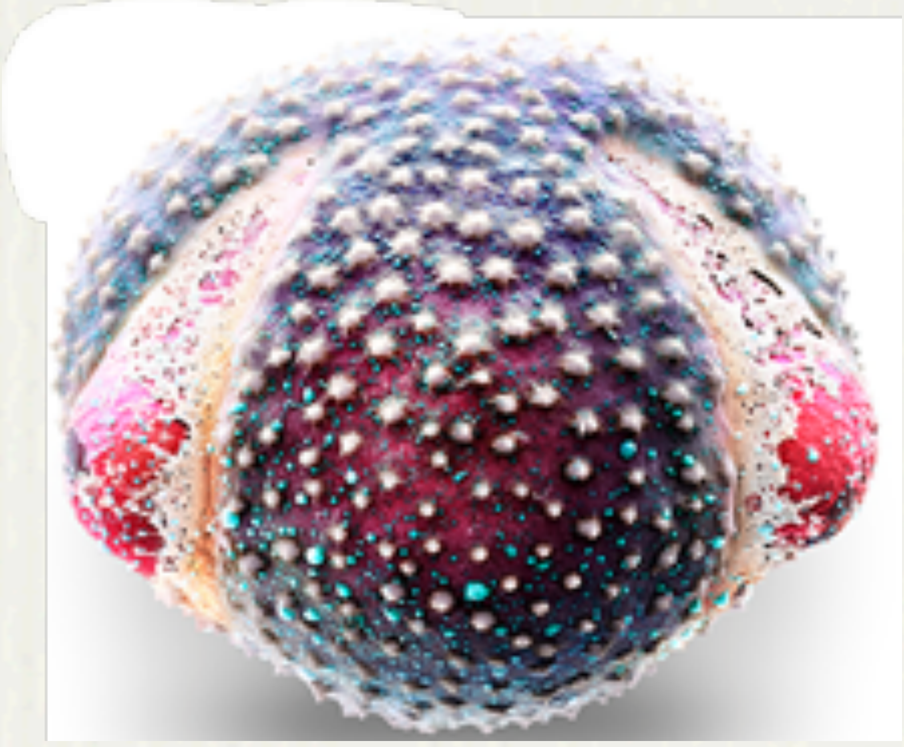
EPFL

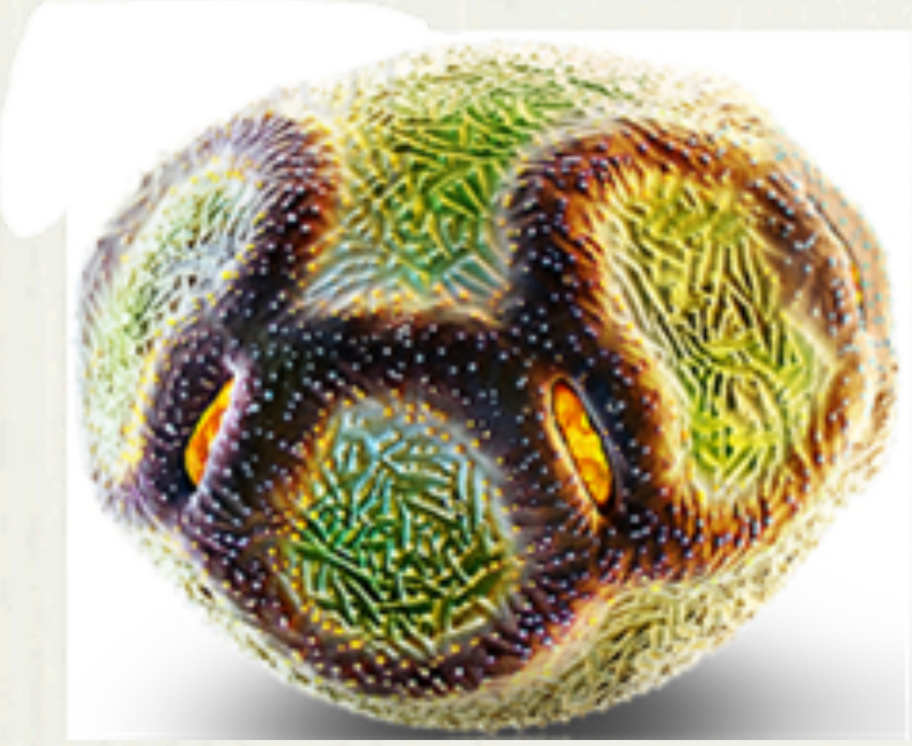
Schweizerische Eidgenossenschaft
Confédération suisse
Confederazione Svizzera
Confederaziun svizra

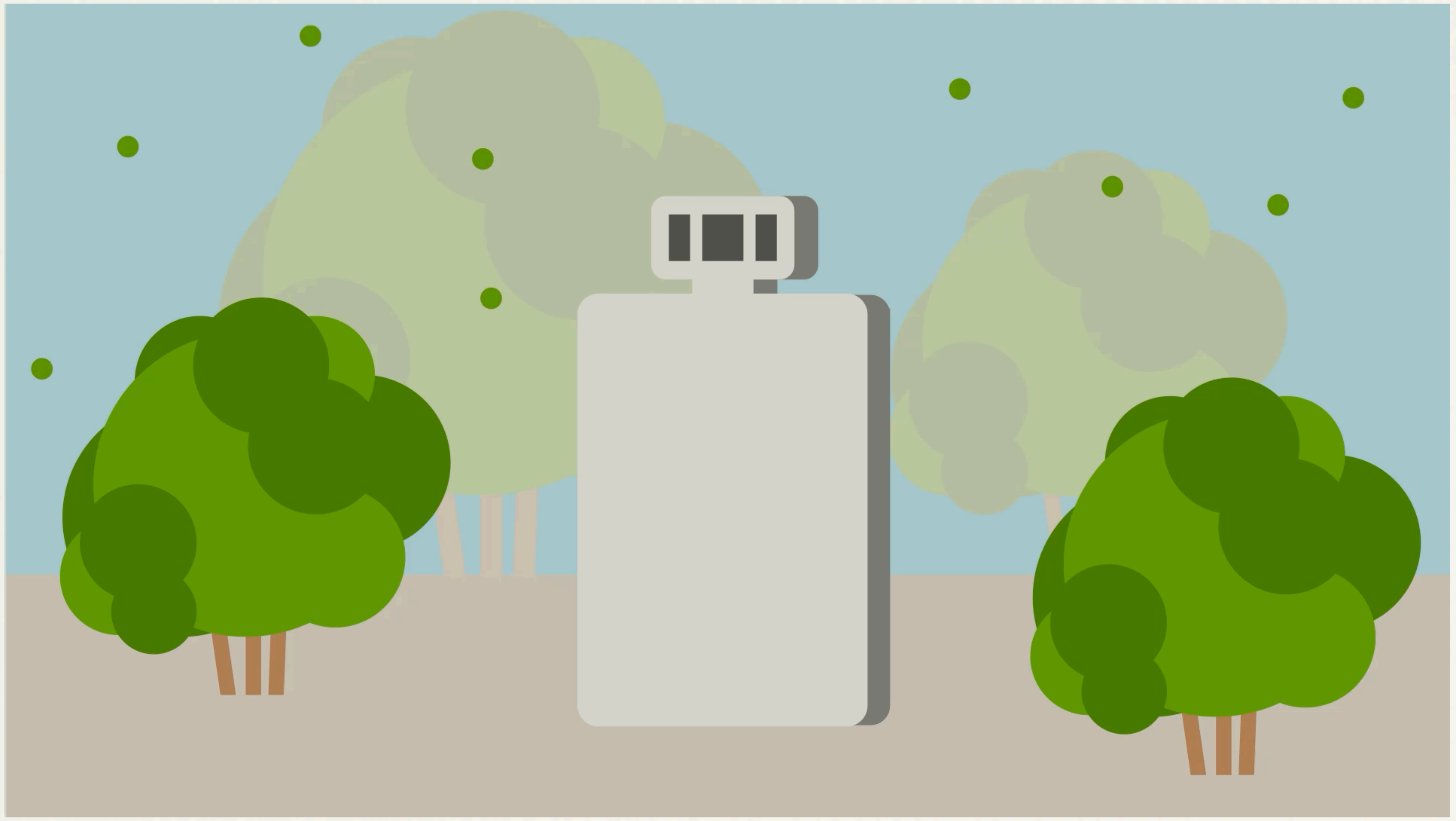
Swiss Confederation

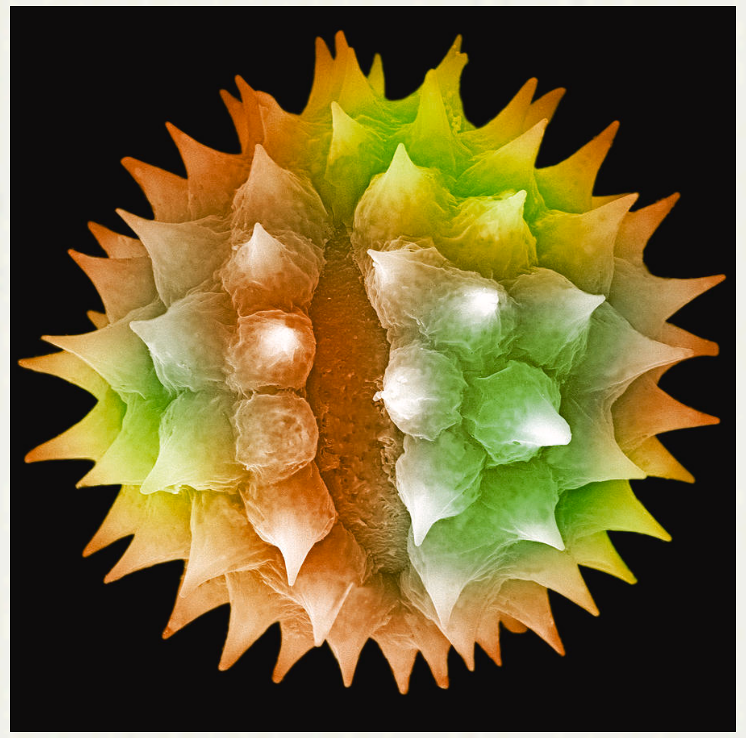
Federal Department of Home Affairs FDHA
Federal Office of Meteorology and Climatology MeteoSwiss

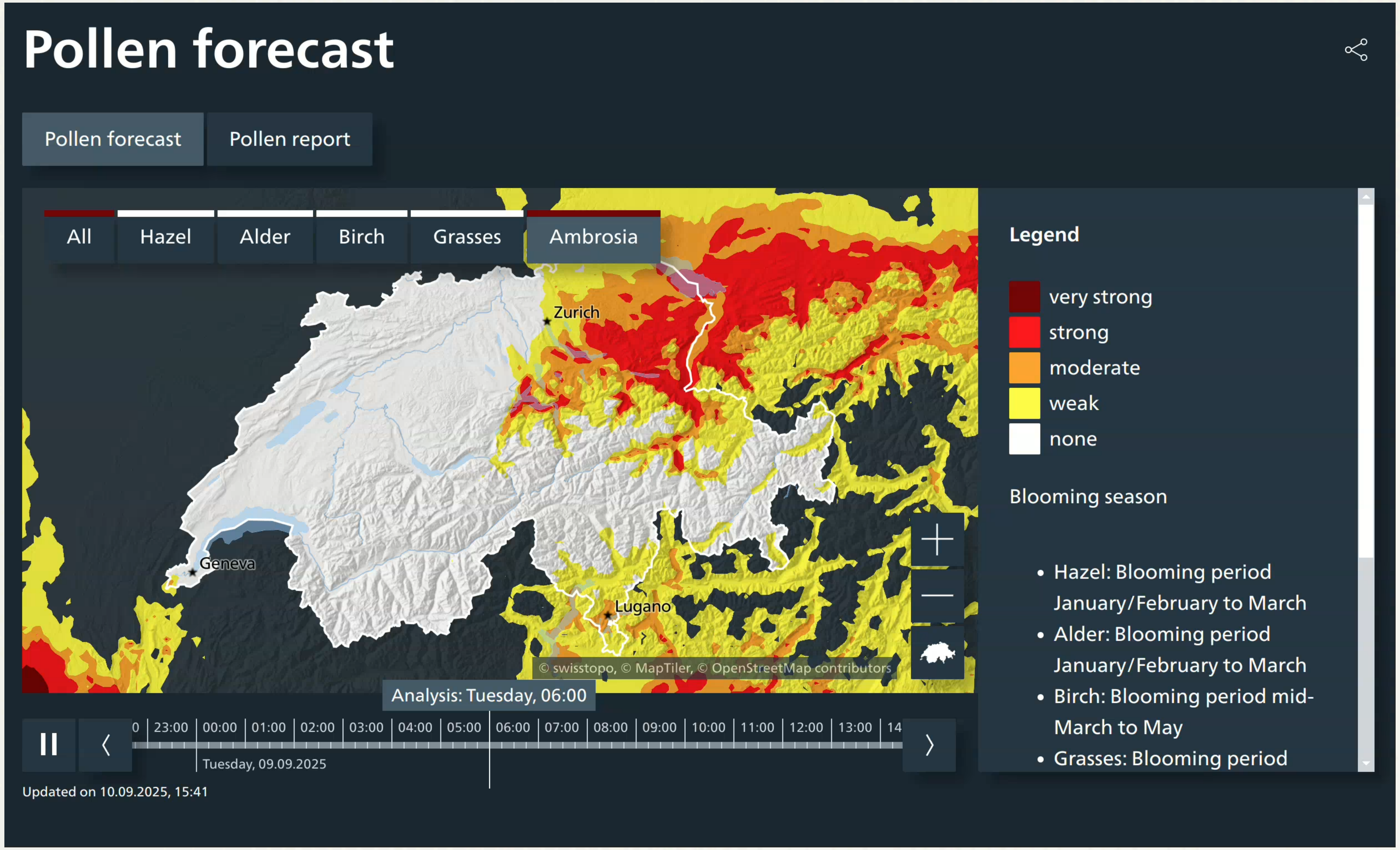
MeteoSwiss











Pollen forecast

© swisstopo, © MapTiler, © OpenStreetMap contributors

Analysis: Tuesday, 06:00

Tuesday, 09.09.2025

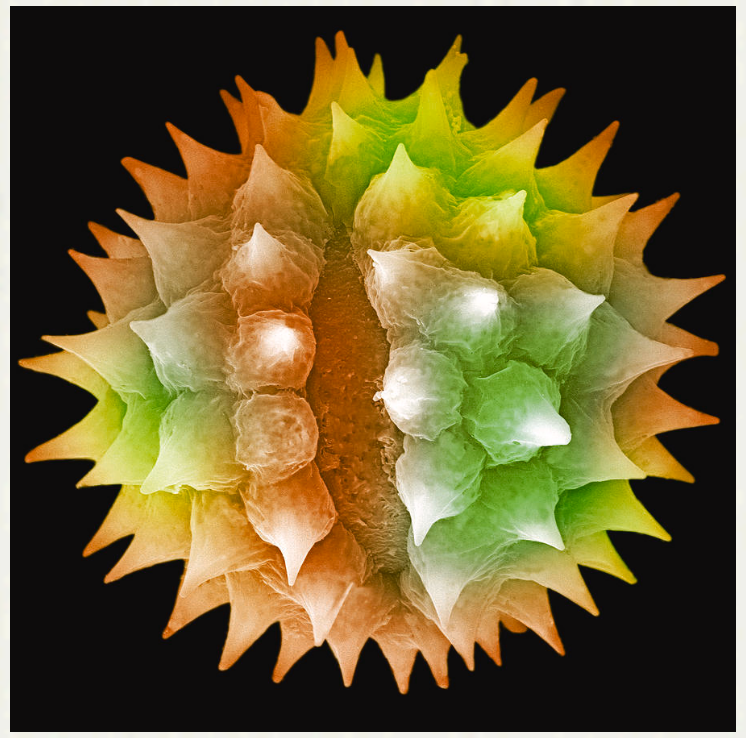
Updated on 10.09.2025, 15:41

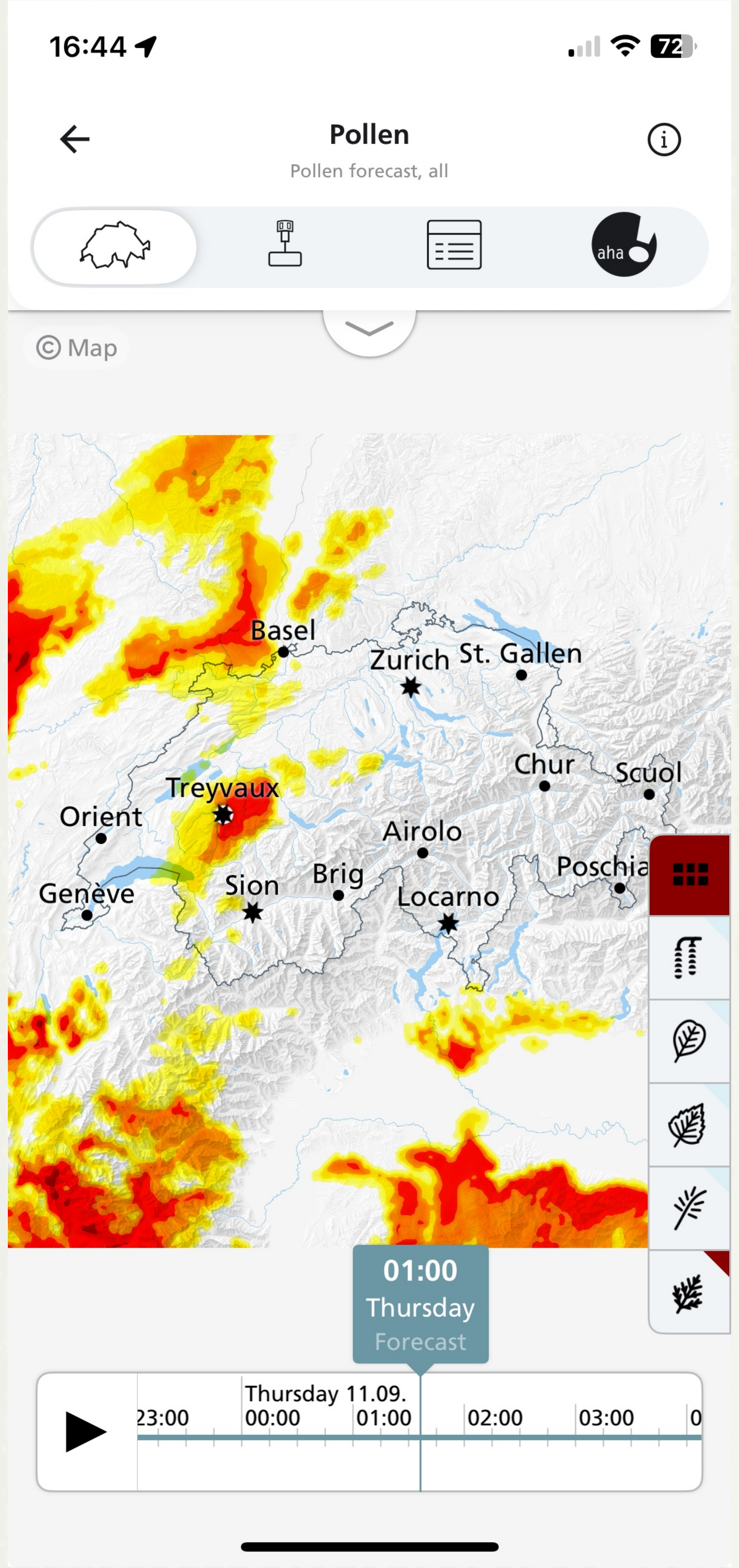
Legend

🟫 very strong
🟥 strong
🟧 moderate
🟨 weak
⬜ none

Blooming season

- Hazel: Blooming period January/February to March
- Alder: Blooming period January/February to March
- Birch: Blooming period mid-March to May
- Grasses: Blooming period





16:44

Pollen

Pollen forecast, all

© Map

01:00
Thursday
Forecast

Thursday 11.09.

