

# Surface-based Molecular Design with Multi-modal Flow Matching

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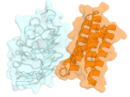
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**Speakers: Tian Zhu & Adrian Dobbelstein**

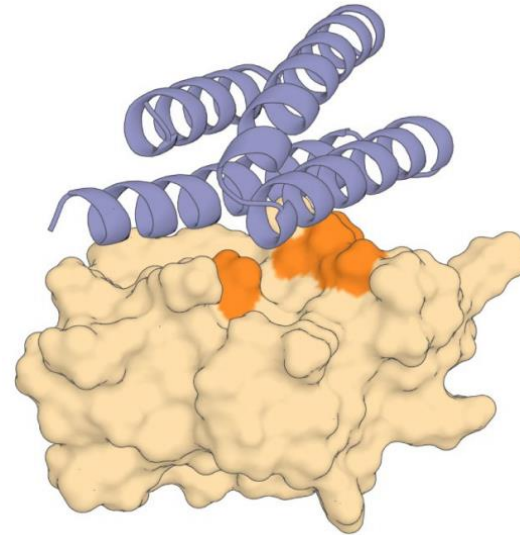
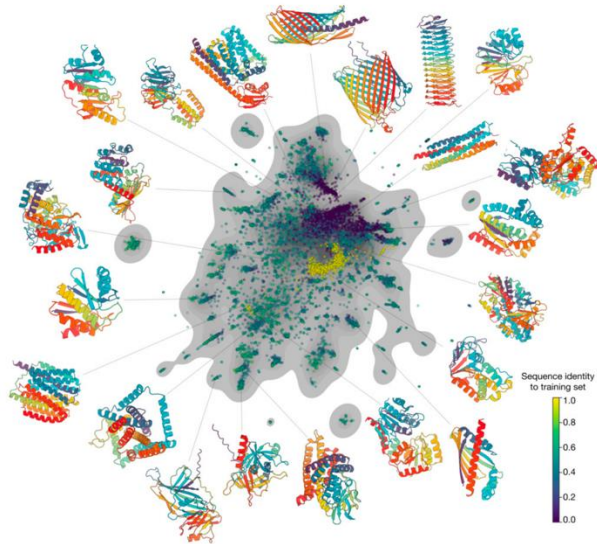
*EE-262 Graph representations for biology and medicine*

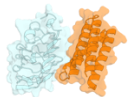
Supervisors: Bruno Correia & Michael Bronstein

# Background: Protein



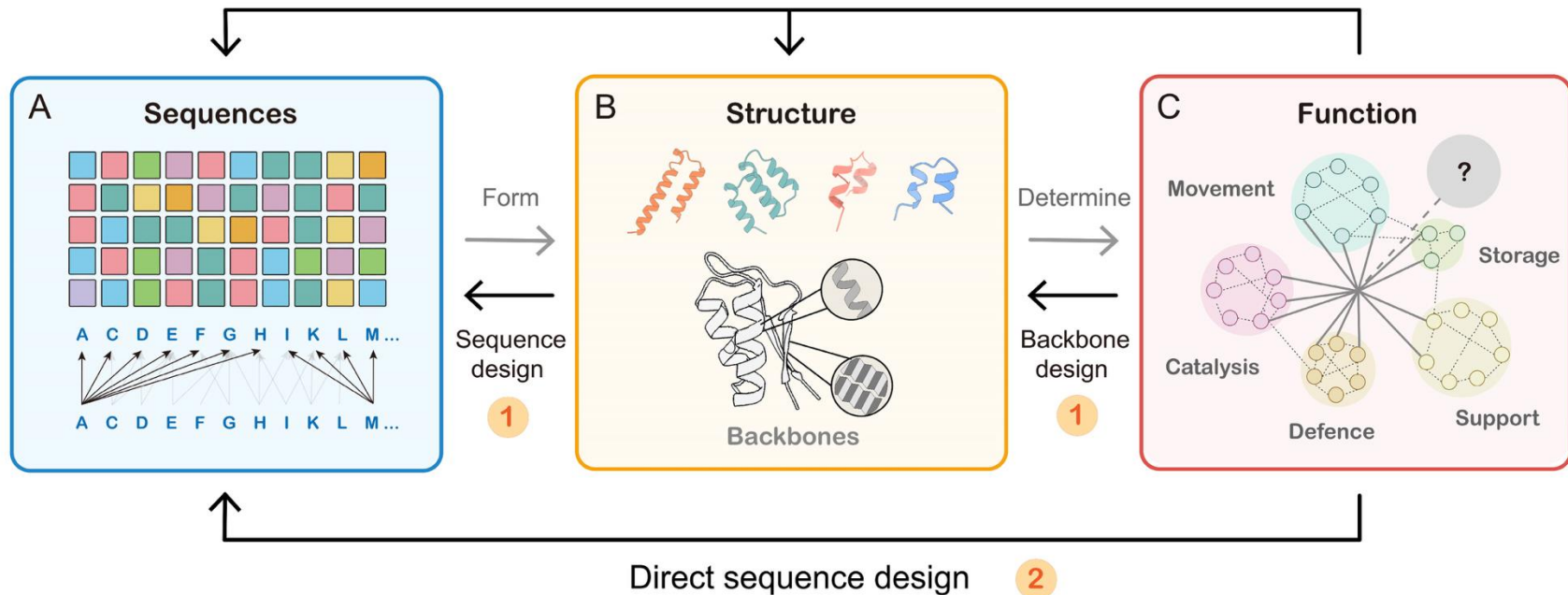
- As the central executioners of genetic information, **proteins govern complex cellular functions**, including molecular recognition, signal transduction, and immune defense.

