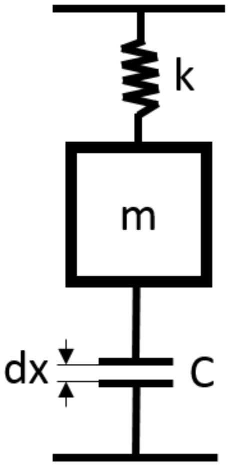
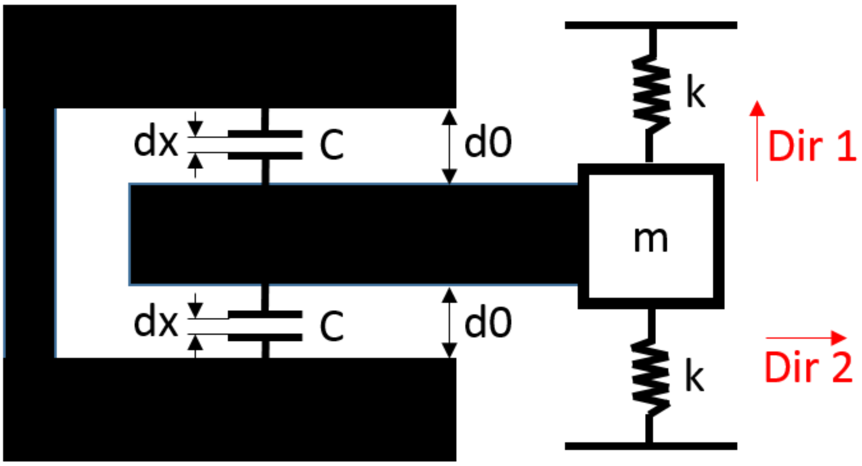
Proposed solution:

Point 1:



Option 1



Option 2

Option 1 is the simplest, but prone to errors because of process variation that impacts the capacitance value in an unpredictable matter.

Option 2 is more complicated, but it's a differential architecture, so process-related capacitance deviations are canceled. We are interested here in the dC, rather than absolute capacitance values.

Option 2 can be used in variable displacement (Dir1) or variable area (Dir2). Difference between these can be seen from the dC/dx function as follows:

$$\Delta C = \varepsilon A \left(\frac{1}{d_0 - \Delta x} - \frac{1}{d_0 + \Delta x} \right)$$

Moving the proofing mass in Direction 1 gives a nonlinear response, while in Direction 2 the difference between the two capacitances is always 0, since both capacitors have the same area (provided there's no vertical displacement). For the rest of the exercise we will use Option 2 in displacement mode.

Point 2.

Sensitivity can be written as:

$$S = \frac{m}{k}$$

But

$$k = \frac{F}{x} = \frac{m \cdot a}{x}$$

So we can express sensitivity as:

$$S = {}^{m}\!\Big/_{\frac{m \cdot a}{x}} = \frac{x}{a} = \frac{10\,nm}{1\,G}$$

We can then derive the spring constant as a function of proof mass and sensitivity, as follows:

$$k = \frac{m}{S}$$

Point 3:

$$C_1 = \varepsilon \frac{A}{d_0 - dx}$$

$$C_2 = \varepsilon \frac{A}{d_0 + dx}$$

The difference between the two will be:

$$\Delta C = N * \varepsilon * A \left(\frac{1}{d_0 - dx} - \frac{1}{d_0 + dx} \right)$$

Where N is the number of capacitor pairs in parallel.

Plugging in the numbers, we get $\Delta C \cong 90 fF$

Point 4:

In order to evaluate the linearity of a given function, we need to look at its second derivative.

We can simplify the dC(dx) function by writing it in the generalized form

$$f(x) = \frac{1}{a - x} - \frac{1}{a + x}$$

Where "a" is a constant.

The second derivative of this function is

$$\frac{d^2 f}{dx^2} = \frac{2}{(a - x)^3} - \frac{2}{(a + x)^3}$$

In the 10-100 nm range, the dx is very small, compared to d0, so the second derivative of dC with respect to dx is almost 0. The curve is almost linear. If, however, the displacement becomes greatly comparable with the neutral gap, the curve would show pronounced nonlinearity.

Point 5:

We start from the minimum acceleration step to be measured, we will name this a_{LSB}. We use the formula for sensitivity to obtain the corresponding displacement Δx_{LSB}, as follows:

$$\Delta x_{LSB} = a_{LSB} * S = 1\, nm$$

After this, we calculate the corresponding capacitance change, using the linearity of the $\Delta C / \Delta x$ curve.

$$\Delta C_{LSB} = \Delta x_{LSB} * {}^{\Delta C}\!/_{\Delta x} = 0.9\, fF$$

After this we apply the transfer function of the charge amplifier, to obtain V_{LSB}

$$V_{LSB} = V_{ref} * {}^{\Delta C_{LSB}}\!/_{C_f} = 10\, mV$$

Finally, we obtain the resolution (number of bits) by:

$$N_b = \log_2 \left(\frac{V_{FS}}{V_{LSB}} \right) = \log_2 200 = 7.6\ \sim 8\, bits$$

In conclusion, we need 8 bits.