Scope of the lecture

"**physical** aspects of **molecular and turbulent diffusion**, as well as of **dispersion** processes, their **mathematical modeling**, solutions and related **environmental applications**"

Objectives

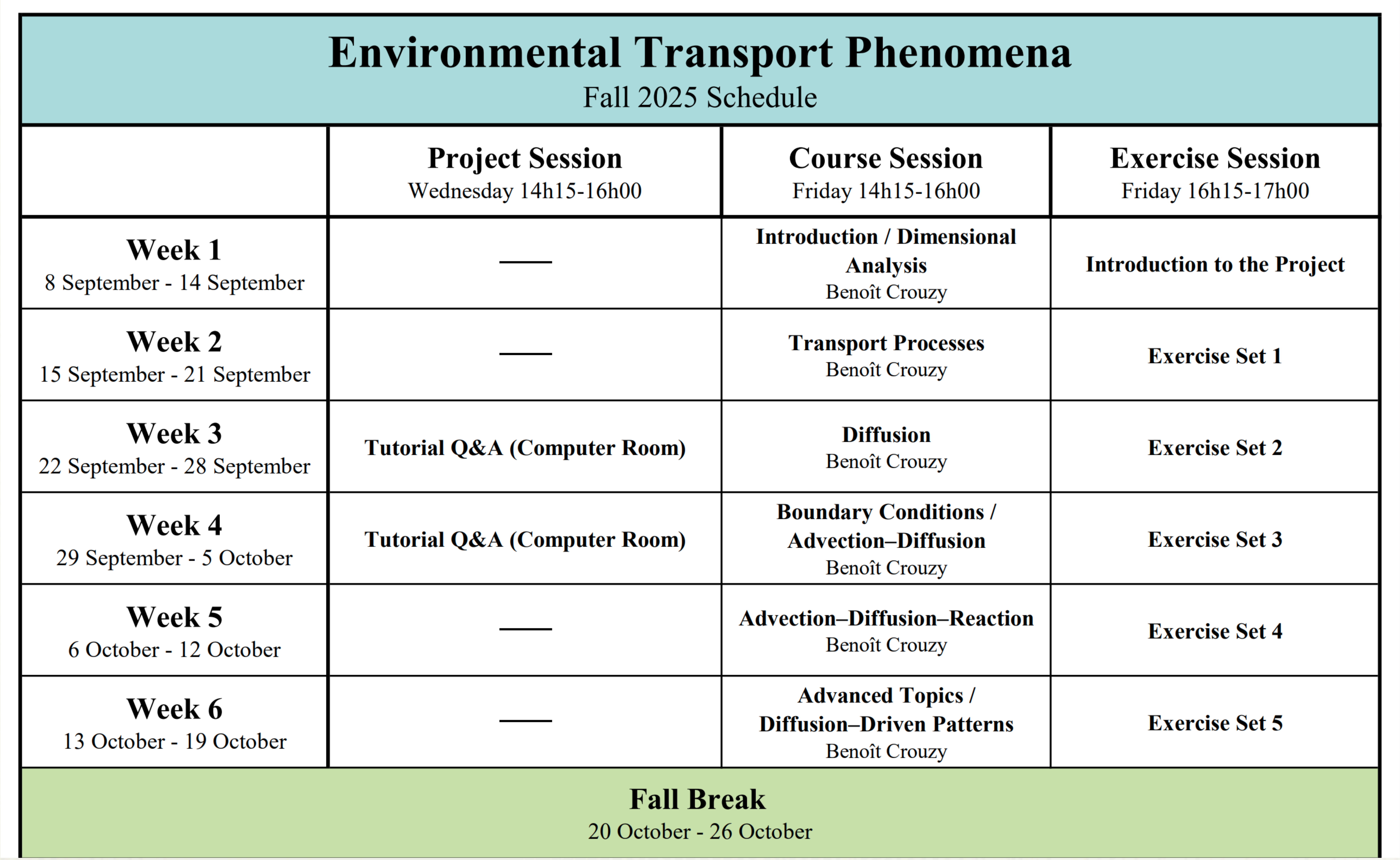
Describe and interpret the physical processes

Solve and elaborate simple physical **models**

Apply computational fluid dynamics (CFD) **models**

Develop numerical transport **models with FLUENT**: problem formulation, **modeling**, and **interpretation** of the results

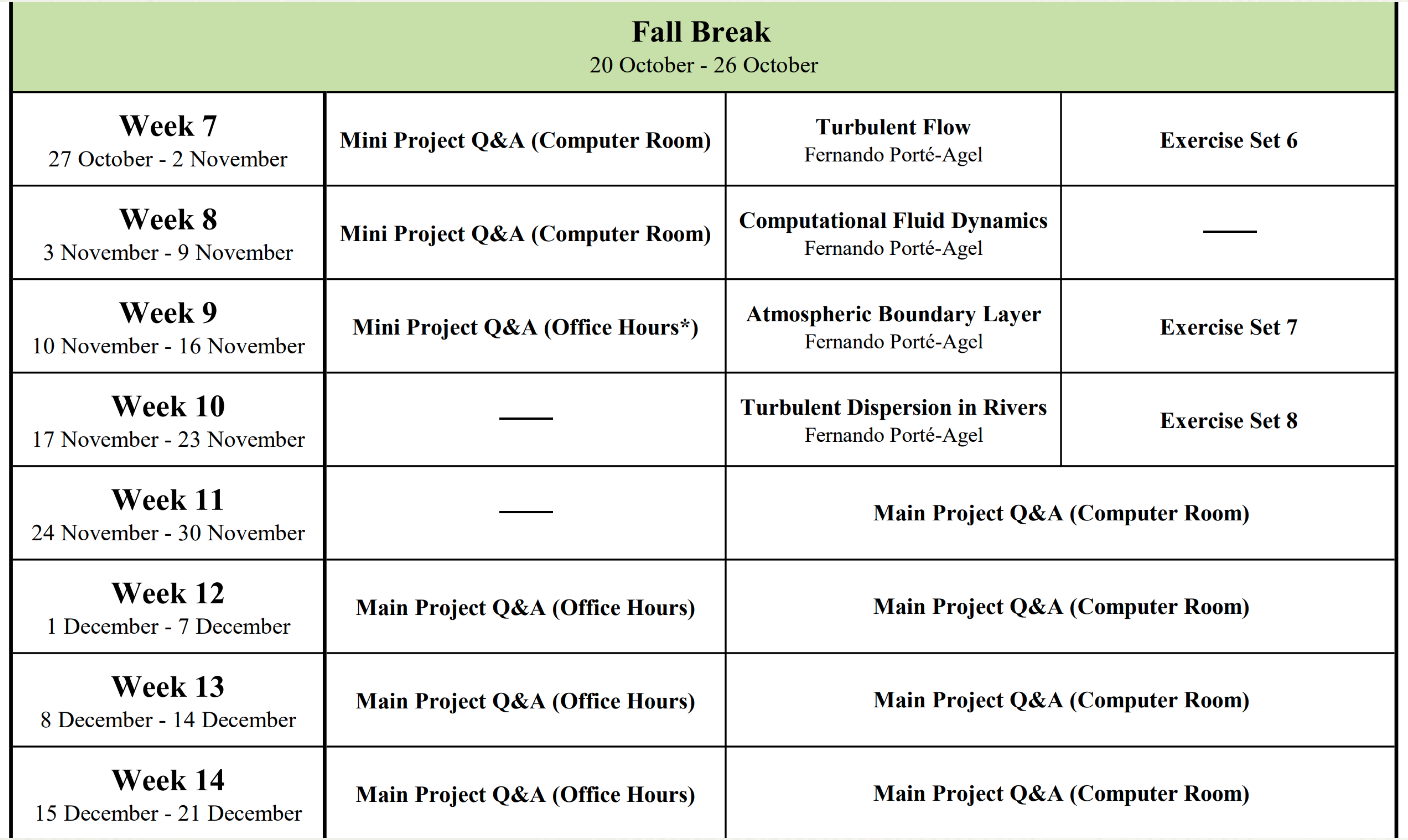
Course organisation



Environmental Transport Phenomena
Fall 2025 Schedule

	Project Session Wednesday 14h15-16h00	**Course Session** Friday 14h15-16h00	**Exercise Session** Friday 16h15-17h00
Week 1 8 September - 14 September	—	**Introduction / Dimensional Analysis** Benoît Crouzy	**Introduction to the Project**
Week 2 15 September - 21 September	—	**Transport Processes** Benoît Crouzy	**Exercise Set 1**
Week 3 22 September - 28 September	**Tutorial Q&A (Computer Room)**	**Diffusion** Benoît Crouzy	**Exercise Set 2**
Week 4 29 September - 5 October	**Tutorial Q&A (Computer Room)**	**Boundary Conditions / Advection–Diffusion** Benoît Crouzy	**Exercise Set 3**
Week 5 6 October - 12 October	—	**Advection–Diffusion–Reaction** Benoît Crouzy	**Exercise Set 4**
Week 6 13 October - 19 October	—	**Advanced Topics / Diffusion–Driven Patterns** Benoît Crouzy	**Exercise Set 5**
Fall Break 20 October - 26 October			

Course organisation



Fall Break 20 October - 26 October			
Week 7 27 October - 2 November	**Mini Project Q&A (Computer Room)**	**Turbulent Flow** Fernando Porté-Agel	**Exercise Set 6**
Week 8 3 November - 9 November	**Mini Project Q&A (Computer Room)**	**Computational Fluid Dynamics** Fernando Porté-Agel	——
Week 9 10 November - 16 November	**Mini Project Q&A (Office Hours*)**	**Atmospheric Boundary Layer** Fernando Porté-Agel	**Exercise Set 7**
Week 10 17 November - 23 November	——	**Turbulent Dispersion in Rivers** Fernando Porté-Agel	**Exercise Set 8**
Week 11 24 November - 30 November	——	**Main Project Q&A (Computer Room)**	
Week 12 1 December - 7 December	**Main Project Q&A (Office Hours)**	**Main Project Q&A (Computer Room)**	
Week 13 8 December - 14 December	**Main Project Q&A (Office Hours)**	**Main Project Q&A (Computer Room)**	
Week 14 15 December - 21 December	**Main Project Q&A (Office Hours)**	**Main Project Q&A (Computer Room)**	

Material

On Moodle

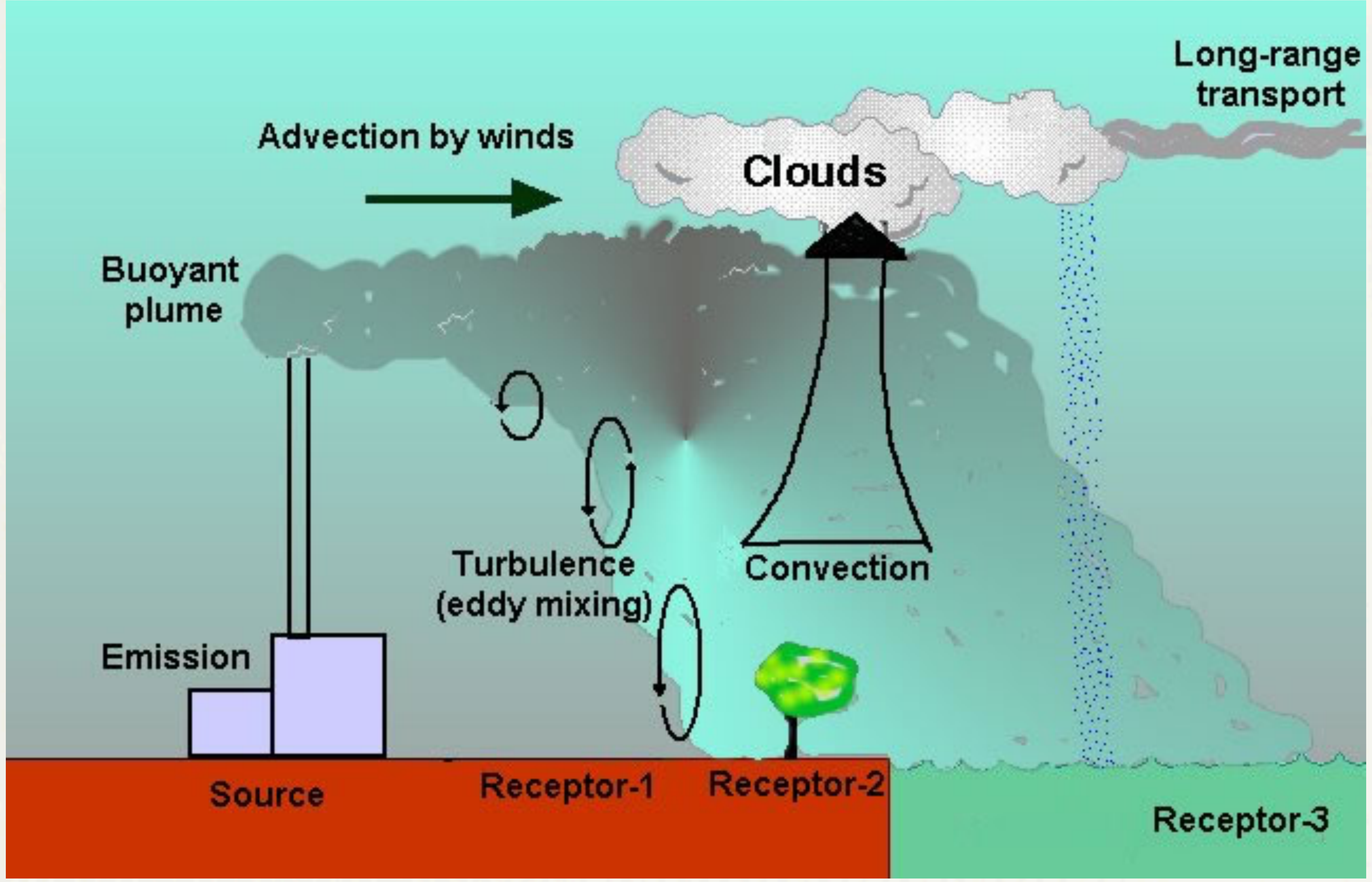
- Slides
- Lecture notes
- Reference (book)

Grading

Course grade: **written** exam 70% + **project** grade 30%

Contents

- **Advection and diffusion**
- **Point source pollution**
- **Introduction to turbulence**
- **Turbulent dispersion**
- Mixing in rivers, lakes and in reservoirs
- Atmospheric boundary layer
- Computational fluid dynamics



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Jérémy Toma

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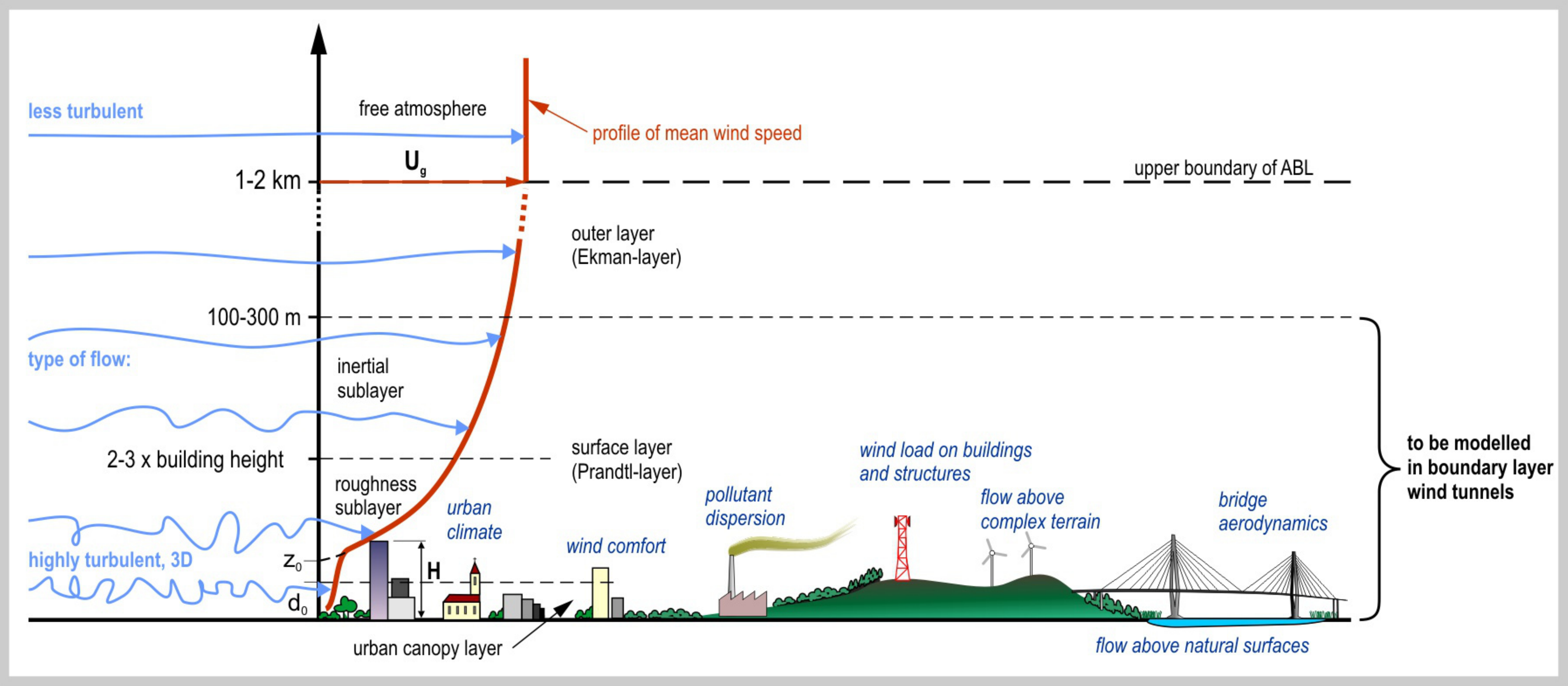

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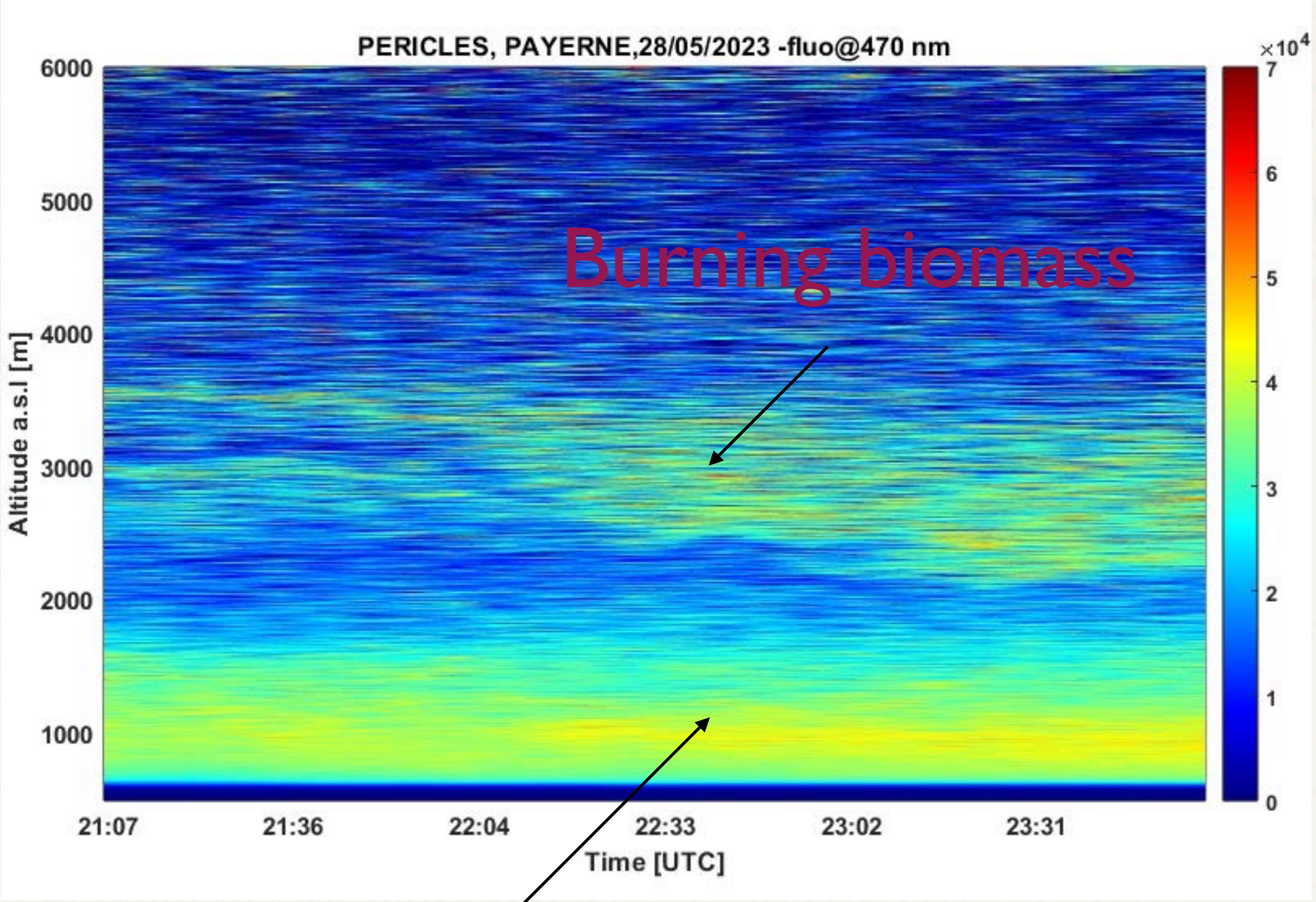
Fluorescent particles