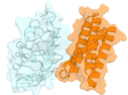




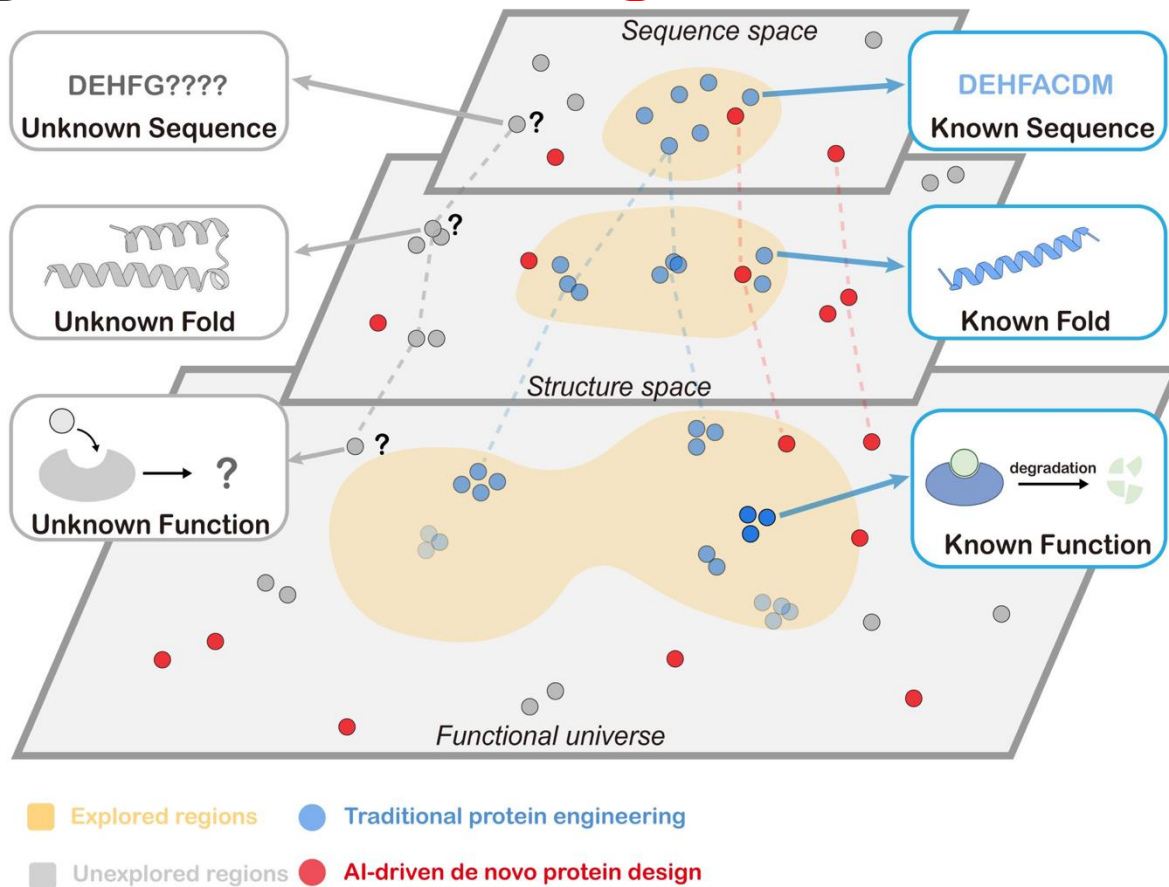
# Protein Anfinsen's dogma

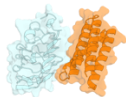
Sequence–structure co-design **3**





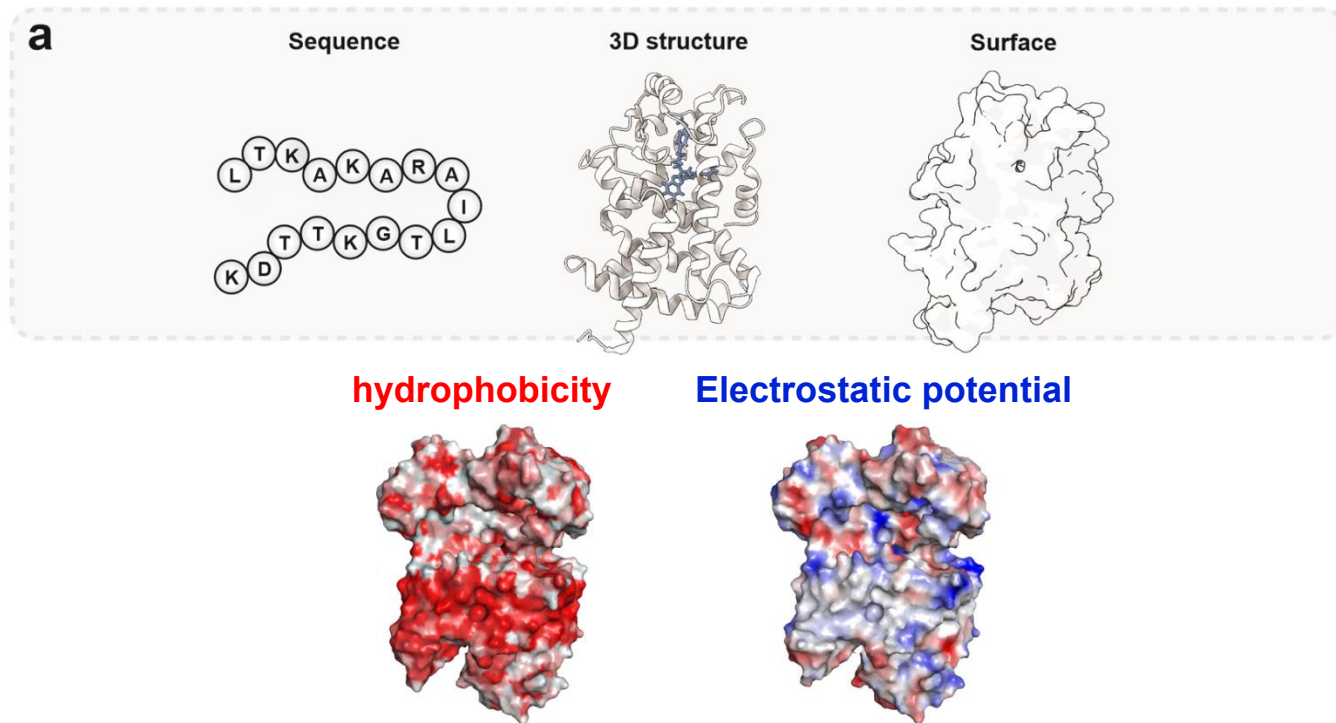
# Background: *de novo* Design



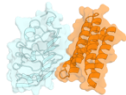


# Background: Protein Surface

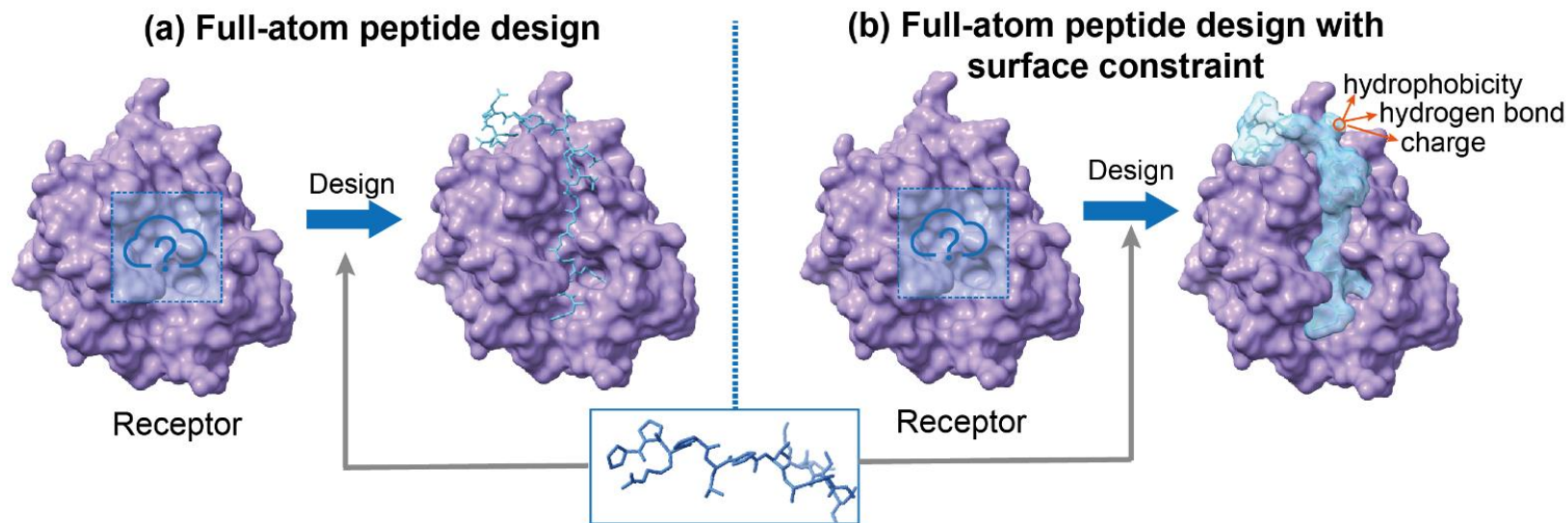
- Beyond primary sequence and 3D topology, the **molecular surface** encodes critical physicochemical properties that dictate binding behaviour.



