Exercice 1

A p-n junction diode obeys the Shockley equation,

$$I_D = I_s\left(e^{V_a/V_T} - 1\right) \approx I_s\, e^{V_a/V_T} \quad \text{for} \;\; V_a \gg V_T, \tag{1}$$

where V_a is the applied voltage, $V_T = kT/q$ is the thermal voltage, and I_s is the reverse saturation current of the diode. As the temperature (T) increases, the exponential factor decreases. However, an increase in T causes I_s to increase since $I_s \propto n_i^2$, where

$$n_i = \sqrt{N_C(T)N_V(T)}\,\exp\left(-\frac{E_g(T)}{2kT}\right) \tag{2}$$

is the intrinsic carrier concentration of the material. As T is increased, the exponential factor in Eq. 2 increases, and so do the effective densities of states N_C and N_V. As a result, n_i increases significantly[1] with T and so does I_s. The increase in I_s more than compensates the decrease in the exponential term in Eq. 1, and the net result is that, for the same applied voltge, the diode current is higher at a higher temperature. In other words, the diode I-V curve shifts left with temperature, as shown in Fig. 1.

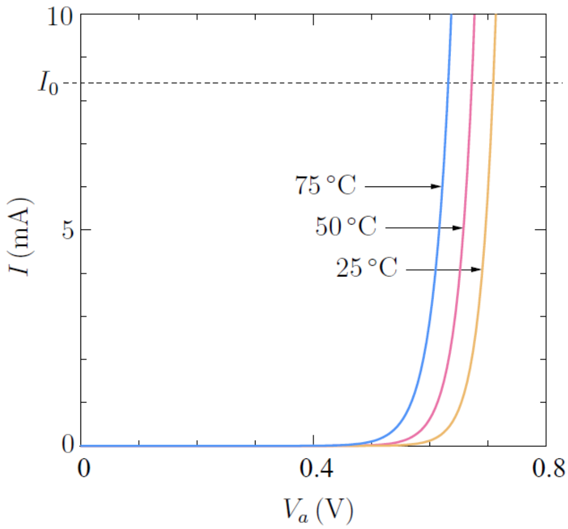

Figure 1: I-V curve of a silicon diode under forward bias at different temperatures (representative plot)

If the diode current is held constant (I in the figure), we can see that the diode voltage decreases as the temperature is increased. For a silicon diode, this change is about −2mV/°C.

1) In silicon, n_i is of the order of 10^{10} carriers/cm^3 at room temperature, T=300K, and doubles with every increase of 10°C. Estimate by calculation what is the increase in the current I_D of a diode supposing an increase of temperature for 50°C, from the reference temperature of 27°C (300K) and a forward bias of 0.6V.

2) In order to make the junction diode a temperature sensor, it has to be biased with a constant current circuit. Demonstrate that circuit of Fig. 2 offers a constant current solution.

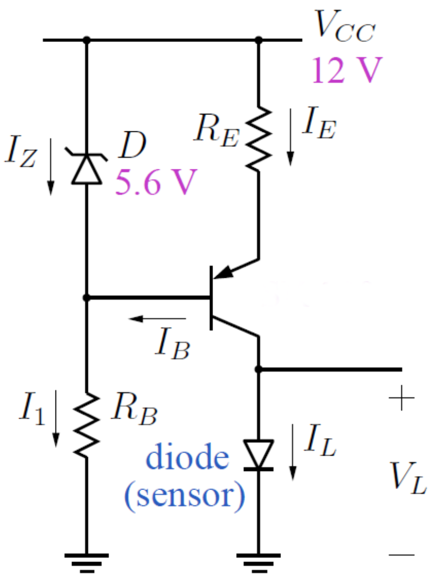

Figure 2: Implementation of a constant current source with a bipolar transistor.

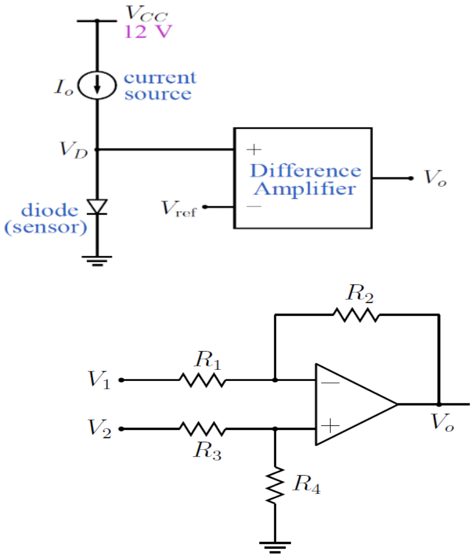

Figure 3: Overall sensor circuit, with diode as sensor and differential amplification.