Burning biomass

Local bioaerosol

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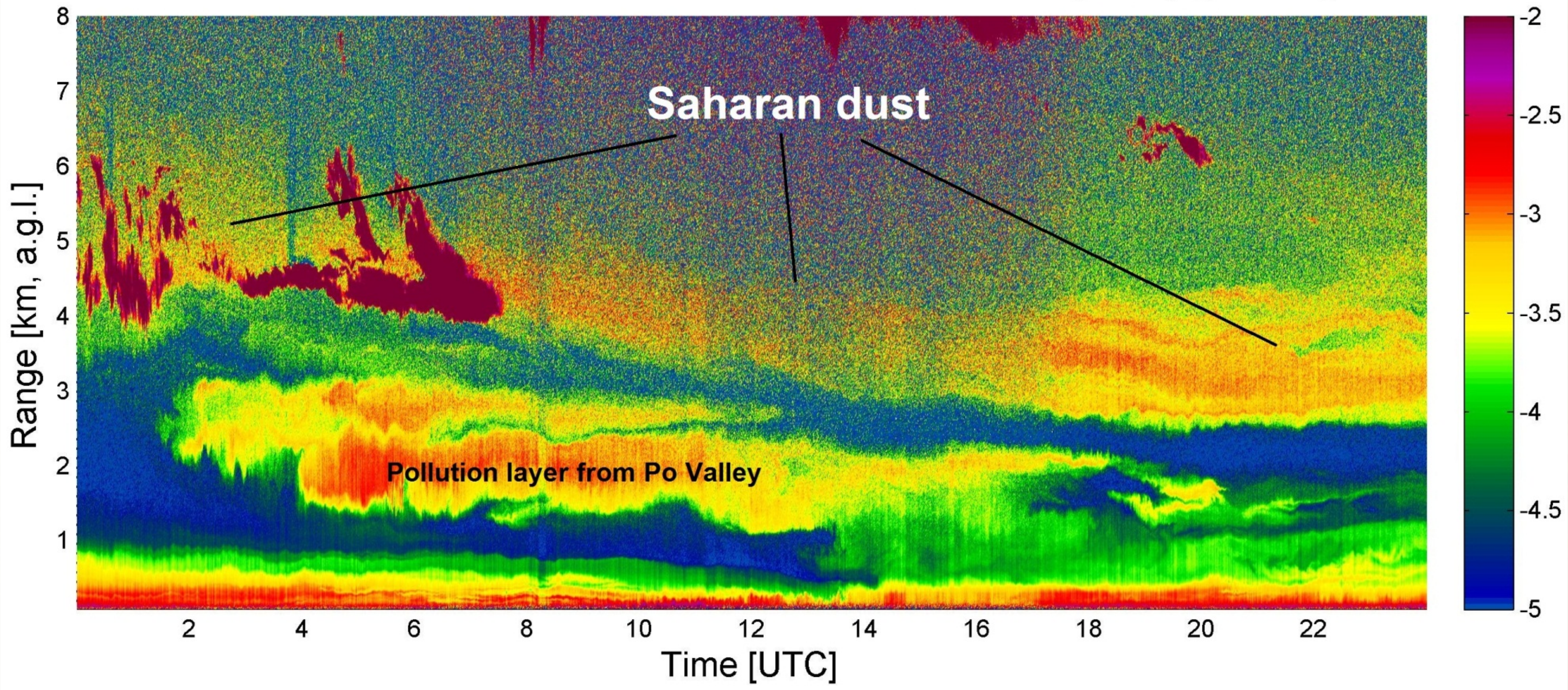

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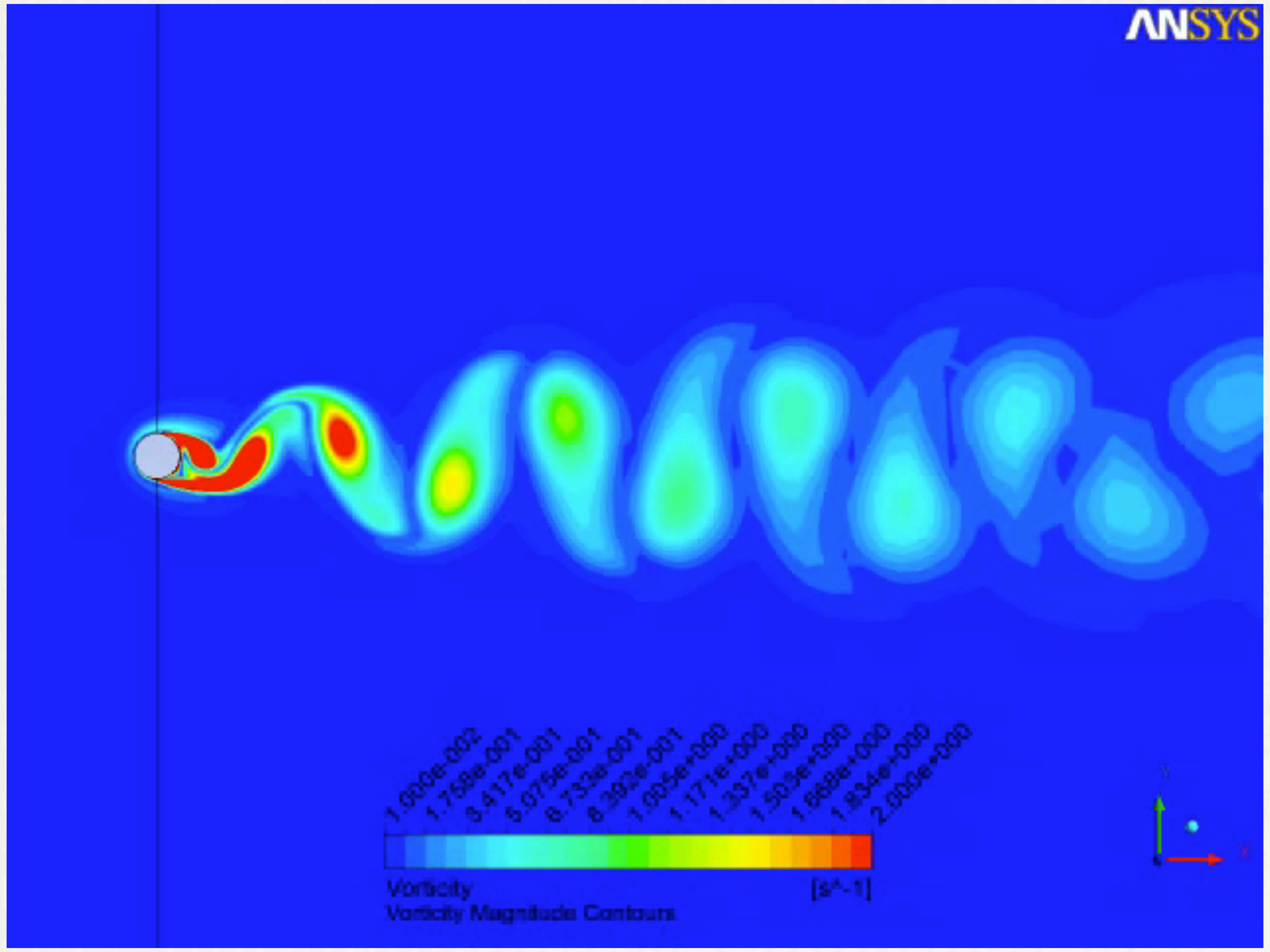
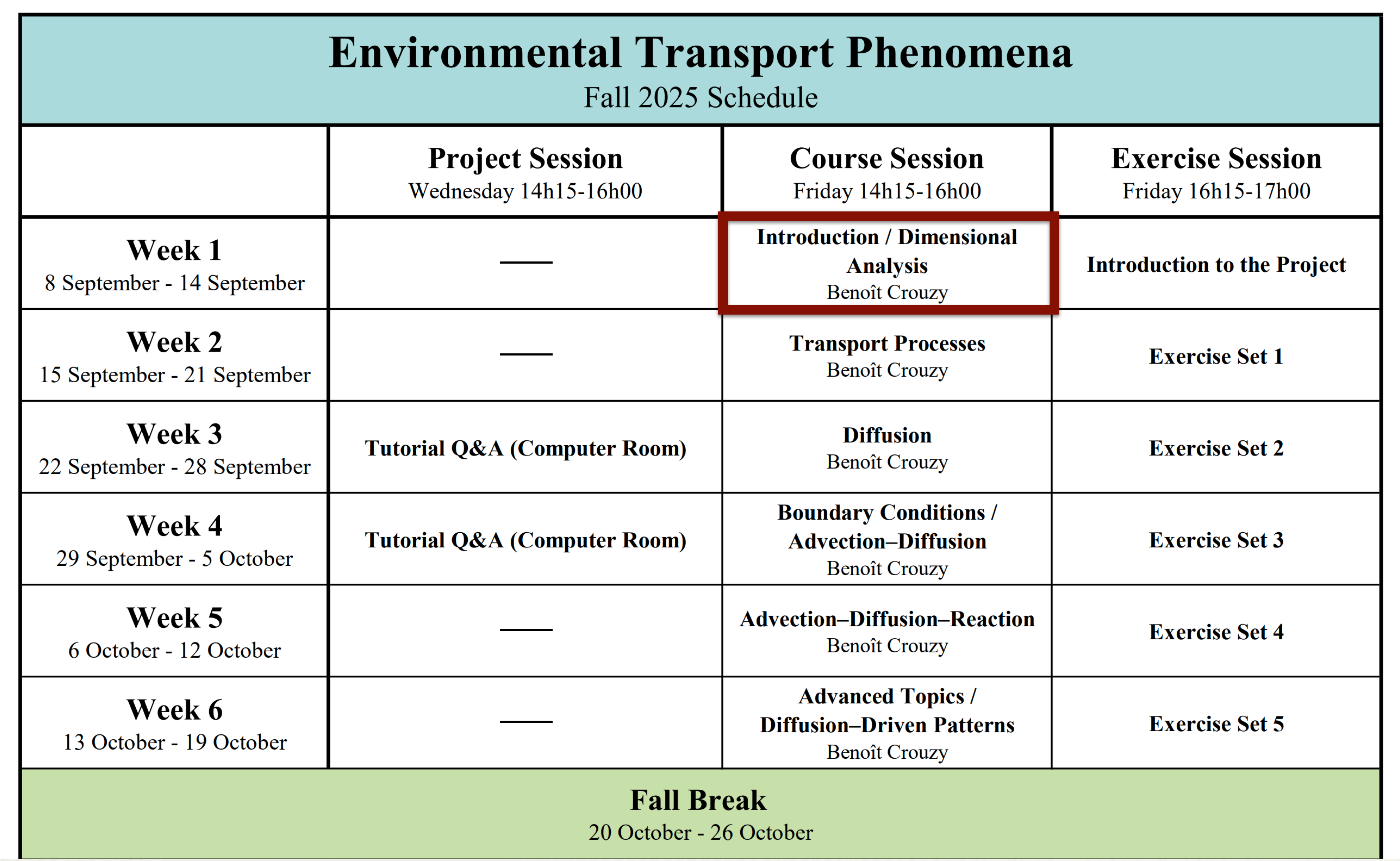


Outline (Part I of the lecture)



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I) Dimensional analysis

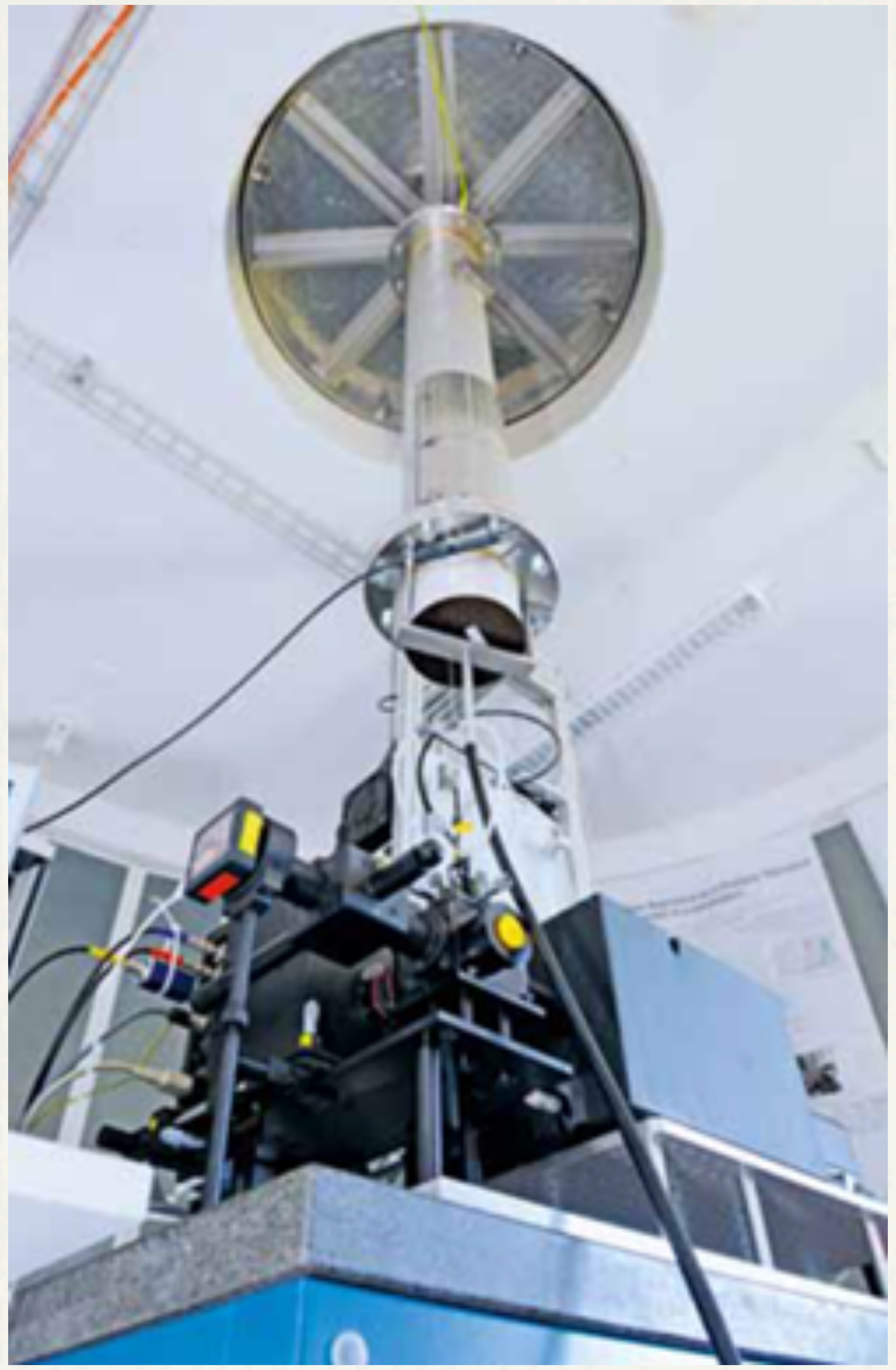
Dimensional analysis

- Review on **units** and **dimensions**
- Simple approach to **tackle complex problems**
- Physical modeling (**scaled** models)
- Interpretation of experimental data (**relative** importance of physical phenomena)

Metrology and environmental sciences

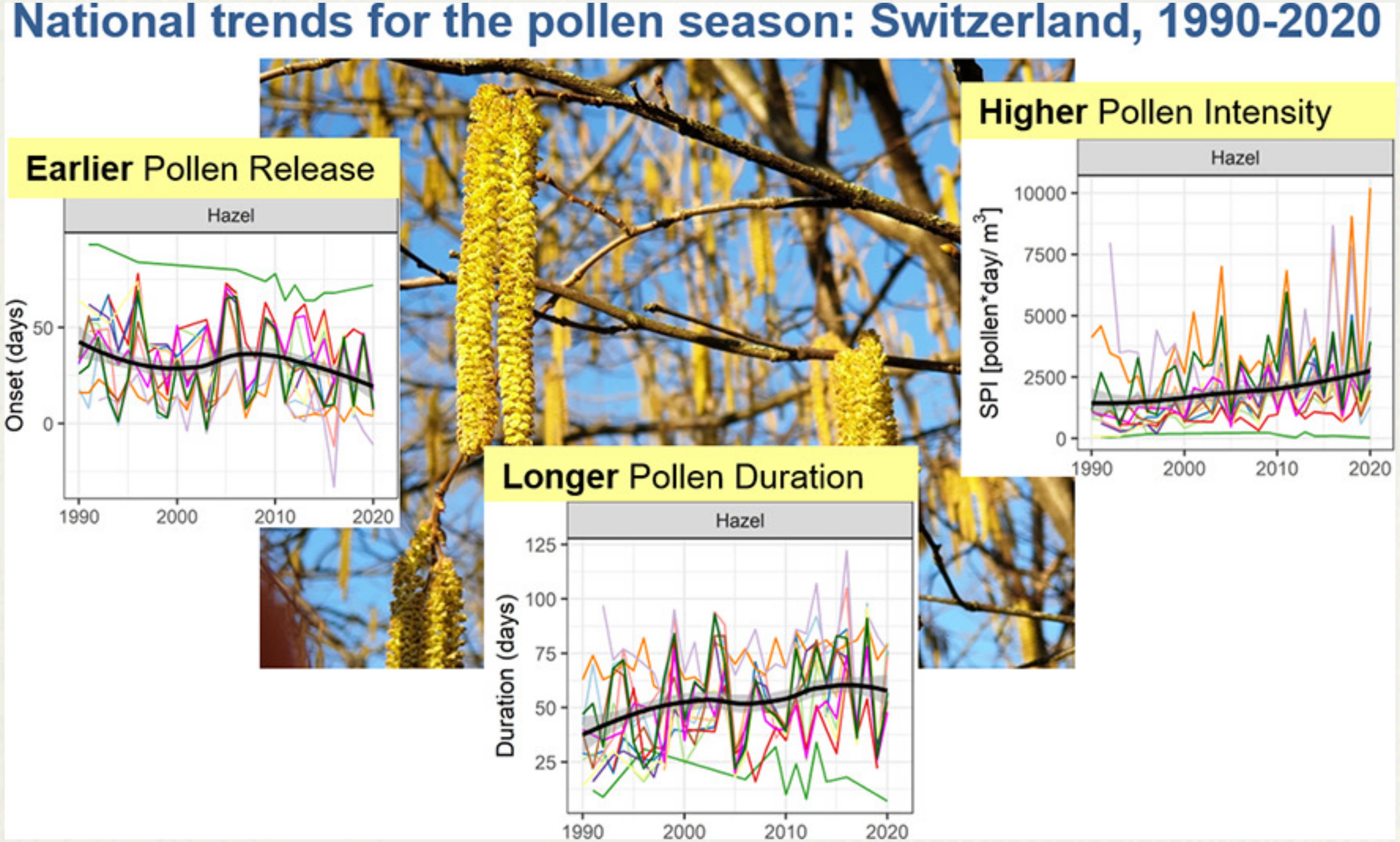
Standardisation of measurement / units assured by metrology institutes traditionally domain of physical sciences and engineering. However,

- need for **traceability** / reproducibility in environmental sciences (e.g. **climate change** studies)
- need to combine **various sources of measurement** for global environmental studies



Calibration of optical particle counters at the Swiss institute for Metrology

Metrology and environmental sciences



National trends for the pollen season: Switzerland, 1990-2020

climate analysis affected by technical changes ?

Dimension and units

Dimension: **type** of physical quantity, **numerical value** assigned by unit (SI units)

- **fundamental** dimensions (eg. mass, length, time, electric current, temperature, amount of substance and luminous intensity) and units (kg, m, s, A, K, mol, cd)
- **derived** dimensions

$$[Q_i] = L^{\alpha_i} M^{\beta_i} T^{\gamma_i}$$

$$[\mu_i] = L^{-1} M^{1} T^{-1}$$

Always monomial power law !