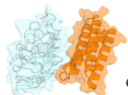
# Task: Peptide Design



- **Goal:** Generate a full-atom peptide binder that complements the surface properties of a given target protein receptor.
- **Key Innovation:** Unlike standard design (a), the approach (b) incorporates **surface representations**—hydrophobicity, hydrogen bonding, and charge—to ensure optimal binding.
- **Definition:** Peptide refers to a short oligomer of fewer than 20 amino acids.



# Task: Motivation

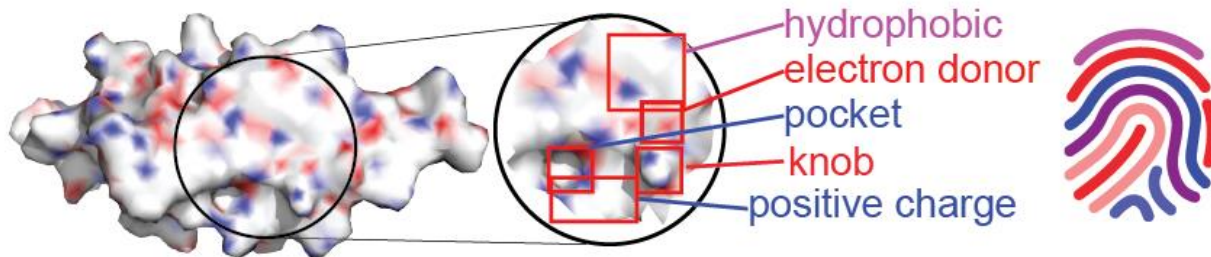


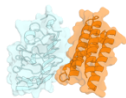
- Protein-protein Interactions (PPIs) are largely dictated by **how complementarily the surfaces of interacting proteins fit together**.
- The surface's electrostatic potential and hydrophobicity are key determinants of PPIs' strength and specificity.
- Surfaces geometries, such as protrusions, grooves, and clefts enable lock-and-key or induced-fit mechanisms essential for specific binding.

 **MaSIF**

Protein molecular surface

Interaction fingerprint





# Multimodal Space for Protein

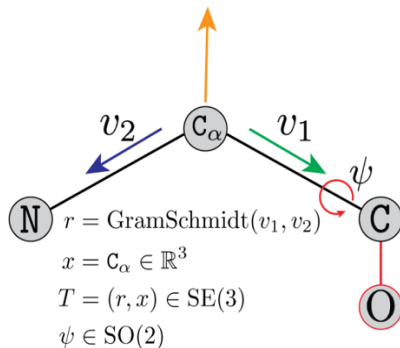
## Sequence

AGVASKS

 $a_i$ 

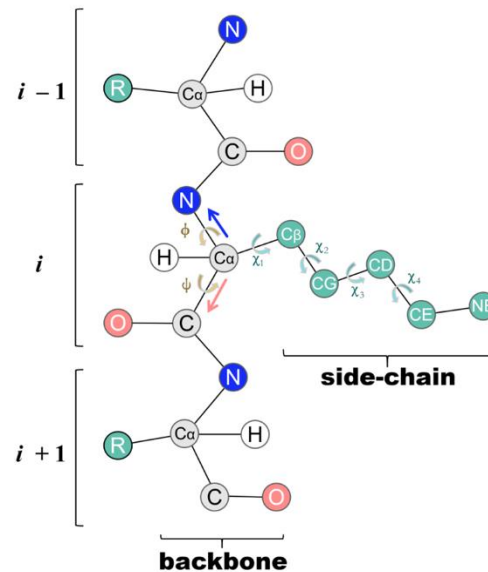
Discrete words  
(Discrete/Simplex Space)

## Backbone


 $x_i + O_i$ 

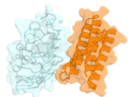
Translation + Rotation  
(SE(3) Space)

## Side-chain

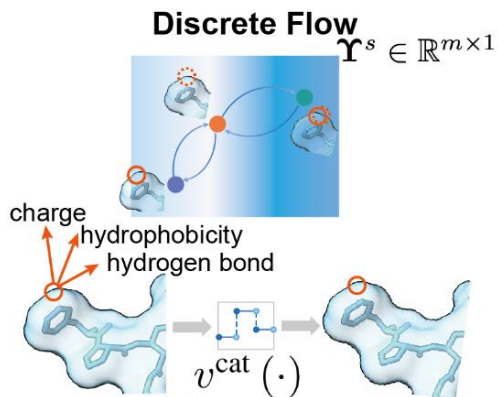

 $\chi_i$ 

Torsion angles  
( $T^4$  Space)

# Multimodal Space for Surface

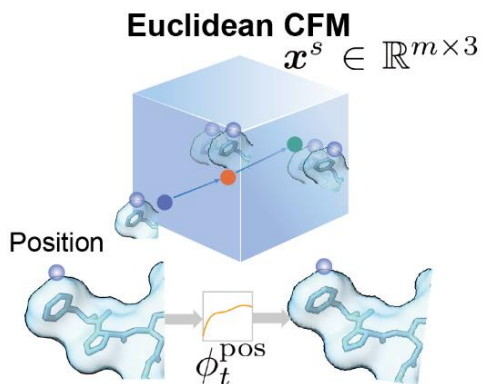


## Physicochemical Properties



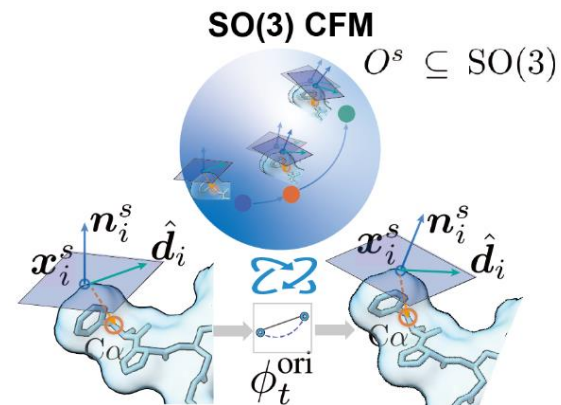
$$\Upsilon_i^s, \tau_i^s$$

## Points

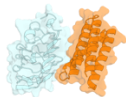


$$\mathbf{x}_i^s$$

## Normals



$$\mathbf{n}_i^s$$



**Goal:** This work designs peptides based on target proteins, so we aim to model the **conditional joint distribution**

$$p(C^{\text{pep}} \mid C^{\text{rec}})$$

where

$$C^{\text{rec}} = \{(a_i, O_i, \mathbf{x}_i, \chi_i)\}_{i=1}^{n_{\text{rec}}}$$

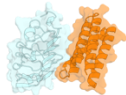
$$C^{\text{pep}} = \left\{ (a_j, O_j, \mathbf{x}_j, \chi_j) \right\}_{j=1}^{n_{\text{pep}}} \cup \left\{ (\mathbf{x}_i^s, \mathbf{n}_i^s, \tau_i^s, \gamma_i^s) \right\}_{i=1}^m \quad m \gg n_{\text{pep}}$$