3) If the Zener diode should operate at a current of 2mA, calculate the value of the resistance R_B in order to impose this value of current.

4) Calculate RE if we want to impose a current of 1mA in the junction diode used as temperature sensor.

5) The circuit works as a constant current source as long as the transistor is in the active operation region (forward mode), VE −VC > 0.3V. Demonstrate that in this case, if V_L =0.7V, this condition is satisfied.

6) Propose a constant current source circuit similar to the one of Fig.2 implemented with a MOSFET transistor.

7) In practice, the temperature-dependent voltage drop on the diode of Fig. 2 is amplified by a differential amplifier scheme as in Fig. 3. Supposing V_2 is the voltage drop on the diode and V1 a reference voltage, calculate the amplified output of the circuit in Fig. 3. Propose values for the resistances so that the output is amplified by 10x compared to the input.

Solution:

1) The diode equation written for the two temperatures are as follows:

$$I_{T0} = I_{S0} e^{V_D q / kT_0} \text{ and } I_{T1} = I_{S1} e^{V_D q / kT_1}$$

Intrinsic carrier concentration increases 2 times every 10 degrees, so for an increase of 50 degrees from 300K to 350K, intrinsic concentration grows by a factor of 32. Since the reverse saturation current of the diode is proportional to the square of carrier concentration, it will see an increase of 1024 times.

The ratio between the two currents can be written as:

$$\frac{I_{T1}}{I_{T0}} = \frac{I_{S1} e^{V_D q / kT_1}}{I_{S0} e^{V_D q / kT_0}} = \frac{1024 * I_{S0} e^{V_D q / kT_1}}{I_{S0} e^{V_D q / kT_0}} = 1024 * e^{\frac{V_D q}{k}\left(\frac{1}{T1} - \frac{1}{T2}\right)}$$

After calculation we obtain that $\frac{I_{T1}}{I_{T0}} \cong 37.3$.

2) $I_L \cong I_E = \frac{V_Z - V_{EB}}{R_E}$. This current does not depend on load voltage, so we can assume the delivered current will be constant across multiple values of load voltage.
3) $I_Z = I_1 = 2mA$

$$I_1 = \frac{V_{CC} - V_Z}{R_B} \rightarrow R_B = \frac{V_{CC} - V_Z}{I_1} = \frac{12V - 5.6V}{2mA} = 3.2\ k\Omega$$

4) $I_E = 1mA = \frac{V_Z - V_{EB}}{R_E} \rightarrow R_E = \frac{V_Z - V_{EB}}{I_E} = \frac{5.6V - 0.6V}{1mA} = 5k\Omega$

5) $\begin{matrix} V_E = V_{CC} - I_E R_E \\ V_C = V_L = 0.7V \end{matrix} \Big| \rightarrow V_E - V_C = 12V - 5k\Omega * 1\text{mA} - 0.7\text{V} = 6.3\text{V}$

6) It is possible to directly replace the PNP BJT in this schematic with a P-type MOSFET.
   Doing so would only change the KVL on the upper loop. Load voltage would still not affect the loop where the current is generated. However, the range of functional gate-source voltages of the MOSFET is very high compared to the relatively tight Base-emitter voltage range of a BJT. This makes for a difficult problem of finding precise resistor values to obtain the wanted currents. Variations from device to device also impacts the MOSFET version heavily.

The most accurate MOSFET-based version of a constant current source is the one presented in previous exercises, based on a single MOSFET + OpAmp.

7) The general transfer equation of the difference amplifier is

$$V_{OUT} = V_2 * \left(\frac{R_4(R_1 - R_2)}{R_1(R_3 + R_4)}\right) - V_1 * \frac{R_2}{R_1}$$

If, however, we choose resistors such that $R_1 = R_3$ and $R_2 = R_4$ then the above equation simplifies to $V_{OUT} = \frac{R_2}{R_1}(V_2 - V_1)$.

In order to obtain a gain of 10 applied to the difference, we can use for example $R_2 = 100k\Omega$ and $R_2 = 10k\Omega$.