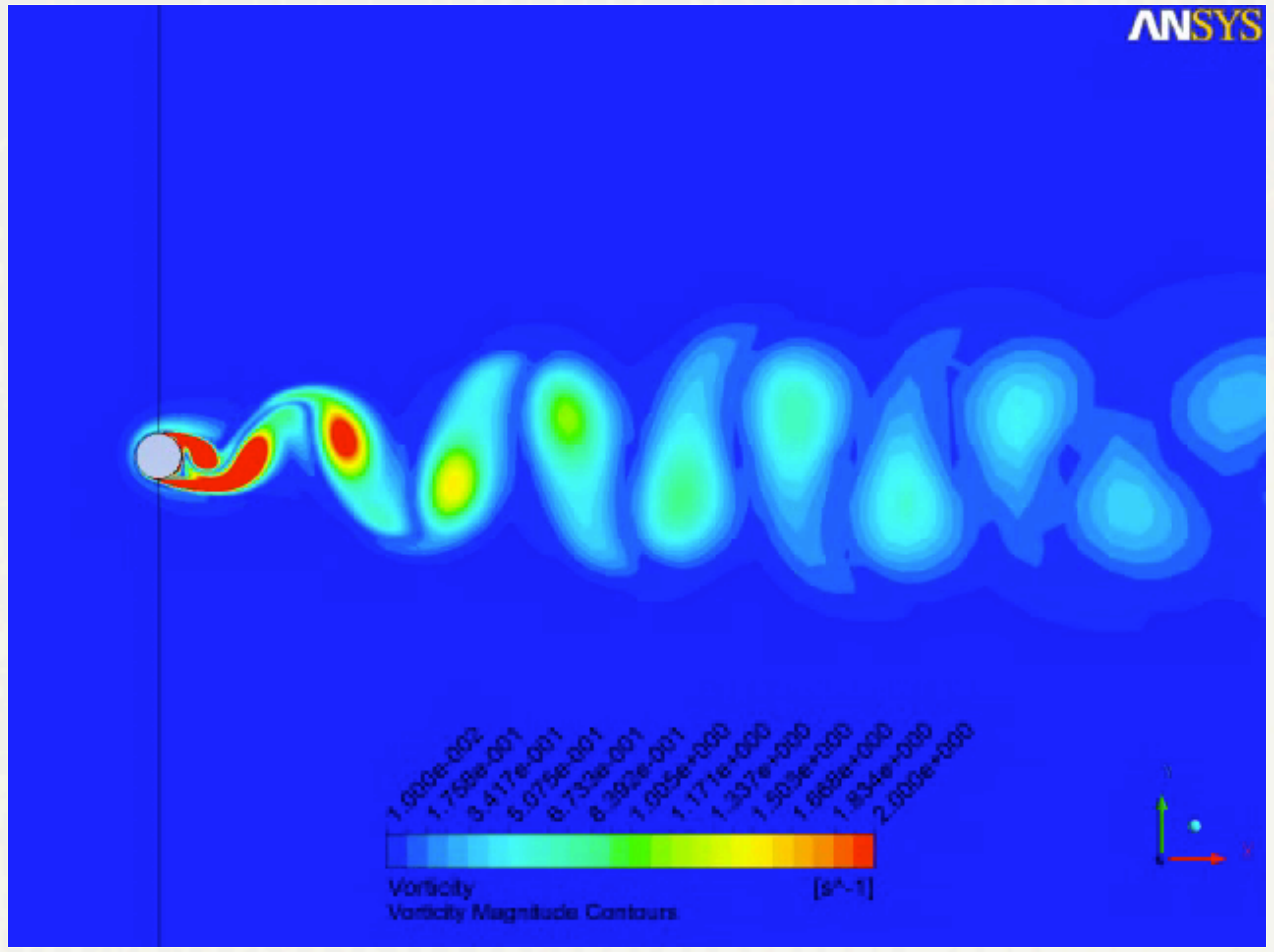
Dimensional **homogeneity**: all additive terms in physical equations must have equal dimensions (check your equations !)

Dimensional independence

Condition for independence of Q_1, Q_2 and Q_3

$$[Q_i] = L^{\alpha_i} M^{\beta_i} T^{\gamma_i}$$

$$det \begin{pmatrix} \alpha_1 & \alpha_2 & \alpha_3 \\ \beta_1 & \beta_2 & \beta_3 \\ \gamma_1 & \gamma_2 & \gamma_3 \end{pmatrix} \neq 0$$



Counter-example: object in a fluid, obstacle dimension **D**, flow velocity **V,** dynamic viscosity μ and fluid density ρ can be combined into a **dimensionless** number.

$$Re = \frac{DV\rho}{\mu}$$

Buckingham π-theorem

Systematic method for computing **dimensionless parameters** from physical variables

- Any physical equation can be written in the form (n physical variables, written in terms of k independent units)

$$f(q_1, q_2, \ldots, q_{n-1}) = q_n$$

- It can be rewritten in the form (**p=n-k**)

$$F(\pi_1, \pi_2, \ldots, \pi_{p-1}) = \pi_p$$

with $\pi_i = q_1^{a_1^i} q_2^{a_2^i} \ldots q_n^{a_n^i}$

form of F a-priori unknown (use experimental data), **exception** for p=1 then F=cste

Buckingham π-theorem

How to proceed to build dimensionless variables:

- List the relevant **physical quantities**

$$[Q_i] = L^{\alpha_i} M^{\beta_i} T^{\gamma_i}$$

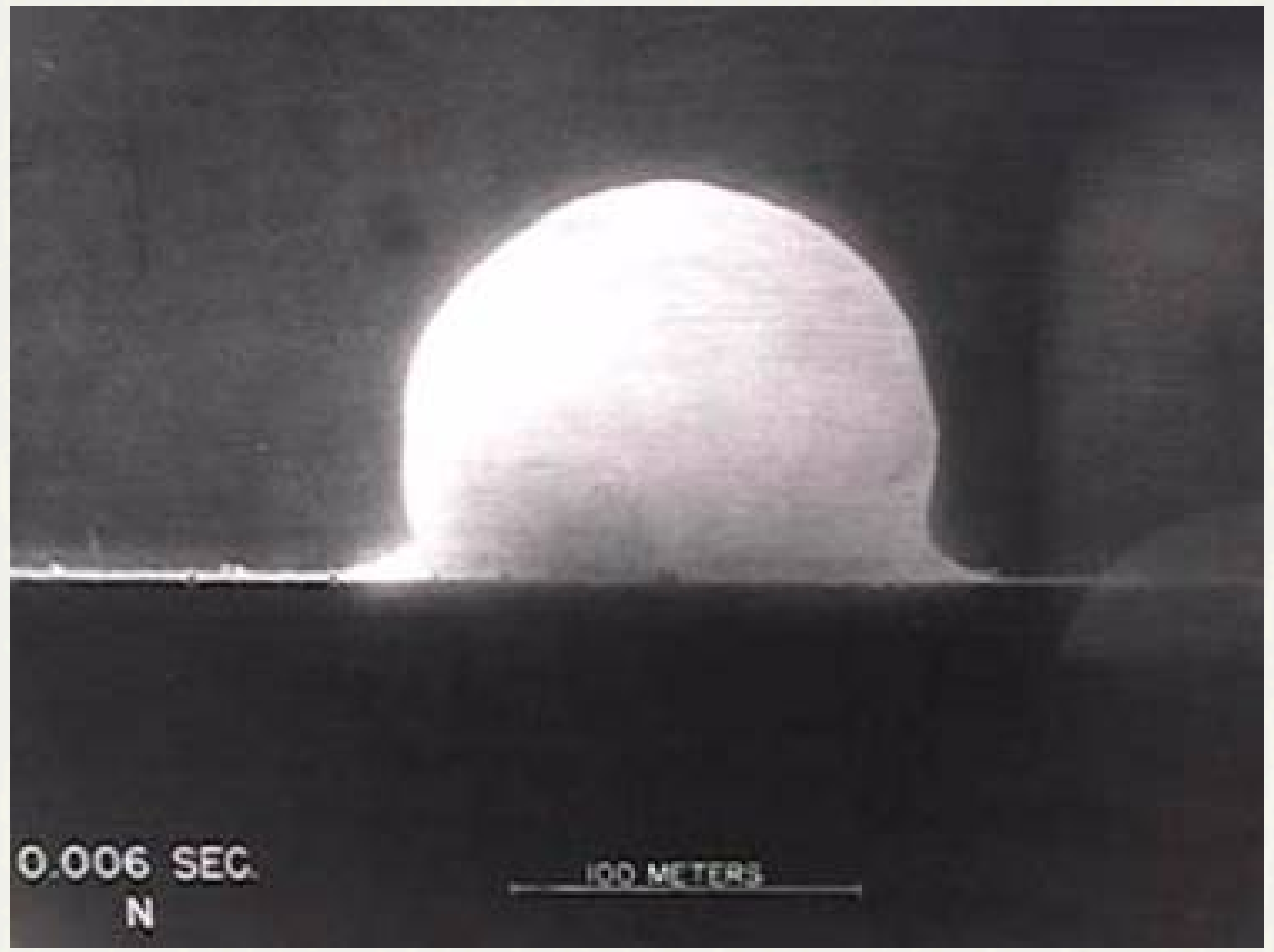
- List the involved **fundamental dimensions**

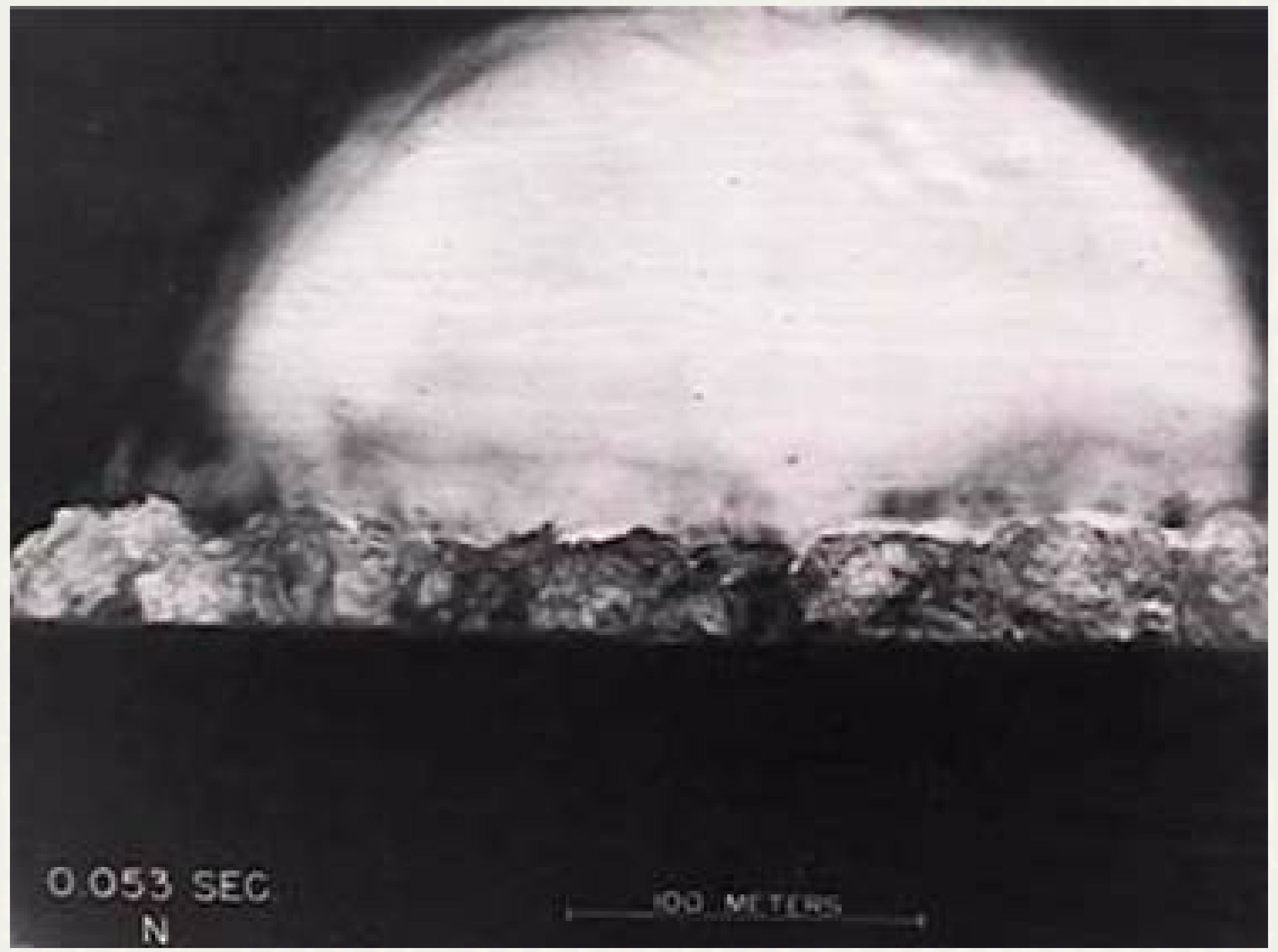
- Write down the **dimensional matrix**

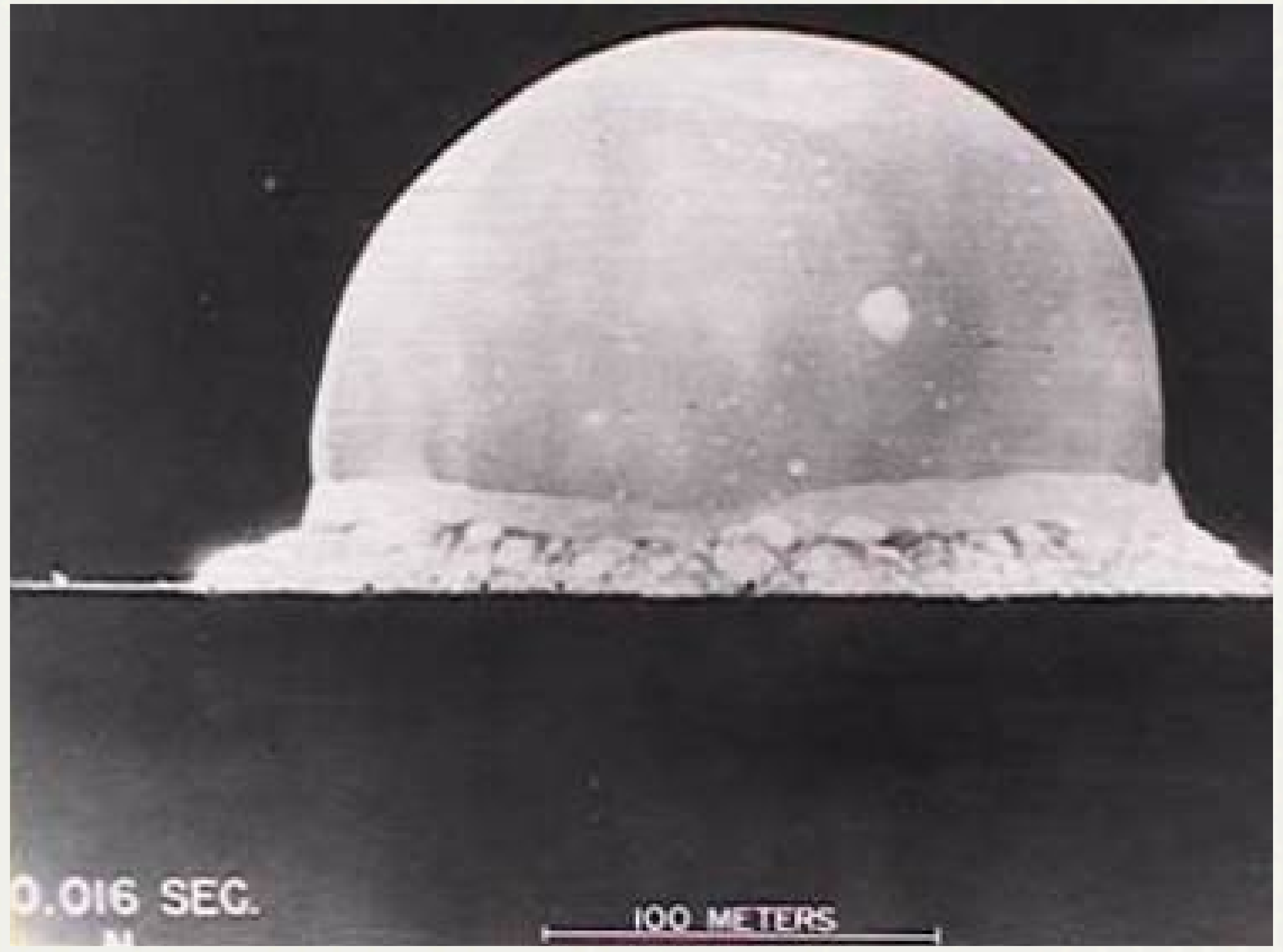
$$\begin{pmatrix} \alpha_1 & \alpha_2 & \alpha_3 \\ \beta_1 & \beta_2 & \beta_3 \\ \gamma_1 & \gamma_2 & \gamma_3 \end{pmatrix}$$

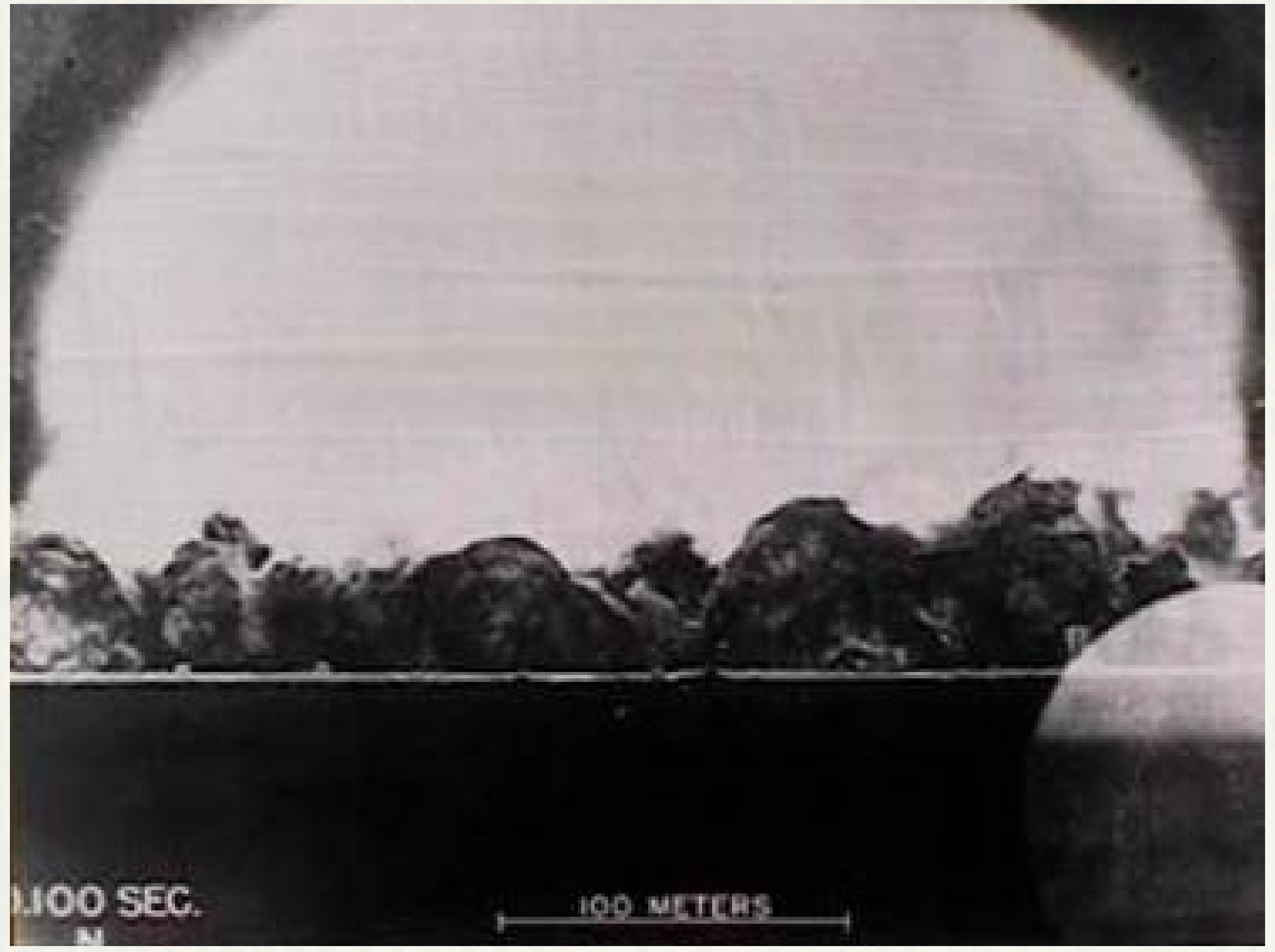
- Find a basis of the **kernel** of this matrix (see basic linear algebra, Gauss-Jordan elimination)