**Structure  
representation**

$a_i \in \{1, \dots, 20\}$  denotes the residue type  
 $\mathbf{x}_i \in \mathbb{R}^3$  and  $O_i \in \text{SO}(3)$  represents the backbone frame  
 $\chi_i \in [0, 2\pi)^4$  denotes the sidechain torsion angles

**Surface  
representation**

$\mathbf{x}_i^s$  is the coordinate of the surface point cloud  
 $\mathbf{n}_i^s$  is the unit normal vector of the surface point cloud  
 $\tau_i^s$  and  $\gamma_i^s$  capture its continuous and categorical physicochemical properties



**Goal:** This work designs peptides based on target proteins, so we aim to model the **conditional joint distribution**

$$p(C^{\text{pep}} | C^{\text{rec}})$$

where

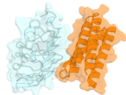
$$C^{\text{rec}} = \{(a_i, O_i, \mathbf{x}_i, \boldsymbol{\chi}_i)\}_{i=1}^{n_{\text{rec}}}$$

$$C^{\text{pep}} = \left\{ (a_j, O_j, \mathbf{x}_j, \boldsymbol{\chi}_j) \right\}_{j=1}^{n_{\text{pep}}} \cup \left\{ (\mathbf{x}_i^s, \mathbf{n}_i^s, \boldsymbol{\tau}_i^s, \boldsymbol{\Upsilon}_i^s) \right\}_{i=1}^m \quad m \gg n_{\text{pep}}$$

Then we can get

$$p(C^{\text{pep}} | C^{\text{rec}}) \propto p \left( \left\{ (a_j, O_j, \mathbf{x}_j, \boldsymbol{\chi}_j) \right\}_{j=1}^{n_{\text{pep}}} \middle| C^{\text{rec}} \right) p \left( \left\{ (\mathbf{x}_i^s, \mathbf{n}_i^s, \boldsymbol{\tau}_i^s, \boldsymbol{\Upsilon}_i^s) \right\}_{i=1}^m \middle| C^{\text{rec}} \right),$$

# Introduction: Flow Matching



**Goal:** Find a velocity field  $u_\theta(x_t, t)$  that, when followed/integrated, transforms  $p_0$  into  $p_{data}$ . The *particularity of CFM* is how the velocity field is learned, as we will detail below.

For unknown target data distribution  $p_{data}$ , it is hard to choose a priori a probability path or velocity field. CFM core idea is to choose a conditioning variable  $z$  and a conditional probability path  $p(x|t, z)$  such that

- the induced global probability path  $p(x|t)$  transforms  $p_0$  into  $p_{data}$
- the associated velocity field  $u_\theta(x_t, t|z)$  has an analytic form.

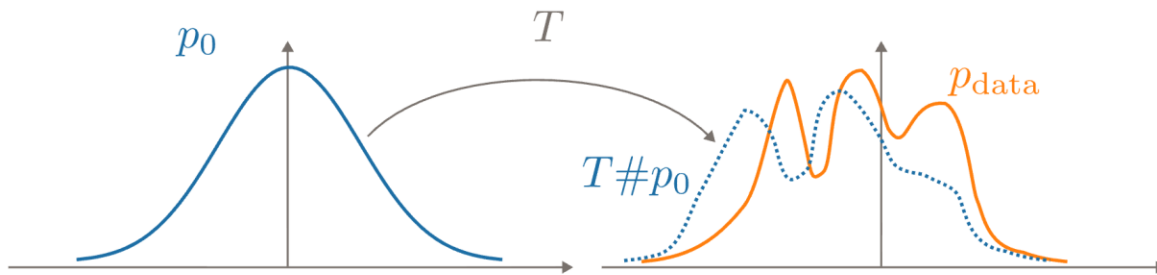
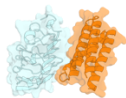


Figure 1. Modern generative modelling principle: trying to find a map  $T$  that sends the base distribution  $p_0$  as close as possible to the data distribution  $p_{data}$ .



# Introduction: Flow Matching

- A first choice is to condition on the base points and the target points, i.e.,  $z$  is a random variable defined as:

$$z \stackrel{\text{choice}}{=} (x_0, x_1) \sim p_0 \times p_{\text{data}}.$$

- Among all the possible probability paths, one can choose to use very concentrated Gaussian distributions and simply interpolate between  $x_0$  and  $x_1$  in straight line: for some fixed standard deviation  $\sigma$ , it writes as

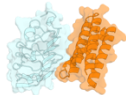
$$p(x|t, z = (x_0, x_1)) \stackrel{\text{choice}}{=} \mathcal{N}((1-t) \cdot x_0 + t \cdot x_1, \sigma^2 \text{Id}).$$

- To recover the correct distributions  $p_0$  at  $t=0$  (resp.  $p_{\text{data}}$  at  $t=1$ ), one must enforce  $\sigma=0$ , finally leading to, where  $\delta$  denotes the Dirac delta distribution.

$$p(x|t, z = (x_0, x_1)) \stackrel{\text{choice}}{=} \delta_{(1-t) \cdot x_0 + t \cdot x_1}(x),$$

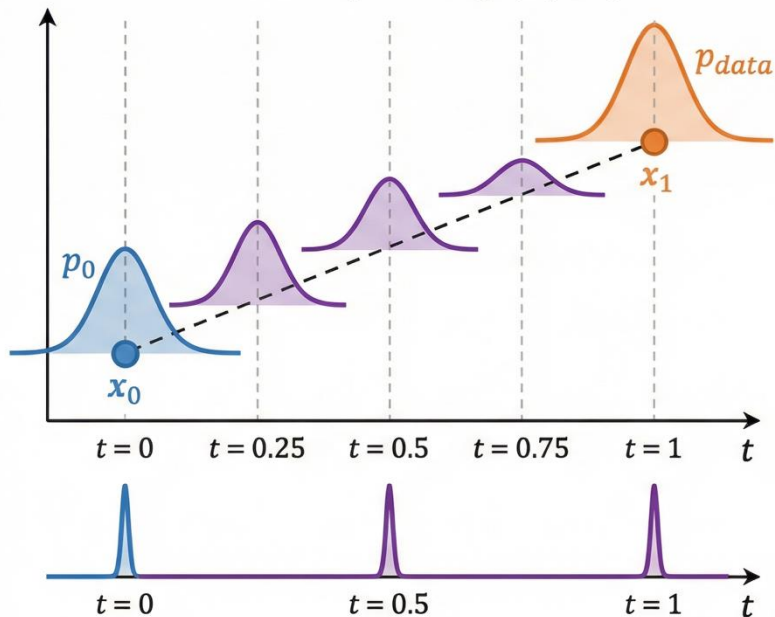
- Conditional vector field:

$$u^{\text{cond}}(x, t, z = (x_0, x_1)) = x_1 - x_0$$



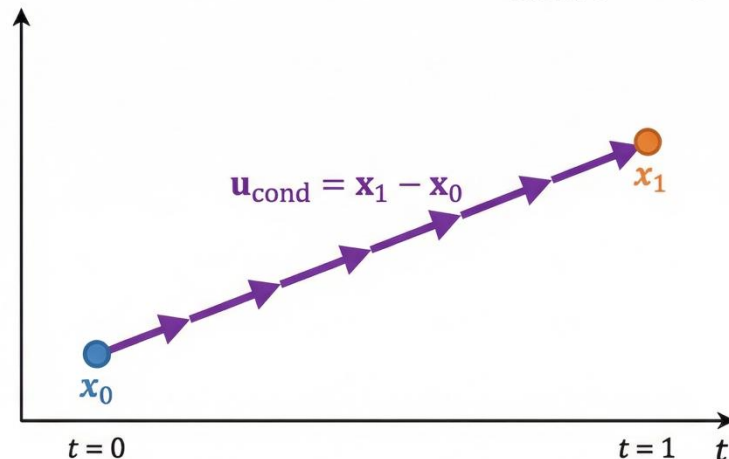
# Introduction: Flow Matching

Probability Path  $p(\mathbf{x}|t, \mathbf{z})$



Path:  $p(\mathbf{x}|t, \mathbf{z}) = \mathcal{N}((1-t)\mathbf{x}_0 + t\mathbf{x}_1, \sigma^2 \mathbf{Id})$   
 $\rightarrow \delta((1-t)\mathbf{x}_0 + t\mathbf{x}_1)(\mathbf{x})$  as  $\sigma \rightarrow 0$