Buckingham π-theorem (Example 1)









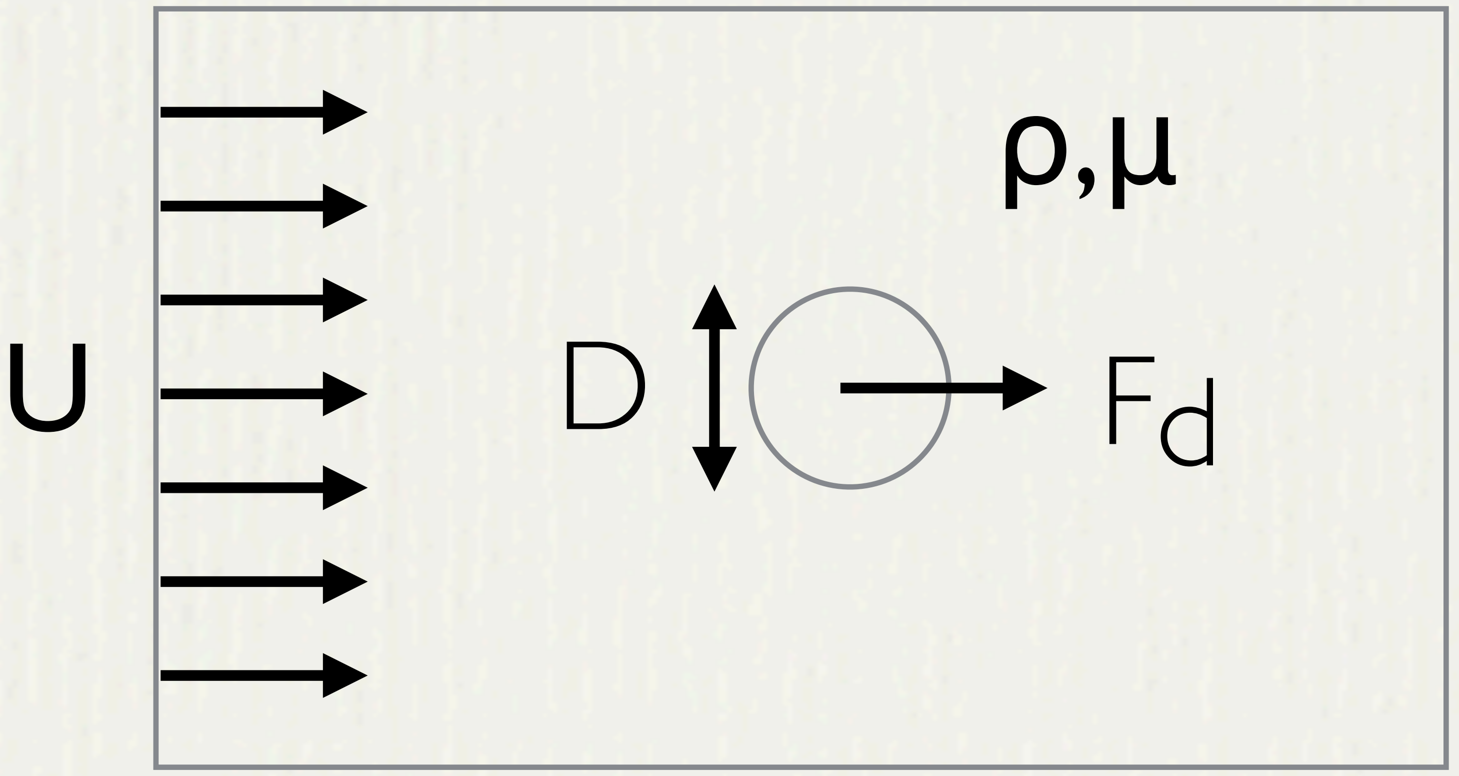
Estimation of Trinity atomic test released **energy** from pictures (blackboard derivation)

Buckingham π-theorem

Remaks on the technique

- Dimensional analysis simply states that there is a relationship between quantities. It doesn't (**except** in the case of a single π, which must, therefore, be constant) states what the relationship is. For the specific relationship one must appeal to theory or, more commonly, to experimental data.
- The choice of the kernel basis is **not unique**, intuition or trial/error is needed to obtain meaningful results (see examples).

Buckingham π-theorem (Example 2)



$$F_d = f(\rho, \mu, U, D)$$

$$\pi_1 = \frac{F_d}{\rho U^2 D^2}$$

p=n-k=5-3=2

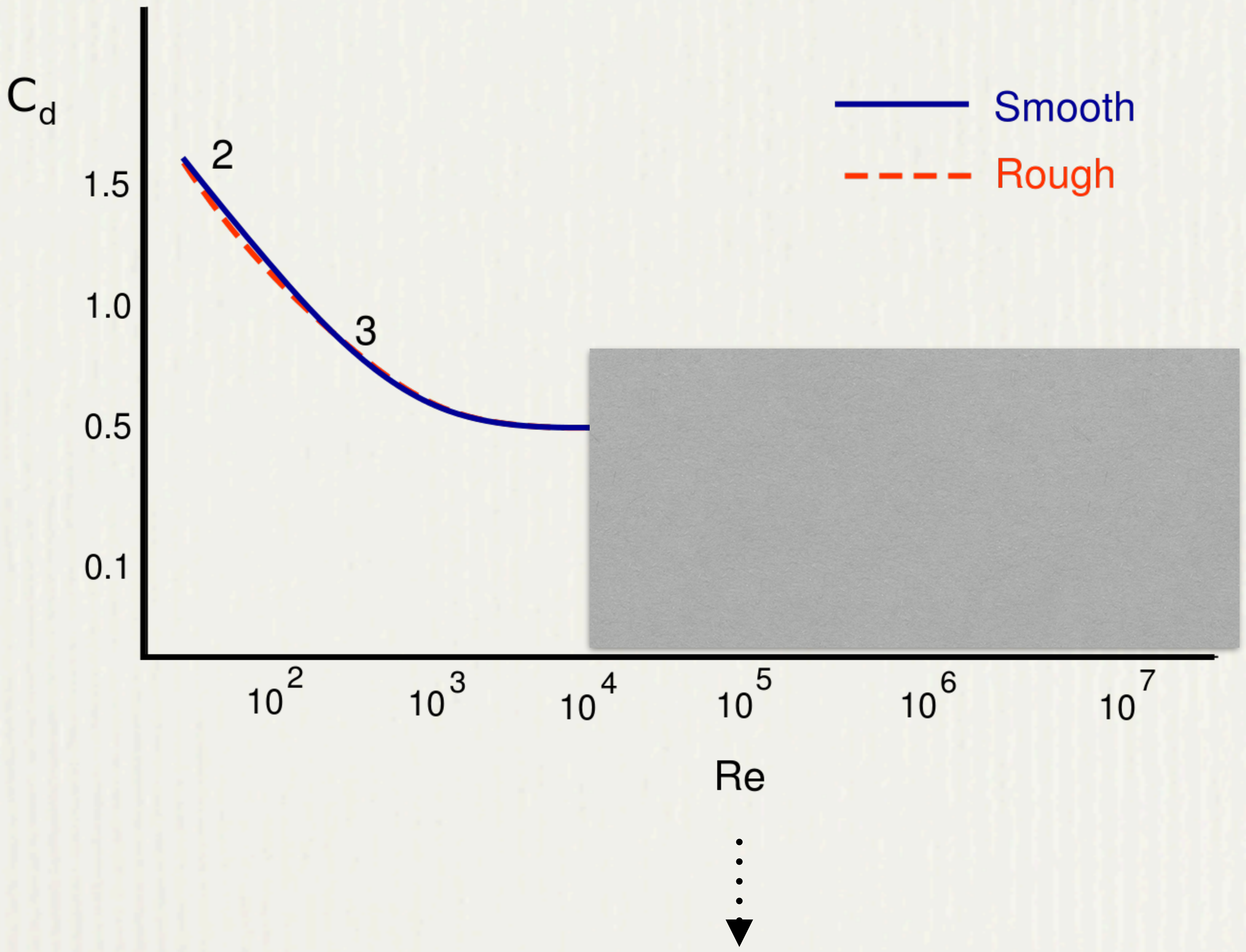
$$\pi_2 = \frac{\rho U D}{\mu} = Re$$

$$\frac{F_d}{\rho U^2 D^2} = F(Re) \boxed{\rightarrow Cst} \quad \cdots\cdots\blacktriangleright$$

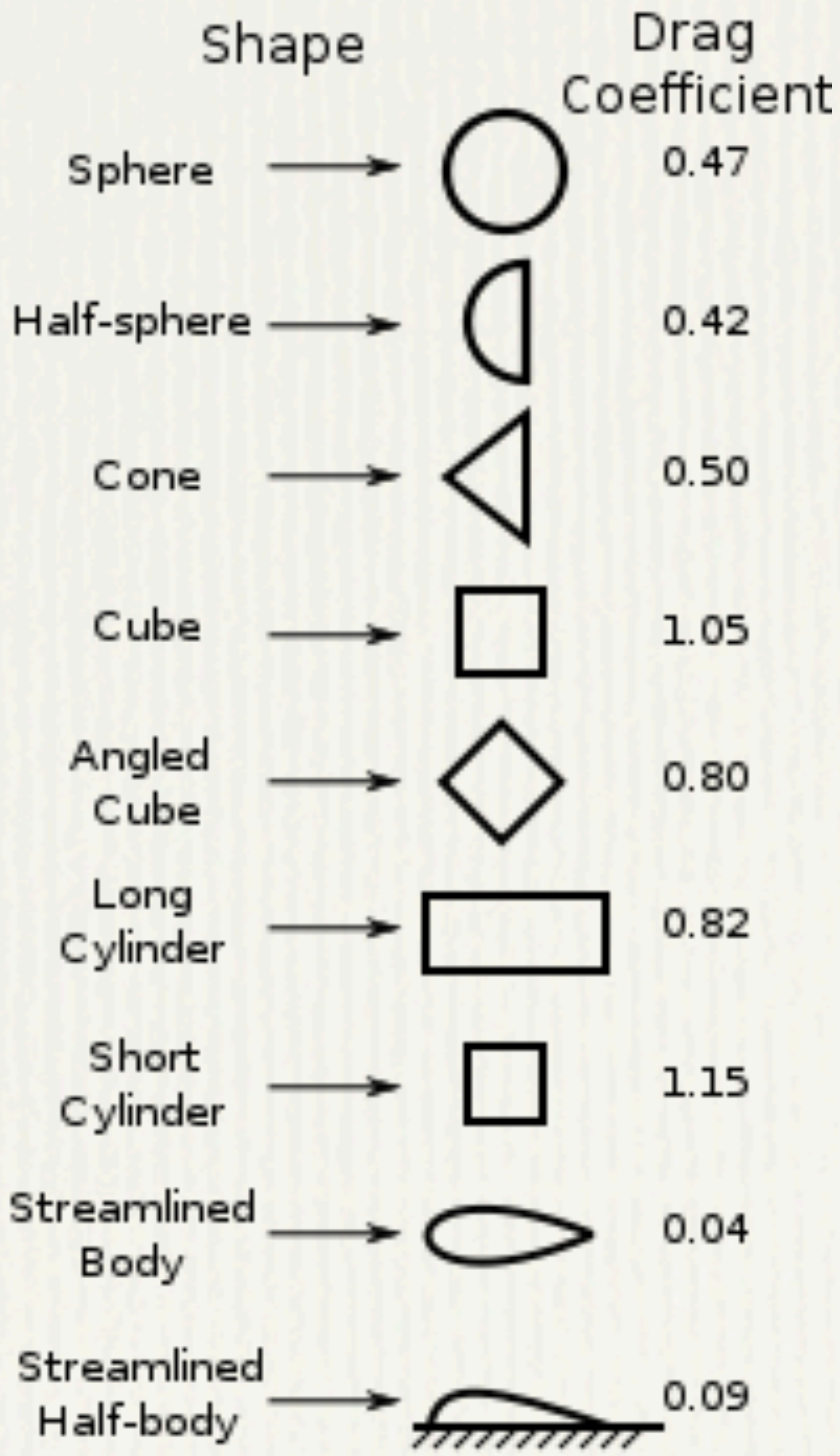
for Re>1000 (viscous forces negligible compared to inertial forces)

Buckingham π-theorem (Example 2)

$$\frac{2F_d}{\rho U^2 A} = C_d$$



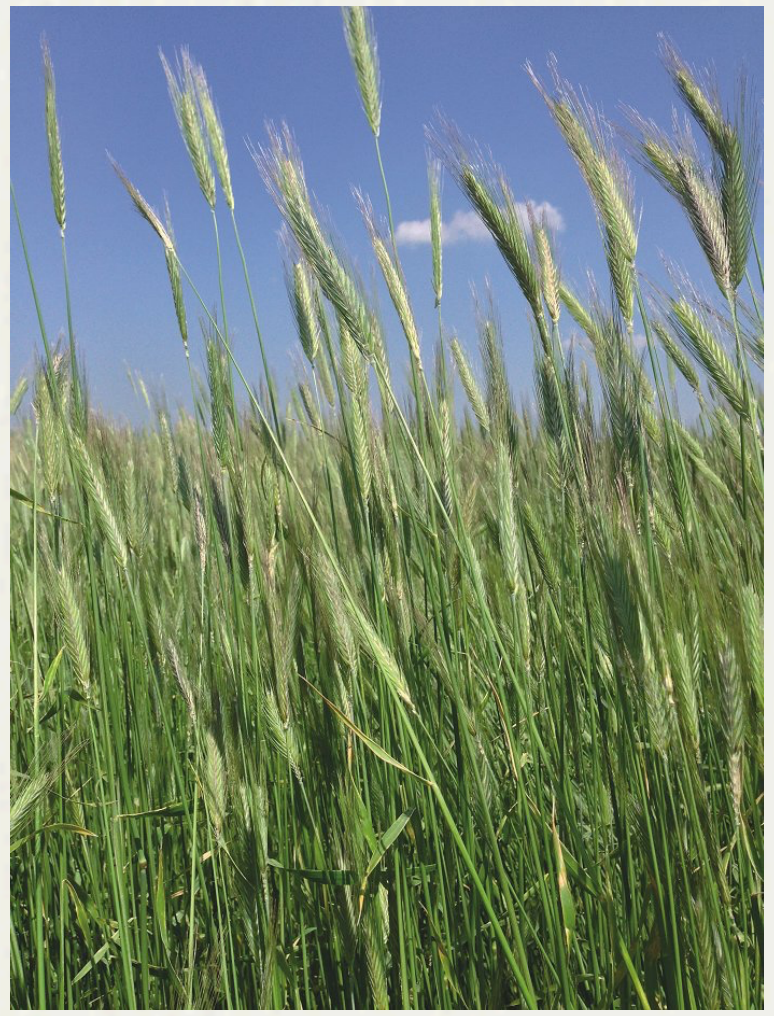
From lab experiments (sphere)

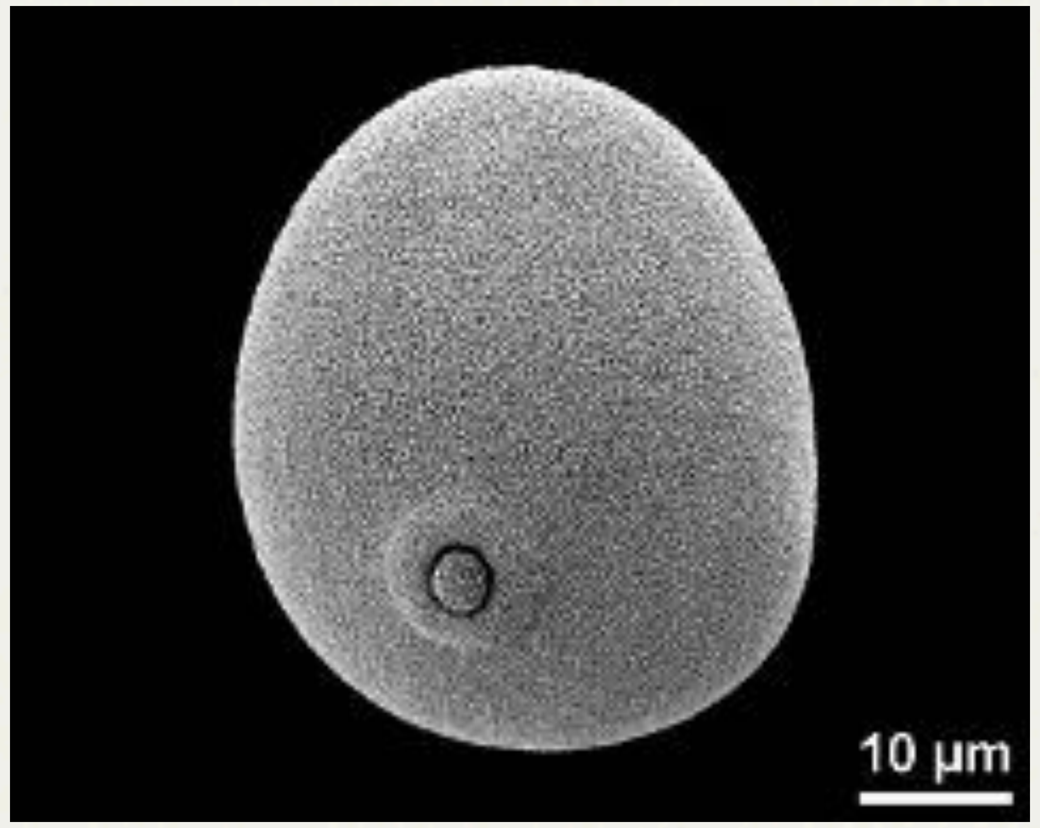


Measured Drag Coefficients

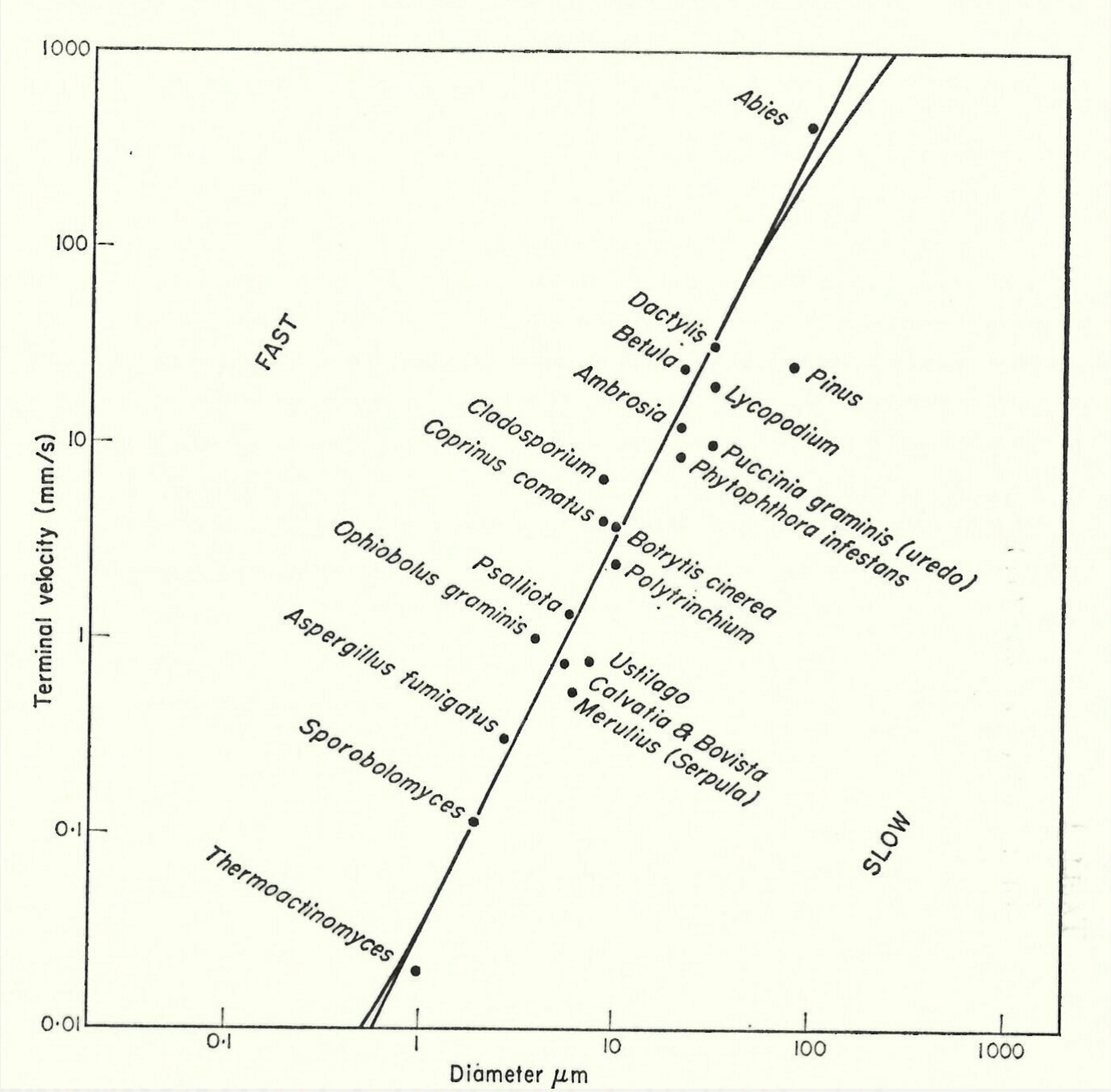
Buckingham π-theorem (Example 3)