Conditional Vector Field  $\mathbf{u}_{\text{cond}}(\mathbf{x}, t, \mathbf{z})$



Constant Velocity:  
 $\mathbf{u}_{\text{cond}}(\mathbf{x}, t, \mathbf{z}) = \mathbf{x}_1 - \mathbf{x}_0$

## Position

Interpolation and vector fields:

$$\phi_t^{\text{pos}}(\mathbf{x}_0^s, \mathbf{x}_1^s) = t\mathbf{x}_1^s + (1-t)\mathbf{x}_0^s,$$

$$u_t^{\text{pos}}(\mathbf{x}_t^s | \mathbf{x}_1^s, \mathbf{x}_0^s) = \mathbf{x}_1^s - \mathbf{x}_0^s = \frac{\mathbf{x}_1^s - \mathbf{x}_t^s}{1-t}.$$

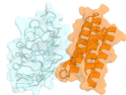
Training objectives:

$$\mathcal{L}_{\text{pos}}(\theta) = \mathbb{E}_{t \sim \mathcal{U}(0,1), p(\mathbf{x}_1^s), p(\mathbf{x}_0^s), p(\mathbf{x}_t^s | \mathbf{x}_0^s, \mathbf{x}_1^s)}$$

$$\|v^{\text{pos}}(\mathbf{x}_t^s, t, C^{\text{rec}}) - (\mathbf{x}_1^s - \mathbf{x}_0^s)\|_2^2,$$

Sampling:

$$\mathbf{x}_{t+\frac{1}{N}}^s = \mathbf{x}_t^s + \frac{1}{N}v^{\text{pos}}(\mathbf{x}_t^s, t, C^{\text{rec}}).$$



## Normal Vector Orientations

Interpolation and vector fields:

$$\phi_t^{\text{ori}}(O_0^s, O_1^s) = \exp_{O_0^s} \left( t \log_{O_0^s} (O_1^s) \right),$$

$$u_t^{\text{ori}}(O_t^s | O_0^s, O_1^s) = \frac{\log_{O_t^s} (O_1^s)}{1 - t},$$

Training objectives:

$$\mathcal{L}_{\text{ori}}(\theta) = \mathbb{E}_{t \sim \mathcal{U}(0,1), p(O_1^s), p(O_0^s), p(O_t^s | O_0^s, O_1^s)} \left\| v_t^{\text{ori}}(O_t^s, t, C^{\text{rec}}) - \frac{\log_{O_t^s} (O_1^s)}{1 - t} \right\|_{\text{SO}(3)}^2,$$

Sampling:

$$O_{t+\frac{1}{N}} = \exp_{O_t^s} \left( \frac{1}{N} v_t^{\text{ori}}(O_t^s, t, C^{\text{rec}}) \right).$$

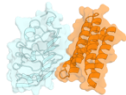
## Continuous Time Markov Chain (CTMC)

- A sequence trajectory  $x_t$  over time  $t \in [0, 1]$  that follows a CTMC alternates between **resting in its current state and periodically jumping to another randomly chosen state**.
- The frequency and destination of the jumps are **determined by the rate matrix**  $R_t$  with the constraint its off-diagonal elements are non-negative.
- We can write the transition probability as

$$p_{t+dt|t}(j|x_t) = \begin{cases} R_t(x_t, j)dt & \text{for } j \neq x_t \\ 1 + R_t(x_t, x_t)dt & \text{for } j = x_t \end{cases} \quad (1)$$

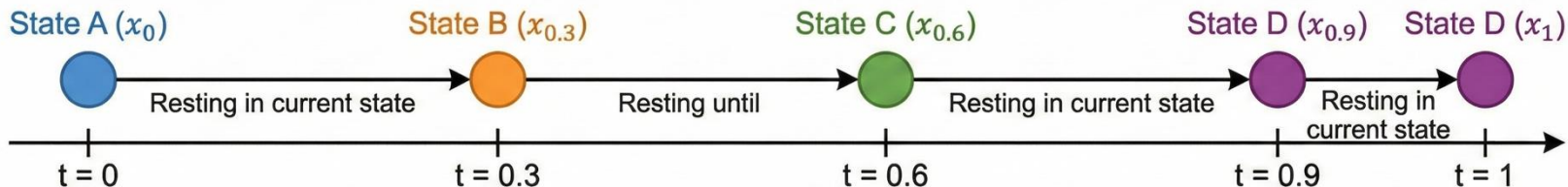
$$= \delta \{x_t, j\} + R_t(x_t, j)dt \quad (2)$$

$$x_{t+\Delta t} \sim \text{Cat}(\delta \{x_t, x_{t+\Delta t}\} + R_t(x_t, x_{t+\Delta t})\Delta t), \quad (3)$$



# Introduction: Discrete Flow Matching (CTMC)

## CTMC Trajectory over Time $t \in [0, 1]$



The sequence trajectory  $x_t$  alternates between resting and periodically jumping to another randomly chosen state.

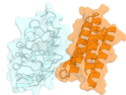
## Determining Jumps with Rate Matrix $R_t$

Rate Matrix  $R_t$  (at time  $t$ )

	State A	State B	State C	State D
State A	$-r_{AA}$	$r_{AB} > 0$	0	0
State B	0	$-r_{BB}$	$r_{BC} > 0$	0
State C	0	0	$-r_{CC}$	0
State D	0	0	0	$-r_{DD}$

Determines Frequency and Destination

Jumps are determined by the rate matrix  $R_t$ . The transition probability over a small time  $dt$  is  $p_{t+dt|t}(j|x_t) = \delta\{x_t, j\} + R_t(x_t, j)dt$  (for  $j \neq x_t$ , prob is  $R_t(x_t, j)dt$ ).