$$V_{set} \propto \frac{gD^2\left(\rho_{pollen} - \rho_{air}\right)}{\mu}$$



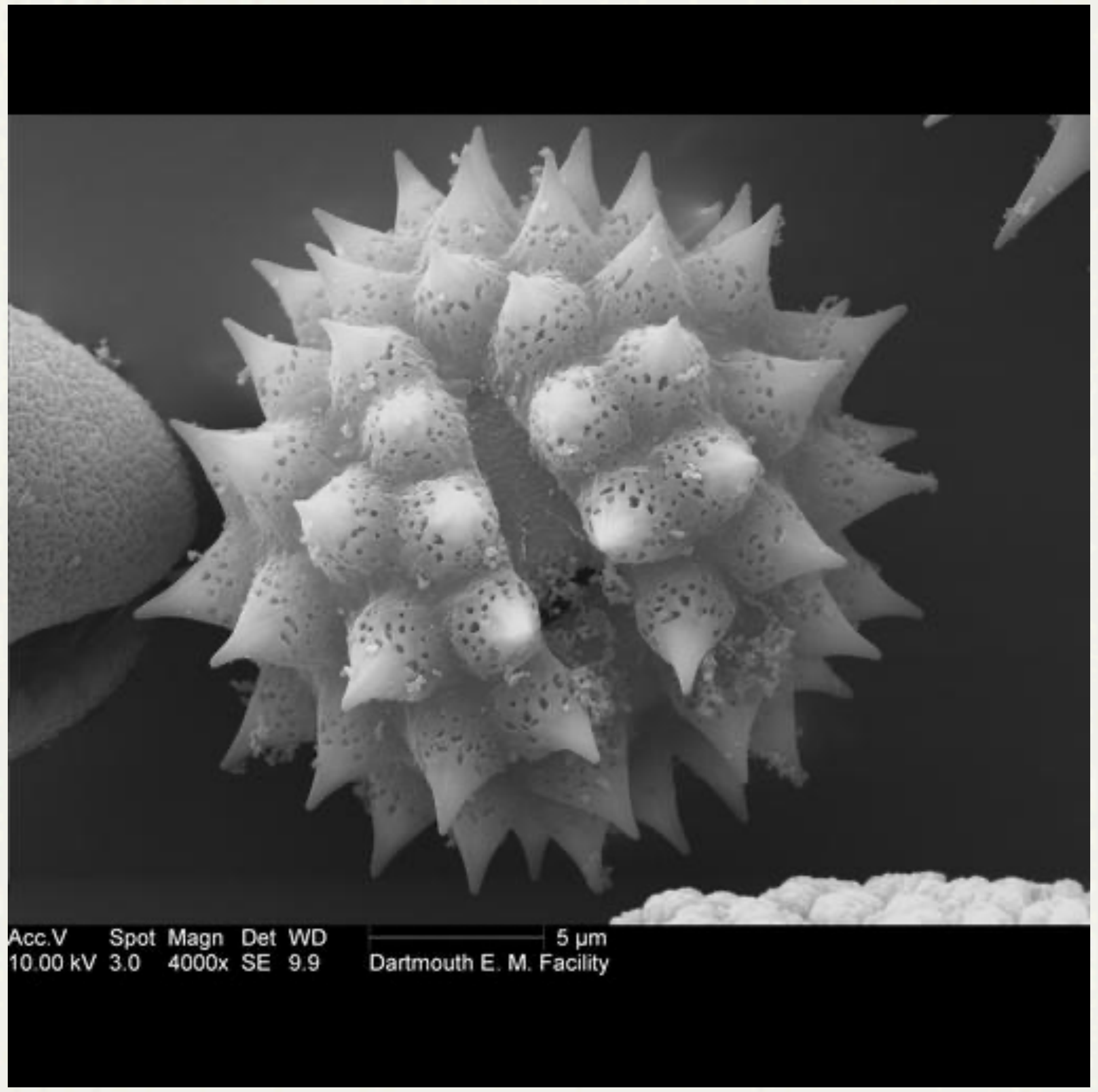


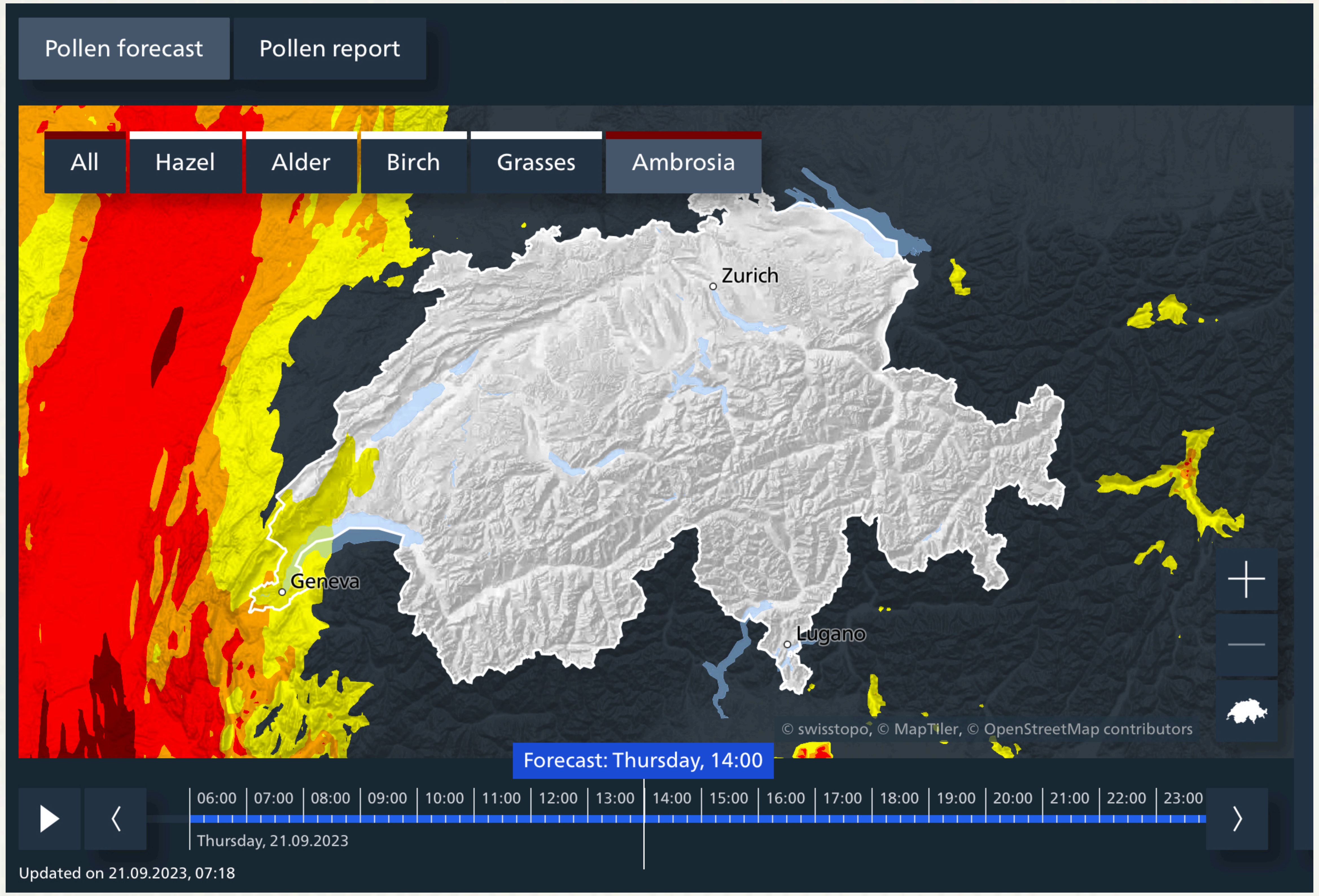
Rye pollen very allergenic but poor flyer due to large size



Estimation of **pollen settling velocity** (blackboard derivation)

Buckingham π-theorem (Example 3)





Use of geometric size for non-spherical pollen grains:
underestimation of settling velocity

Complete self similarity

Complete similarity occurs if the following condition is satisfied.

$$\lim_{\pi_{n-1} \to 0, \infty} F(\pi_1, \pi_2, \ldots, \pi_{n-1}) = Cst$$

Then π_{n-1} can be removed from the functional link the problem can be simplified.

In example 2 this is the case in the limit of **large Reynolds number.**

$$\pi_2 = \frac{\rho U D}{\mu} = Re$$

Incomplete self-similarity

In limits where

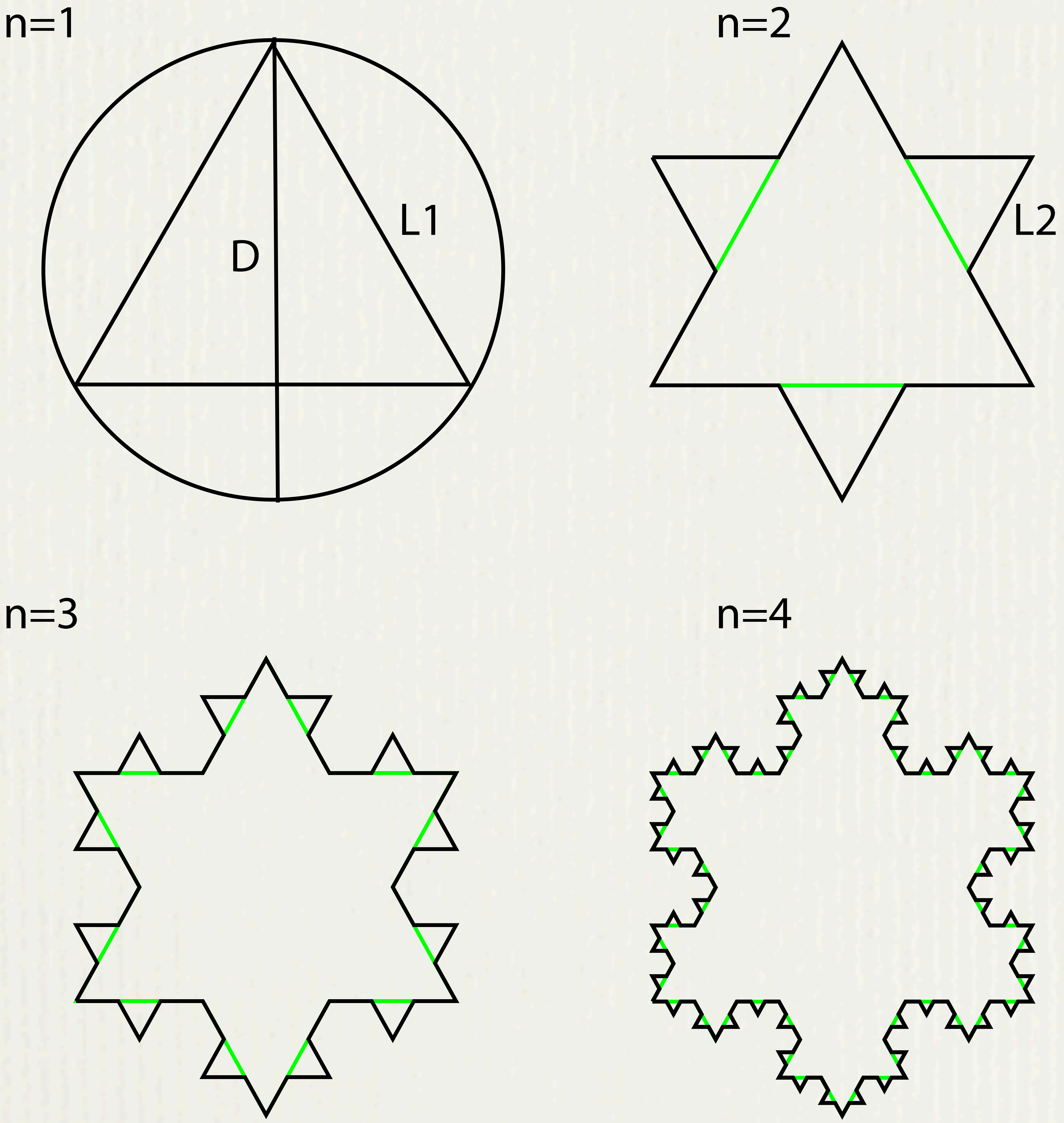
$$\lim_{\pi_{n-1}\to 0,\infty} F(\pi_1, \pi_2, \ldots, \pi_{n-1}) = 0, \infty$$

π_{n-1} cannot removed from the functional link but the problem **may still be simplified** for π_{n-1} >>1 or for π_{n-1} ~ 0 where the following approximations can be made

$$F(\pi_1, \pi_2, \ldots, \pi_{n-1}) \approx \pi_{n-1}^{\alpha} \tilde{F}(\pi_1, \pi_2, \ldots, \pi_{n-2})$$

Additional insight (theory, experiment) is needed to obtain the coefficient α. See following example Koch snowflake.

Incomplete self-similarity (example: Koch flake)



When n goes to infinity the resulting object has an infinite perimeter (dimension between 1 and 2, **fractal**).

$$\frac{P_n}{D} = \left(\frac{L_n}{D}\right)^{-\alpha} \cdot Cst \qquad \alpha > 0$$

Fractals: real-world relevance

Massstab 200 km
-> 2400 km

Massstab 100 km
-> 2800 km

Massstab 50 km
-> 3450 km

What is the length of the coastline of Britain ?

Alaska-Canada boundary dispute:

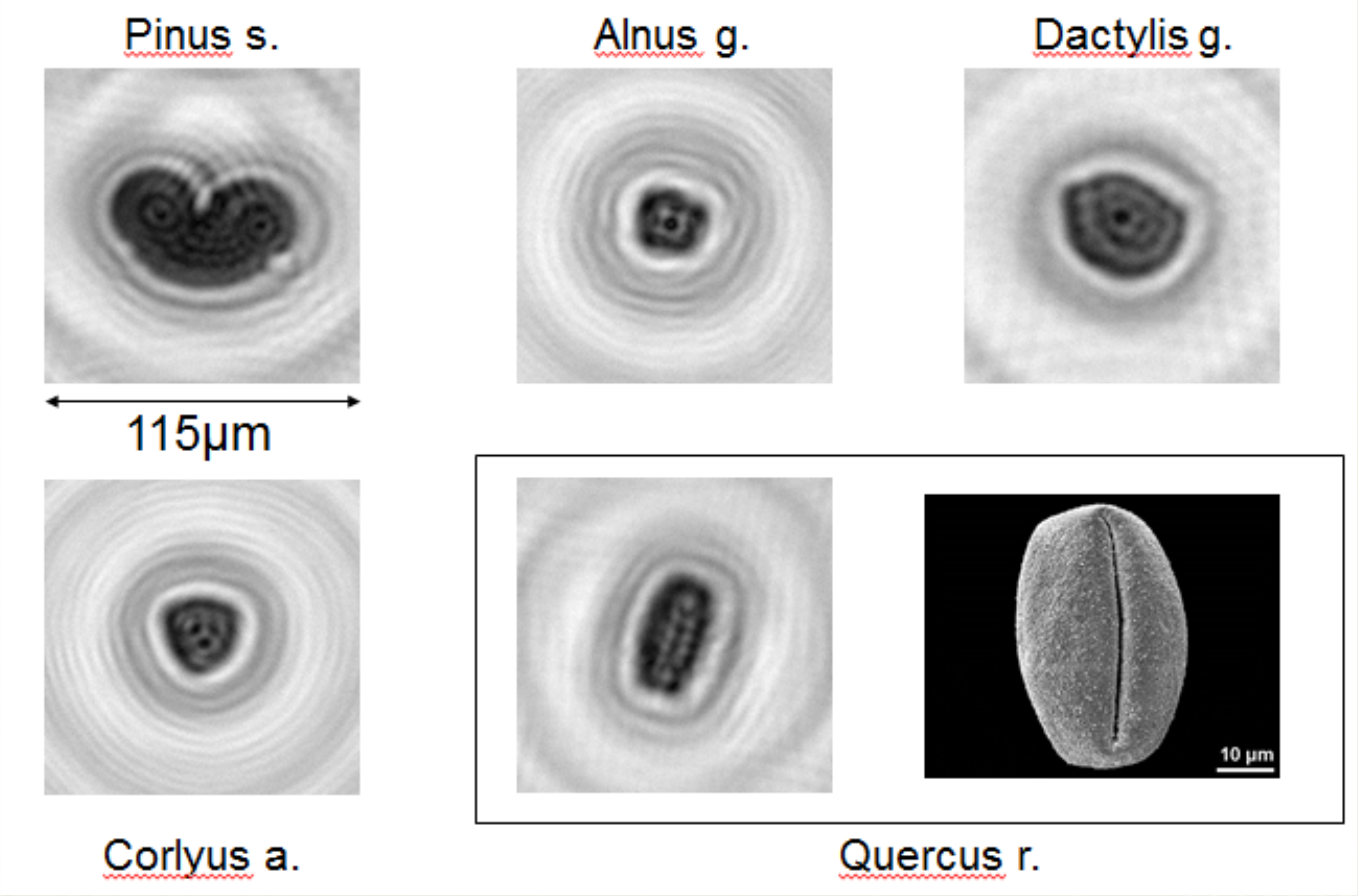
“the limit... shall be formed by a line parallel to the winding of the coast, and which shall never exceed the distance of ten marine leagues therefrom”

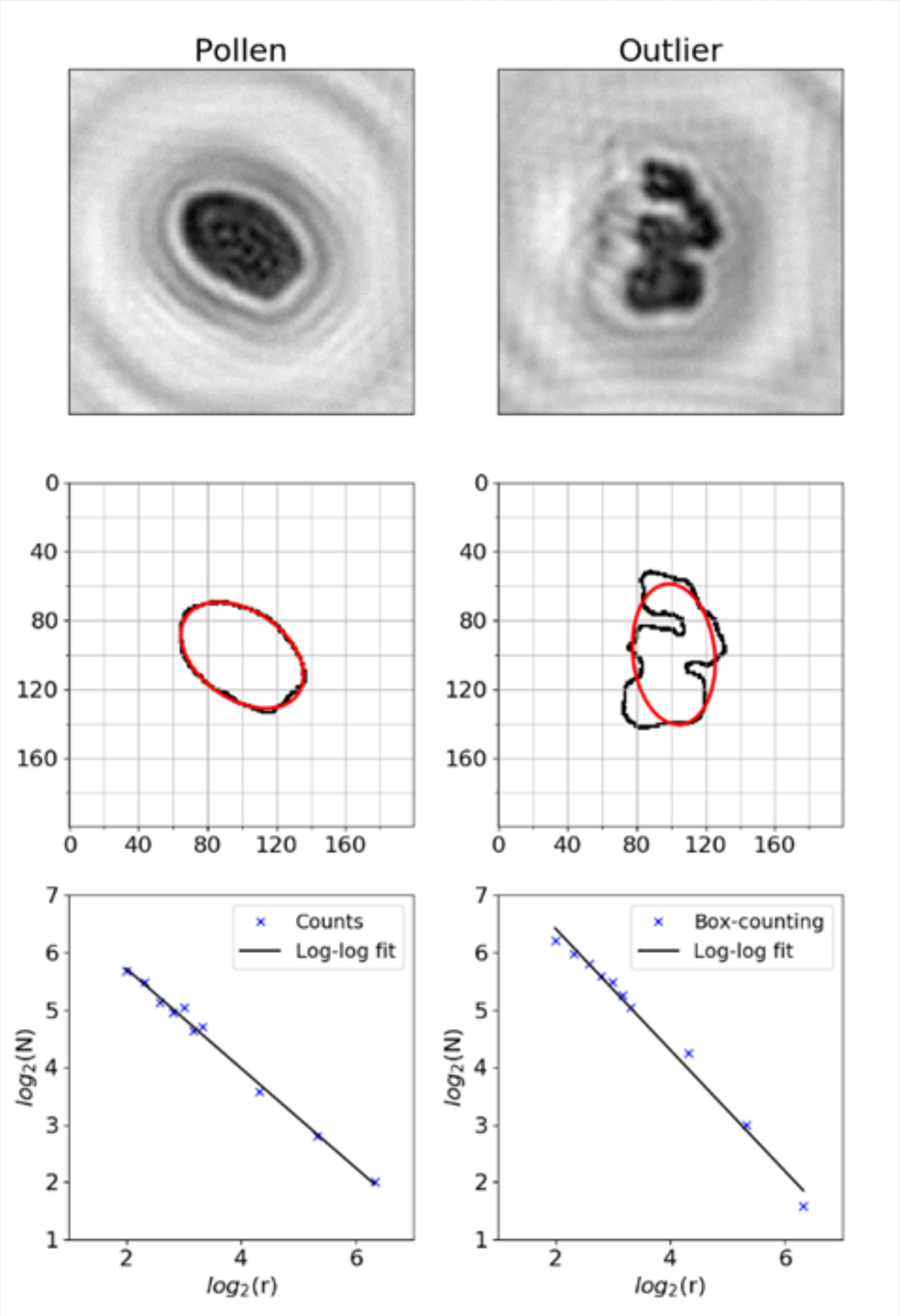
https://www.google.ch/maps/@57.2685656,-135.2953876,482068m/data=!3m1!1e3

Fractals: identification of bioaerosols

Digital holography used for on-line pollen monitoring

Fractal dimension allows to distinguish coarse particulate matter from bioaerosols

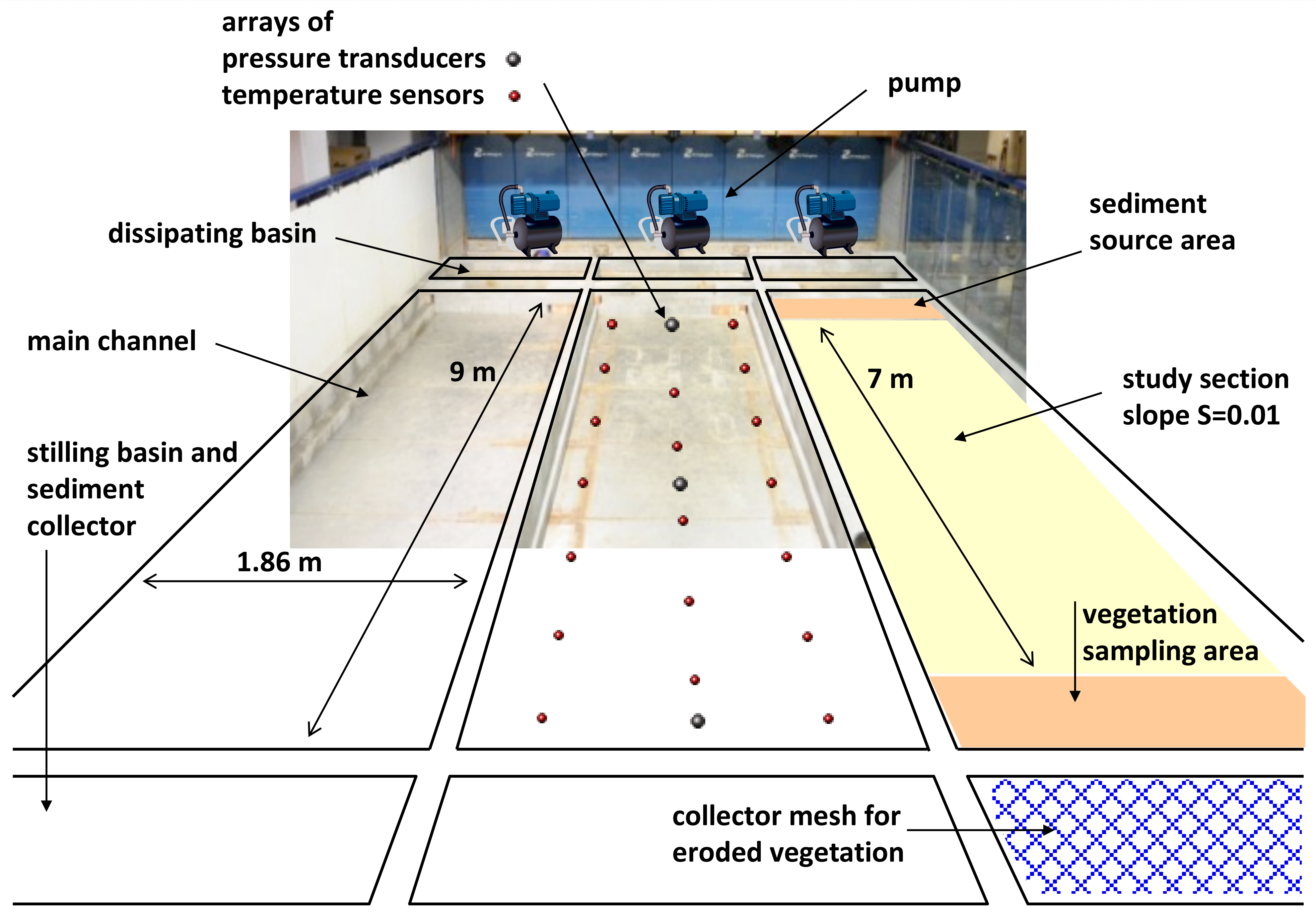


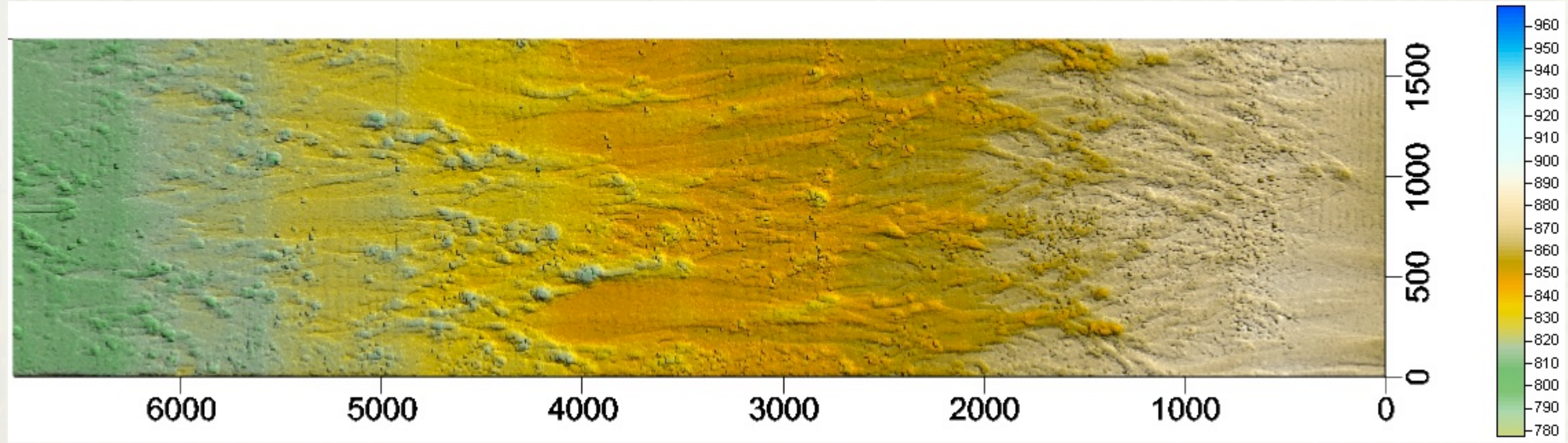


https://amt.copernicus.org/articles/13/1539/2020/

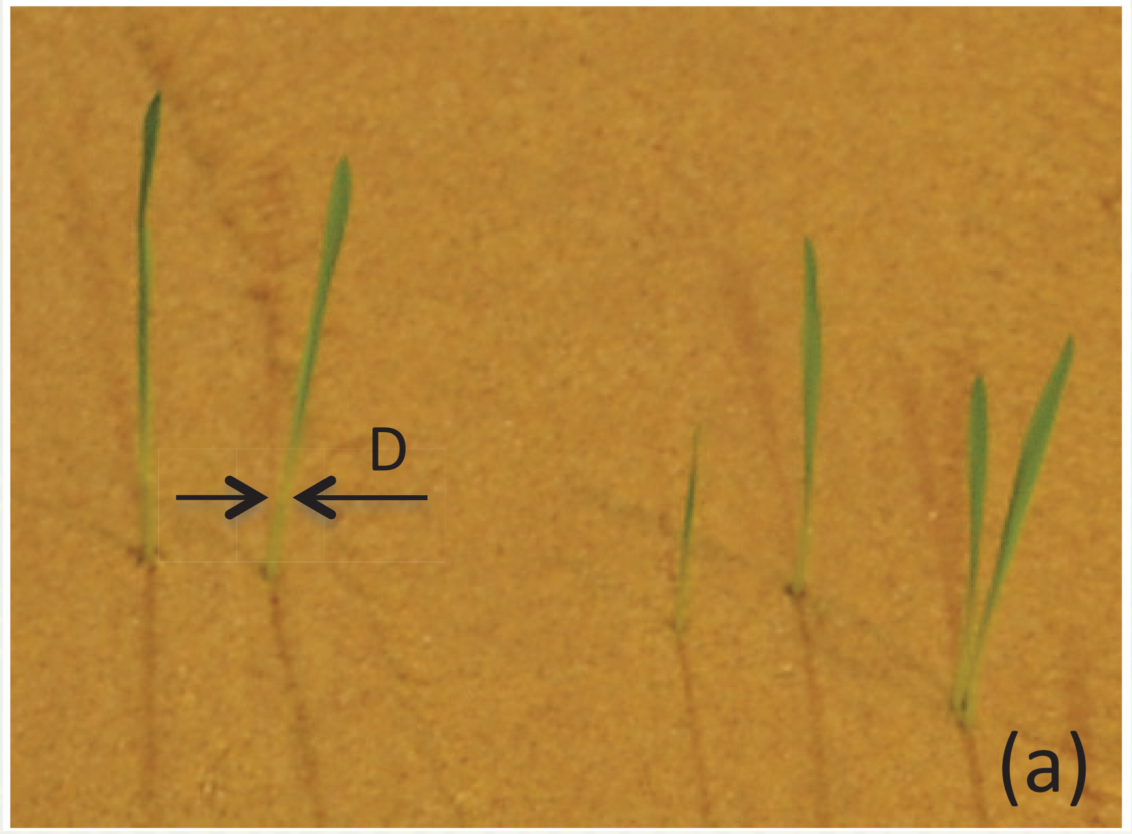
Model Theory (environmental fluid mechanics)

- How to make a model (e.g. laboratory) of a real-world system ?





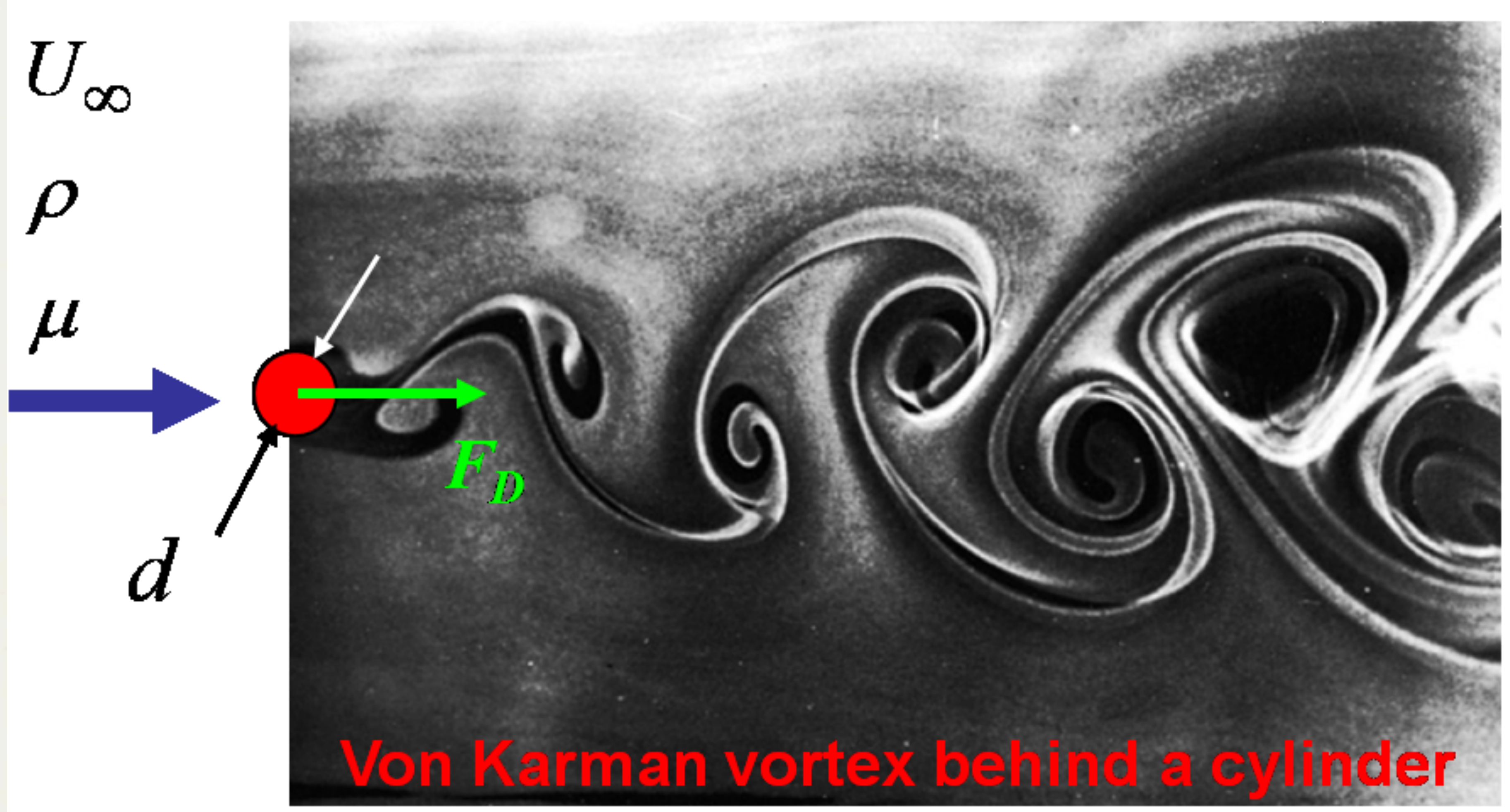
laboratory



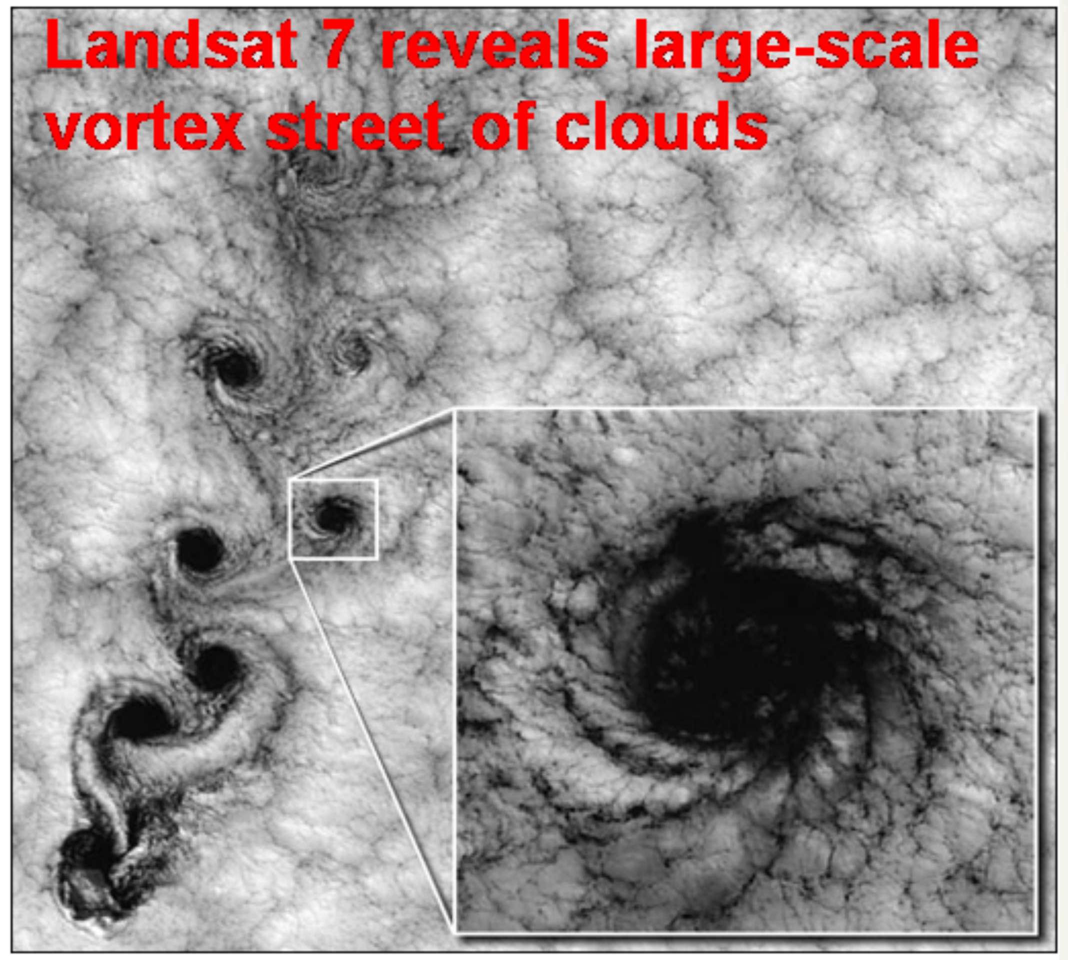
real-world

Model Theory (environmental fluid mechanics)

- Good model has comparable behavior to original
- Physical behavior governed by physical equations
- π theorem states that physical equations can be rewritten in dimensionless form.
- Model and original are **similar** if they have the same dimensionless numbers.