Interpolation Stage:

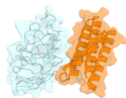
$$p_{t|1}^{\text{mask}}(\mathbf{Y}_t^s | \mathbf{Y}_1^s) = \text{Cat}(t\delta\{\mathbf{Y}_1^s, \mathbf{Y}_t^s\} + (1-t)\delta\{M, \mathbf{Y}_t^s\}).$$

Rate Matrix:

$$R_t^*(\mathbf{Y}_t^s, j | \mathbf{Y}_1^s) := \frac{\text{ReLU}\left(\partial_t p_{t|1}(j | \mathbf{Y}_1^s) - \partial_t p_{t|1}(\mathbf{Y}_t^s | \mathbf{Y}_1^s)\right)}{S \cdot p_{t|1}(\mathbf{Y}_t^s | \mathbf{Y}_1^s)},$$

Training Objective:

$$\mathcal{L}_{\text{cat}}(\theta) = \mathbb{E}_{t \sim \mathcal{U}(0,1), p(\mathbf{Y}_1^s), p_{t|1}(\mathbf{Y}_t^s | \mathbf{Y}_1^s)} \left[ \log v^{\text{cat}}(\mathbf{Y}_t^s, t, C^{\text{rec}}) \right].$$



# Overall Training Objective

$$\mathcal{L}_{\text{CFM}} = \lambda_{\text{pos}}\mathcal{L}_{\text{pos}} + \lambda_{\text{ori}}\mathcal{L}_{\text{ori}} + \lambda_{\text{cat}}\mathcal{L}_{\text{cat}} + \lambda_{\text{con}}\mathcal{L}_{\text{con}} + \lambda_{\text{str}}\mathcal{L}_{\text{str}}$$

$\lambda_*$ : Hyperparameters that balance the contribution of different objective terms.

$\mathcal{L}_{\text{pos}}$ : Supervises the spatial coordinates of the generated surface point cloud.

$\mathcal{L}_{\text{ori}}$ : Aligns the generated surface normals with the target orientation.

$\mathcal{L}_{\text{cat}}$ : Enforces consistency in discrete surface properties.

$\mathcal{L}_{\text{con}}$ : Enforces consistency in continuous surface properties.

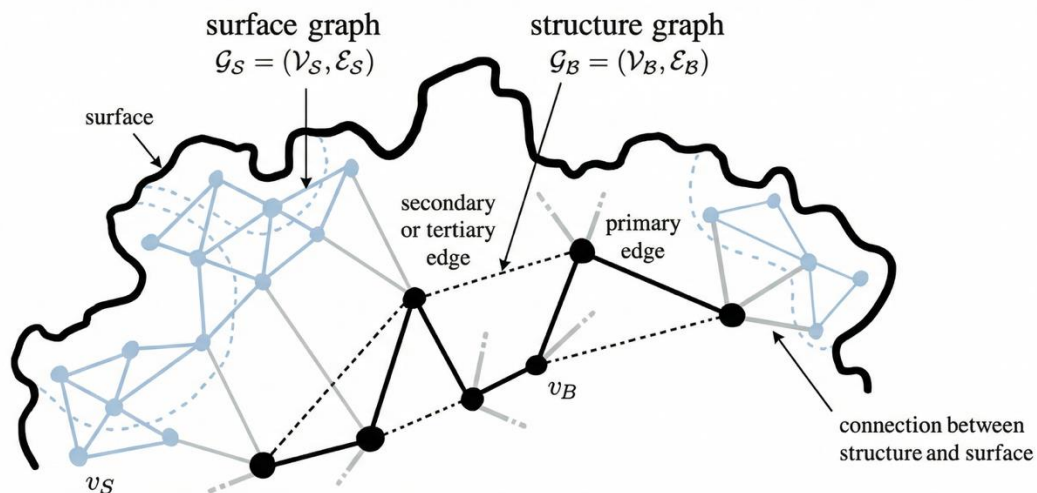
$\mathcal{L}_{\text{str}}$ : Flow Matching objective modeling the joint distribution of residue positions ( $a$ ), orientations ( $O$ ), amino acid types ( $x$ ), and torsion angles ( $\chi$ ).

# Equivariant Surface Geometric Network

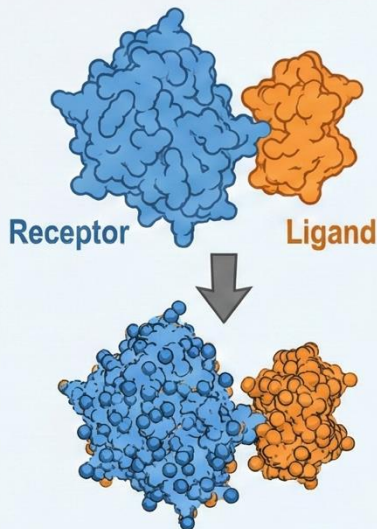
**Graph Construction:** we can build a dynamic heterogeneous surface graph

$$\mathcal{G} = (\mathcal{G}_{\text{rec}}, \mathcal{G}_{\text{pep}})$$

at different timestep  $t$ , where the intra and inter-point connectivities  $(\mathcal{E}_{\text{rec}}, \mathcal{E}_{\text{pep}}, \mathcal{E}_{\text{inter}})$  are determined based on a spatial distance threshold cutoff  $r$ .



## 1. Input & Discretization



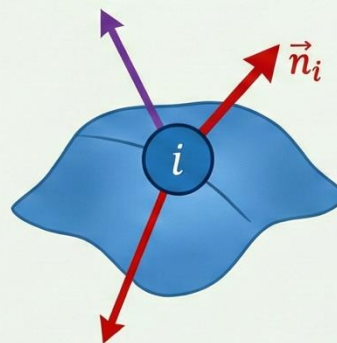
**Input:** 3D Protein Complex Structure

**Process:** Generate discrete surface nodes ( $\mathcal{V}$ ) based on residue positions or surface sampling (e.g., Connolly surface).

## 2. Node Feature Initialization

**Scalar Features ( $h_i^{(0)}$ )**

Invariant properties (e.g., Amino acid type, Charge, Hydrophobicity).  
Cannot rotate.



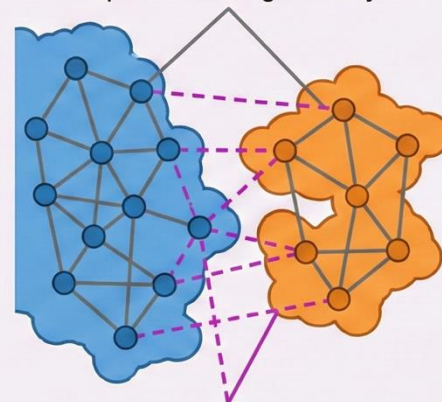
**Vector Features ( $x_i^{(0)}, \vec{n}_i$ )**

Geometric properties: 3D Coordinate ( $x_i$ ) & Surface Normal vector ( $\vec{n}_i$ ).  
Rotates with structure.

## 3. Dual-Graph Connectivity

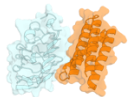
**Intra-graph Edges ( $\mathcal{E}_{intra}$ )**

Connect nodes within SAME protein based on local distance ( $d < R_{intra}$ ).  
Captures local geometry.