Von Karman vortex behind a cylinder



Landsat 7 reveals large-scale vortex street of clouds

Model Theory (environmental fluid mechanics)

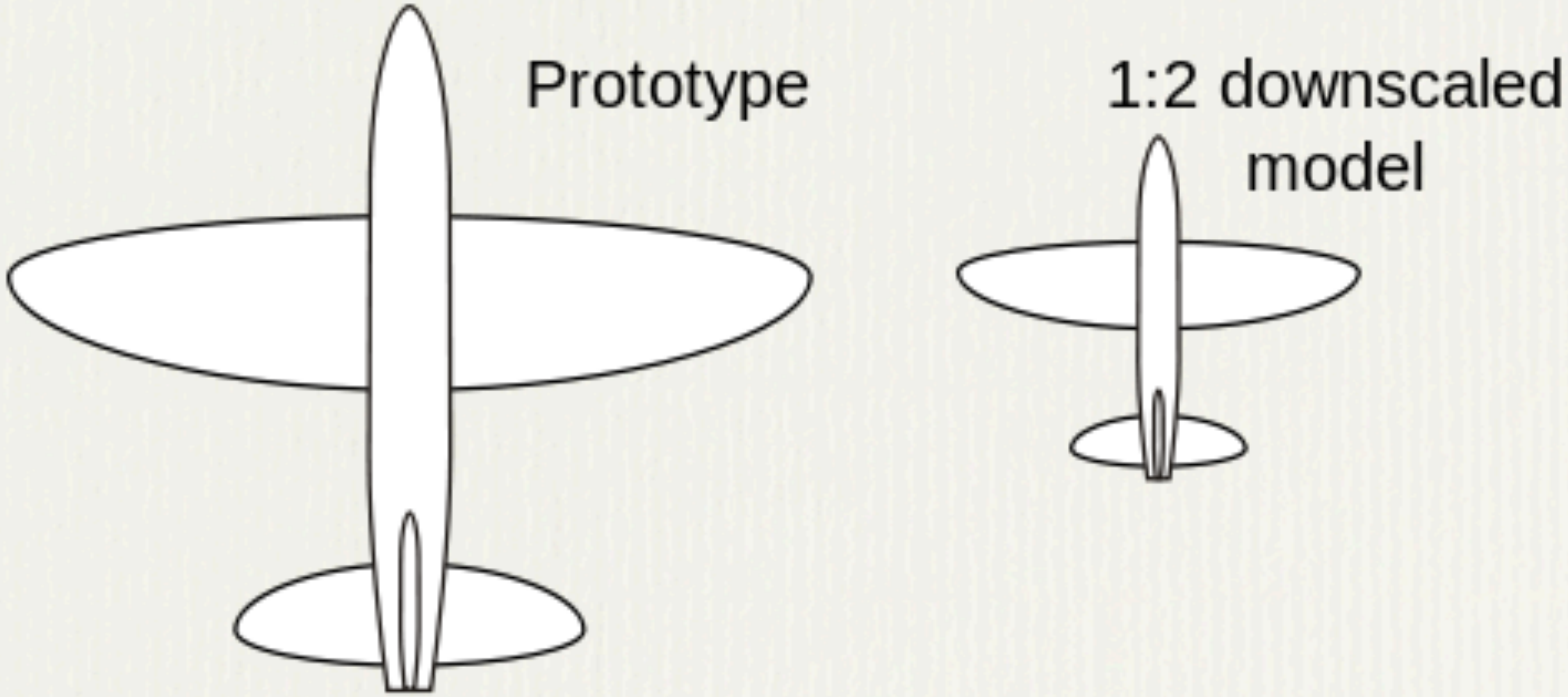
In environmental fluid mechanics we distinguish between

- **complete** similarity
- **incomplete** similarity: geometric (identical shapes), kinematic (ratio between lengths and times are identical), dynamic (ratio of all forces identical, e.g. Reynolds -> inertial vs. viscous or Froude -> inertial vs. gravitational, numbers are the same)

Reynolds similarity often **difficult to achieve**: if the velocity is fixed by the Froude similarity then viscosity has to be adjusted.

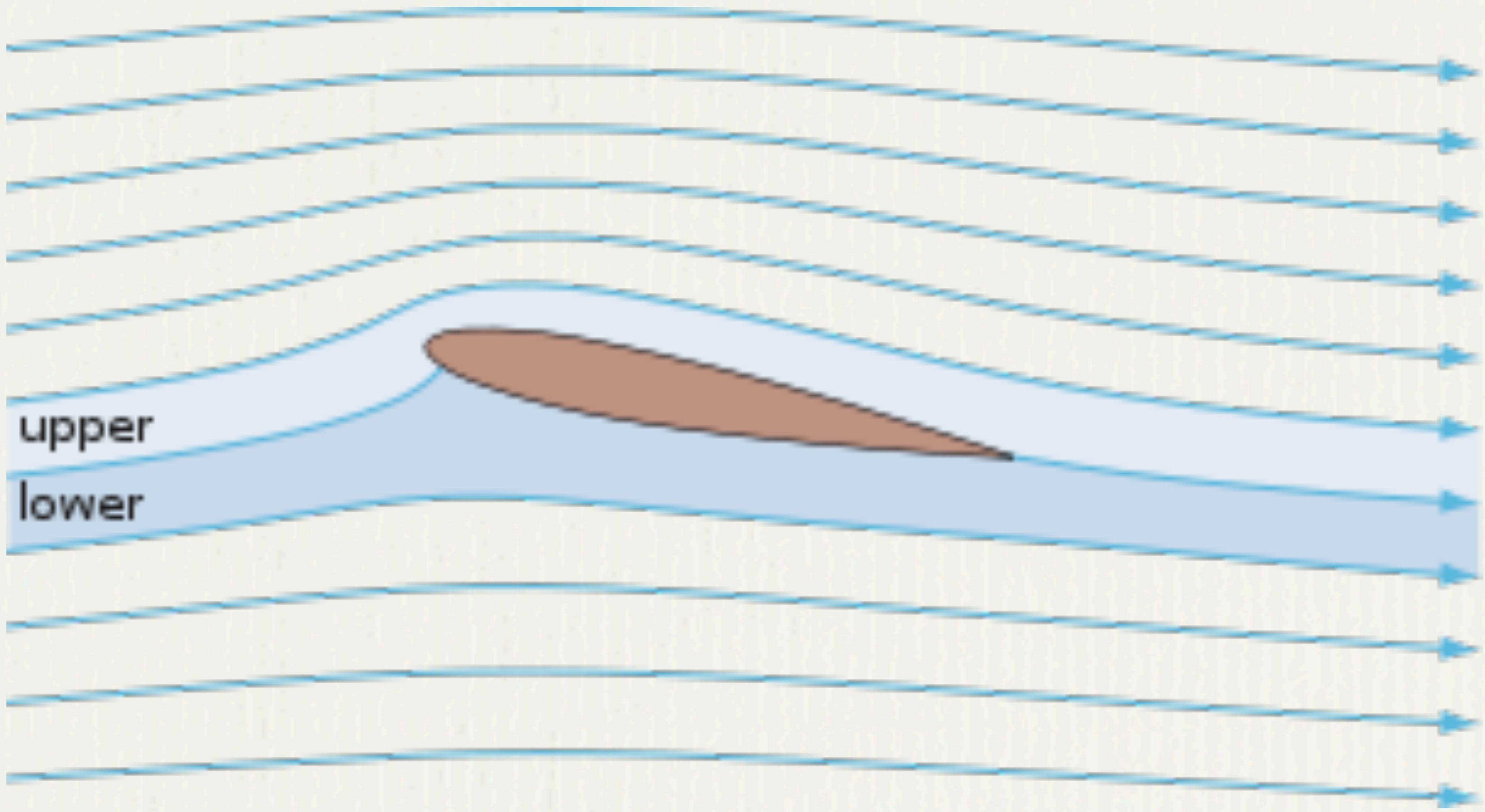
- as long as the flow is fully turbulent (viscosity forces negligible) this similarity is often not necessary

Model Theory: geometric similarity



Ratios between corresponding **lengths** in model and application (or prototype) are the same.

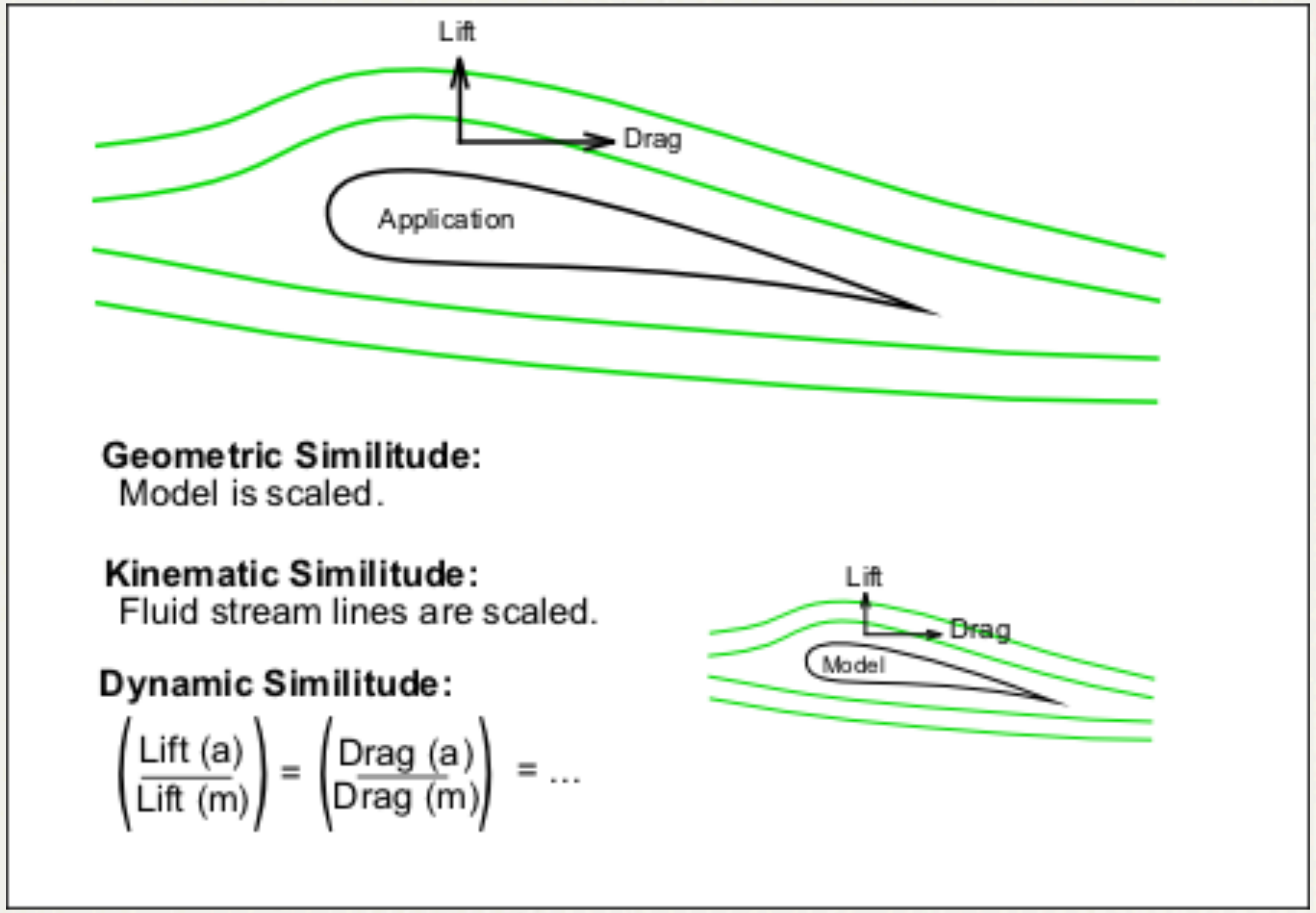
Model Theory: kinematic similarity



Flow field in prototype and model have the same shape and the ratios of corresponding velocities and accelerations are the same.

Geometrically similar streamlines are kinematically similar.

Model Theory: dynamic similarity



Geometric Similitude:
Model is scaled.

Kinematic Similitude:
Fluid stream lines are scaled.

Dynamic Similitude:

$$\left(\frac{\text{Lift (a)}}{\text{Lift (m)}}\right) = \left(\frac{\text{Drag (a)}}{\text{Drag (m)}}\right) = \ldots$$

Ratio between forces in application and model must be constant.

Geometric and kinematic similarity necessary but insufficient conditions

Summary of Lecture I: Uses of dimensional analysis

- Recognise (self)-**similarity** -> model theory

- **Reduce** the number of variables

- Variables known, physics unknown (equations, boundary conditions): **simple approach** to tackle complex problems

- Interpretation of experimental data: from the dimensionless variables identify the **relative importance** of physical phenomena

Summary of Lecture I: **common pitfalls**

- Review on **units and dimensions**: check your equations for consistency

- Beware of **incomplete** set of independent quantities

- **Superfluous** independent quantities complicate the result (be pragmatic !)

- Functional dependence can be more or less difficult to recognise depending on the choice of dimensionless variables

II) Fluid Mechanics

Fluid mechanics: review

- Basis for transport phenomena

- Velocity field specified by

$$\mathbf{V}(x,y,z,t) = u(x,y,z,t)\mathbf{e}_x + v(x,y,z,t)\mathbf{e}_y + w(x,y,z,t)\mathbf{e}_z$$

- **State of the fluid** specified by the velocity components u, v, w, pressure p and density ρ.

- 5 equations (and appropriate **initial** and **boundary** conditions) are needed to describe the evolution of the fluid

- In the presence of additional substances, their **concentration** C(x,y,z,t) has to be added. Correspondingly, an additional equation is needed.

Fluid mechanics: review

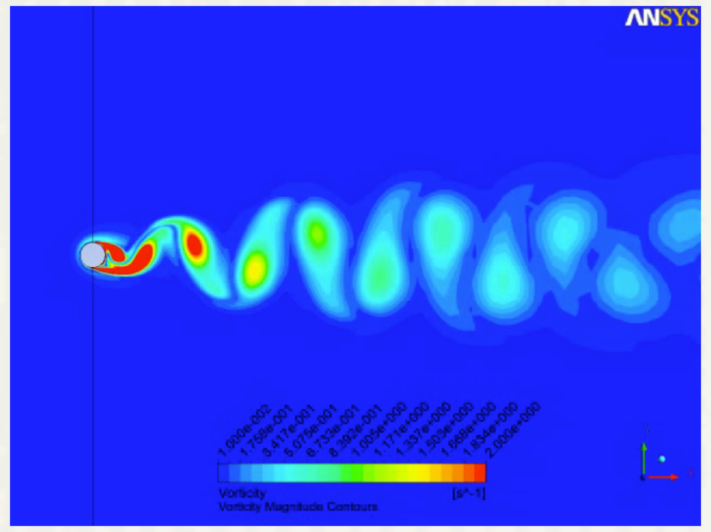
- Regardless of the form of the fluid mechanics physical equations, dimensional analysis can be applied and relevant dimensionless quantities can be found.

- Velocity U, viscosity μ, density ρ, characteristic length L, gravitational acceleration g, time t, pressure p and diffusion coefficient D (incompressible fluid).

$$Re = \frac{\rho L U}{\mu} \qquad Fr = \frac{U}{\sqrt{gL}} \qquad Pe = \frac{UL}{D}$$

$$Ne = \frac{p}{\rho U^2} \qquad St = \frac{fL}{U}$$

inverse characteristic time e.g.
vortex shedding frequency



$$\dots$$

Fluid mechanics: review

- **Newtonian** fluid: shear stress $\tau = \mu \dfrac{du}{dy}$, with u velocity component parallel to the direction of shear and y displacement in perpendicular direction.

- **Navier-Stokes equation** for incompressible flow of Newtonian fluid reads (momentum conservation)

inertia

divergence of stress

$$\rho \left(\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla)\mathbf{v} \right) = -\nabla p + \mu \nabla^2 \mathbf{v} + \mathbf{f}$$

variation

convection

source

Left hand side material derivative $\rho \dfrac{D\mathbf{v}}{Dt}$

With $\dfrac{D}{Dt} \equiv \dfrac{\partial}{\partial t} + \mathbf{v} \cdot \nabla$ (local+convective term)

Fluid mechanics: review

- Continuity equation (mass conservation) $\frac{\partial \rho}{\partial t} + \nabla \cdot \rho \mathbf{v} = 0$

incompressible flow $\boxed{\nabla \cdot \mathbf{v} = 0}$ Solenoidal flow (e.g. magnetic field)

- Compressible flow p and ρ are linked (state equation energy conservation)

$$\frac{D\rho}{Dt} = \frac{\rho}{\epsilon}\frac{Dp}{Dt} \qquad Ma = U\sqrt{\frac{\rho}{\epsilon}}$$

Mach number

- Bernoulli equation along a streamline (curve tangent to velocity field), valid for steady, incompressible flow of perfect fluid (no viscosity).

$$\frac{1}{2}v^2 + \frac{P}{\rho} + gh = Cste.$$

Fluid mechanics: review

- Navier-Stokes equation can be rewritten in dimensionless form

$$\mathbf{f} = \rho g \mathbf{e}_z$$

$$\left(\frac{\partial \mathbf{v}^\star}{\partial t^\star} + (\mathbf{v}^\star \cdot \nabla)\mathbf{v}^\star \right) = -\nabla p^\star + \frac{1}{Re}\nabla^2 \mathbf{v}^\star + \frac{1}{Fr^2}\mathbf{e}_z^\star$$

- with the **scales**

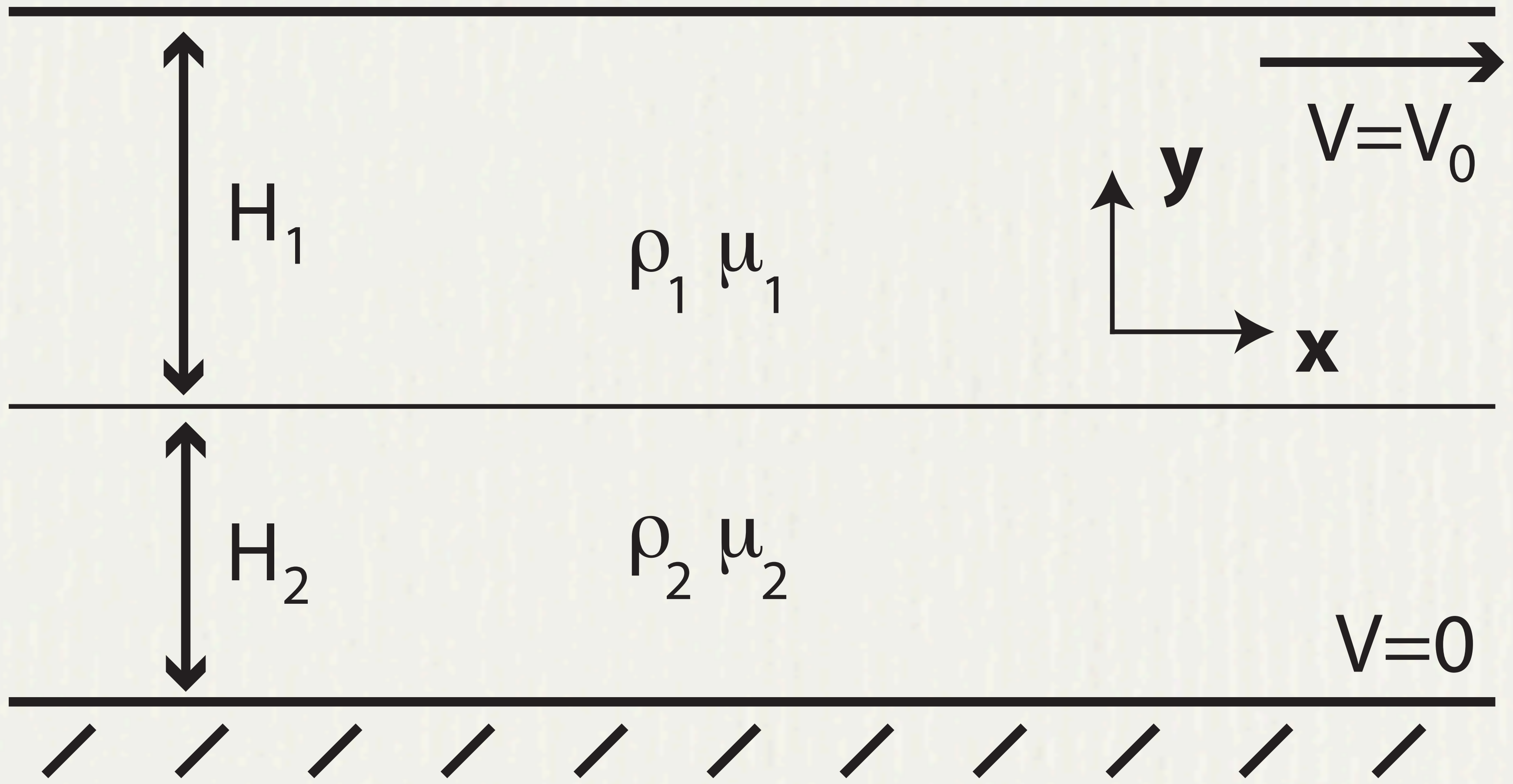
characteristic length L time $\frac{L}{U}$

flow velocity U $p^\star = \frac{p}{\rho U^2}$

- Stokes regime (Re<<1), Euler regime (Re>>1, inertia dominates)

Fluid mechanics: example

- Example (exercise set): laminar motion of two layers of Newtonian fluids (stationary process or steady flow)



$$\mathbf{v} = u(y)\mathbf{e}_x$$

$$\frac{\partial^2 u}{\partial y^2} = 0$$

- Boundary conditions: no-slip condition (zero velocity relative to boundary and equality of stresses + velocities at the boundary)

$$\begin{aligned} u(0) &= 0 \\ u(H_1 + H_2) &= V_0 \\ \mu_1 \frac{du}{dy}|_{y=H_2+\epsilon} &= \mu_2 \frac{du}{dy}|_{y=H_2-\epsilon} \\ u(H_2+\epsilon) &= u(H_2-\epsilon) \end{aligned}$$

- No initial condition (boundary condition in time) due to stationarity

Fluid mechanics: turbulent regime