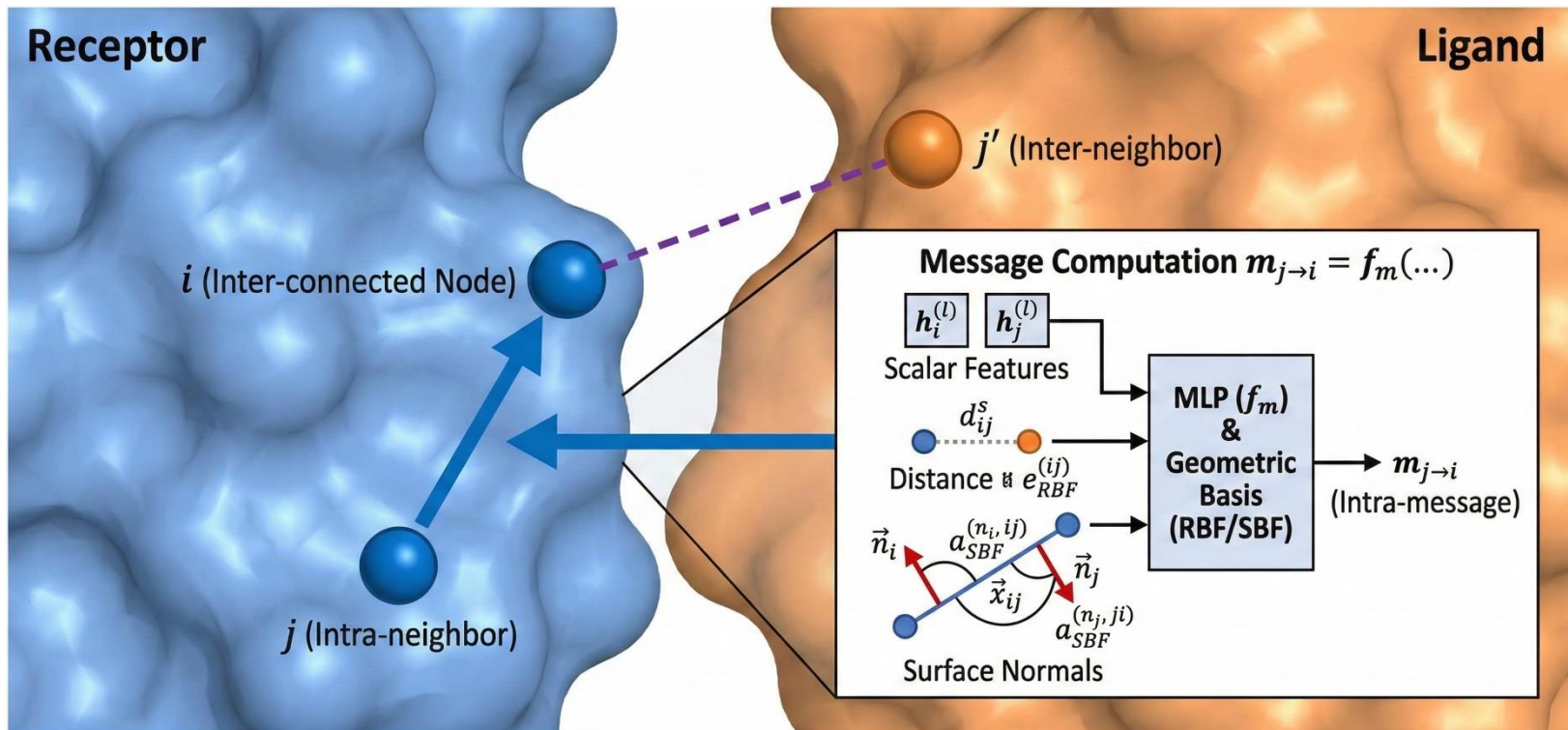


**Inter-graph Edges ( $\mathcal{E}_{inter}$ )**

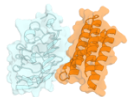
Connect nodes across INTERFACE based on proximity ( $d < R_{inter}$ ).  
Captures interaction.



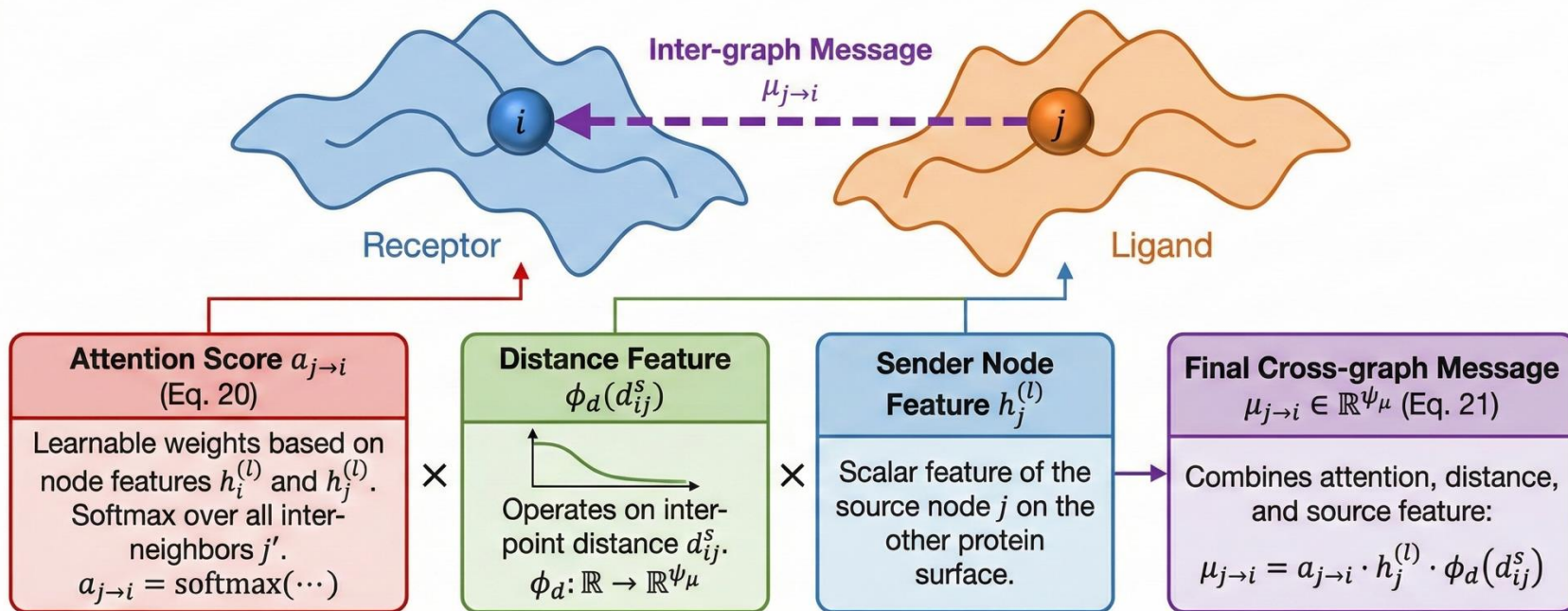
# Intra-graph Message Passing



Node  $i$ , located at the interface, receives a message from its internal neighbor  $j$ . This message  $m_{j \rightarrow i}$  encodes local geometry (distance  $d_{ij}$ , angles with normals  $\vec{n}_i$ ,  $\vec{n}_j$ ) processed by an MLP, as defined in Eq. (18).



# Cross-graph Message Passing

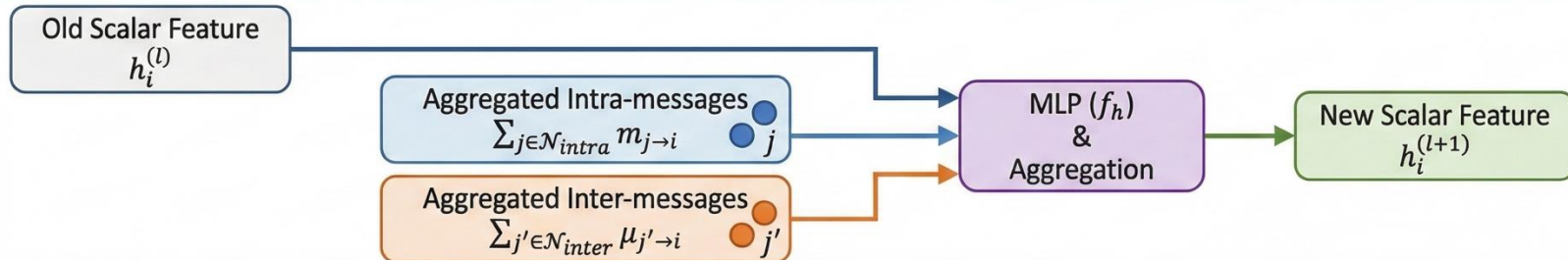


The **cross-graph message**  $\mu_{j \rightarrow i}$  aggregates information from inter-connected neighbor  $j$ , weighted by an attention mechanism  $a_{j \rightarrow i}$  and modulated by the spatial distance  $d_{ij}^s$ .

# Aggregation for intra- and inter-messages

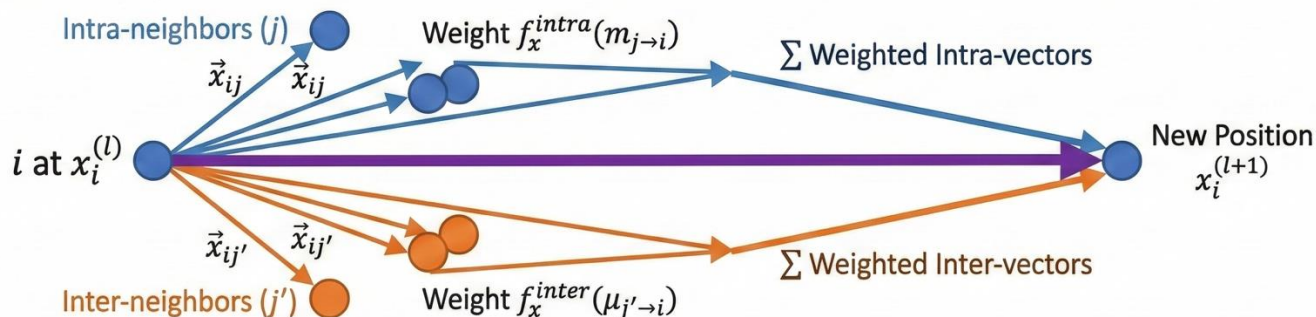
Scalar Feature Update ( $h_i^{(l+1)}$ ) - Eq. (22)

$$h_i^{(l+1)}(h_{h \rightarrow i}) + ai(\mathcal{N}_{intra}(\mu_{i'j' \rightarrow i})m_{j' \rightarrow i}) \quad (22)$$

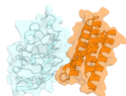


Coordinate Update ( $x_i^{(l+1)}$ ) - Eqs. (23, 24)

$$i - \vec{x}_i(x_i^{(l+1)}) \quad (23) \quad \vec{x}_i - f_{intra}(x_i^{(l+1)}) \quad (24)$$



Coordinates are updated by adding weighted relative vectors from both intra- and inter-neighbors, ensuring E(3)-equivariance.



# Experiments

**Training set:** 8,365 non-redundant protein-peptide complexes across 292 clusters.

**Test set:** 10 clusters and 158 complexes.

## Evaluation Metrics

### Geometry:

- *Amino Acid Recovery rate (AAR)*: Quantifies sequence identity between the generated peptides and the native reference.
- *Root-mean-square deviation (RMSD) of C $\alpha$  atoms*: Assesses structural deviation of the peptide backbone after alignment with the native complex.
- *Secondary-structure similarity ratio (SSR)*: Measures the consistency of secondary structure elements between generated and native conformations.
- *Binding Site Ratio (BSR)*: Evaluates the spatial overlap of interfacial residues between the generated and native ligands.

### Energy (Using Rosetta):

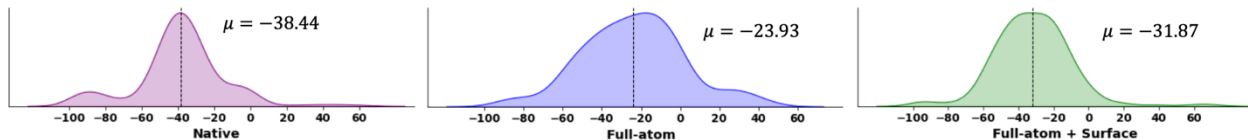
- *Affinity*: The percentage of generated peptides exhibiting lower binding energy than the native ligand.
- *Stability*: The proportion of complexes achieving a lower total energy score compared to the native state.

### Design:

- *Designability*: The fraction of sequences that fold into their designed structures  $C\alpha$  RMSD  $< 2 \text{ \AA}$ , validated via ESMFold.
- *Diversity*: The average pairwise structural dissimilarity (calculated as TM-score) within the generated samples.

**Table 1: Evaluation of different methods in the sequence-structure co-design task and ablation studies on key components of SurfFlow. The best and suboptimal results are labeled **boldly** and underlined.**

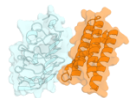
	Geometry				Energy		Design	
	AAR % $\uparrow$	RMSD $\text{\AA}$ $\downarrow$	SSR % $\uparrow$	BSR % $\uparrow$	Stb. % $\uparrow$	Aff. % $\uparrow$	Des. % $\uparrow$	Div. $\uparrow$
RFdiffusion [61]	40.14	4.17	63.86	26.71	<b>26.82</b>	16.53	<b>78.52</b>	0.38
ProteinGen [37]	45.82	4.35	29.15	24.62	23.48	13.47	71.82	0.54
Diffusion [39]	47.04	3.28	74.89	49.83	15.34	17.13	48.54	<u>0.57</u>
PepGLAD [26]	50.43	3.83	80.24	19.34	20.39	10.47	<u>75.07</u>	0.32
PPIFlow [34]	48.35	3.59	68.13	25.94	15.77	12.08	46.53	0.51
PepFlow [33]	<u>51.25</u>	<u>2.07</u>	<u>83.46</u>	<u>86.89</u>	18.15	21.37	65.22	0.42
SurfFlow (w/o ESGN)	52.59	2.05	83.77	86.91	19.42	19.82	68.41	0.60
SurfFlow (w/o Position)	53.26	1.99	84.79	87.15	21.30	22.38	72.09	0.60
SurfFlow (w/o Orientation)	53.04	2.00	84.60	87.04	20.79	22.46	72.36	0.60
SurfFlow (w/o Biophysical Prop.)	52.31	2.03	83.96	86.98	19.55	20.25	70.83	0.58
<b>SurfFlow</b>	<b>54.07</b>	<b>1.96</b>	<b>85.11</b>	<b>87.38</b>	<u>22.46</u>	<b>22.51</b>	73.60	<b>0.61</b>



**Figure 3: Binding energy distributions of designed and native peptides, where the lower is better.**

**Conclusion:** SurfFlow can achieve better performance than other peptide design models

# Takeaway



## Pros.:

- This work presents SurfFlow, a novel model that produces all protein modalities – sequence, structure, and surface – concurrently.
- Empirical results prove the reasonability and promise of considering molecular surfaces for protein discovery

## Cons.:

- No experiments to ablate all peptide surface component in its entirety and fair comparison to other seq-structure codesign methods.
- The surface of a peptide is determined entirely by its amino acid sequence. Therefore, it is no sense to jointly generating both the amino acid sequence and surface shape

**Thank you!**

**Any questions?**