Smart Sensors for IoT
Exercise 7 (15.12.2021)

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Problem 1 The Transducer Bridge

Transducer resistances are expressed in the form $R+\Delta R$, where R is the resistance at some reference condition, such as 0 °C in the case of temperature transducers, or the absence of strain in the case of strain gauges, and ΔR represents the deviation from the reference value as a consequence of a change in the physical condition affecting the transducer. Transducer resistances are also expressed in the alternative form $R(1+\delta)$, where $\delta = \Delta R/R$ represents the fractional deviation. Multiplying δ by 100 yields the percentage deviation.

Platinum resistance temperature detectors (Pt RTDs) have a temperature coefficient $\alpha = 0.003\,92\,°\mathrm{C}^{-1}$. Usually, the Pt RTD reference value is given at 0 °C and it is 100 Ω. The expression for the resistance R as a function of the temperature T is:

$$R(T) = R(0\,°\mathrm{C}) \cdot (1+\alpha T) = 100 \cdot (1+\alpha T) = 100 \cdot (1+\delta).$$

An implementation of the Platinum resistance temperature detectors is shown in Fig. 1 and let $V_{\mathrm{REF}} = 15\,\mathrm{V}$.

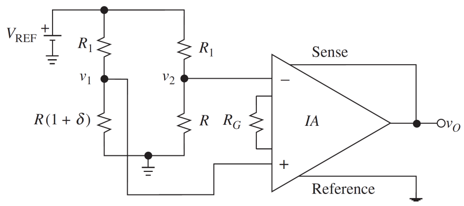

Figure 1: Transducer bridge and IA used as Pt RTD.

- Specify values for R_1 and A suitable for achieving an output sensitivity of 0.1 V/°C near 0 °C. To avoid self-heating in the RTD, limit its power dissipation to less than 0.2 mW.
- Compute $v_O(100\,°\mathrm{C})$.

Problem 2 Wheatstone bridge linearization

The circuit in Figure Fig. 2 is a linearized bridge. Assume the op amps to be ideal:

- show that the output voltage is directly proportional to the measured quantity.

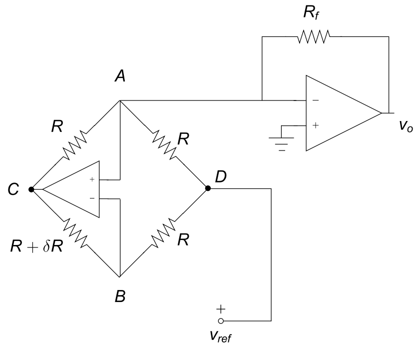

Figure 2: Wheatstone bridge linearization by two op amps.

Problem 3 Switched Capacitor circuit

Consider the Switched Capacitor circuit in Fig. 3, where the amplifier is ideal except it has a small offset voltage V_{os}.

- Evaluate the effect the offset voltage on the output.

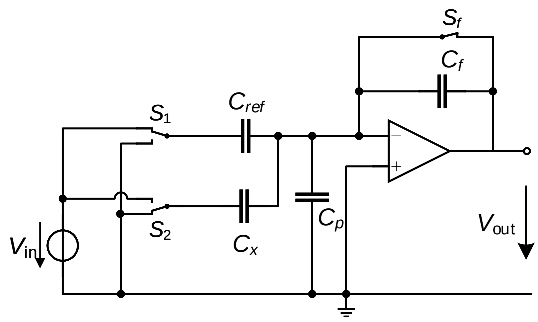

Figure 3: Switched Capacitor circuit.

Solutions to Exercise 7 (15.12.2021)

Problem 1 The Transducer Bridge

Platinum resistance temperature detectors (Pt RTDs) have a temperature coefficient $\alpha = 0.003\,92\,^{\circ}\mathrm{C}^{-1}$. Usually, the Pt RTD reference value is given at $0\,^{\circ}\mathrm{C}$ and it is $100\,\Omega$. The expression for the resistance R as a function of the temperature T is:

$$R(T) = R(0\,^{\circ}\mathrm{C}) \cdot (1 + \alpha T) = 100 \cdot (1 + \alpha T) = 100 \cdot (1 + \delta).$$

An implementation of the Platinum resistance temperature detectors is shown in Fig. 1 and let $V_{\mathrm{REF}} = 15\,\mathrm{V}$.

- Specify values for R_1 and A suitable for achieving an output sensitivity of $0.1\,\mathrm{V}/^{\circ}\mathrm{C}$ near $0\,^{\circ}\mathrm{C}$. To avoid self-heating in the RTD, limit its power dissipation to less than $0.2\,\mathrm{mW}$.

The voltage at the node v_1 is given by:

$$v_1 = \frac{R(1+\delta)}{R(1+\delta)+R_1} V_{\mathrm{REF}}; \tag{1}$$

whereas the voltage at the node v_2 is given by:

$$v_2 = \frac{R}{R+R_1} V_{\mathrm{REF}}; \tag{2}$$

The voltage v_O is given, with some manipulation, by:

$$v_O = (v_2 - v_1)A = A V_{\mathrm{REF}} \frac{\delta}{1 + R_1/R + (1 + R_1/R)(1+\delta)}. \tag{3}$$

Denoting the transducer current as i, the power dissipation of the resistance temperature detector (RTD) is:

$$P_{\mathrm{RTD}} = R i^2; \tag{4}$$

therefore, in order to meet the power dissipation requirement, $i^2 \leq P_{\mathrm{RTD}}/R = 0.2\,\mathrm{mW}/100\,\Omega$, i.e. $i \leq 1.4\,\mathrm{mA}$.

To be on the safe side, let us impose $i = 1\,\mathrm{mA}$. Since the current flowing in R is:

$$i = \frac{V_{\mathrm{REF}}}{(R_1 + R)}; \tag{5}$$

this yields to $R_1 = 15\,\mathrm{k}\Omega$.

For $\Delta T = 1\,^{\circ}\mathrm{C}$, we have $\delta = \alpha \cdot 1\,^{\circ}\mathrm{C} = 0.00392$, and we want $\Delta v_O = 0.1\,\mathrm{V}$.

We can use the following equation since $\delta \ll 1$:

$$v_O \approx \frac{AV_{\text{REF}}}{2 + R_1/R + R/R_1}\delta; \tag{6}$$

we need $A = 258.5\,\text{V/V}$.

- Compute $v_O(100\,°\text{C})$.

For $\Delta T = 100\,°\text{C}$ we have $\delta = \alpha\Delta T = 0.392$, which is not much smaller than 1 and therefore the following equation should be better used:

$$v_O = AV_{\text{REF}}\frac{\delta}{1 + R_1/R + (1 + R_1/R)(1 + \delta)}; \tag{7}$$

from which we get $v_O(100\,°\text{C}) = 9.974\,\text{V}$. Equation (6) predicts that $v_O(100\,°\text{C}) = 100\,\text{V}$, , which exceeds the actual value by 0.026 V. Since 0.1 V corresponds to 1 °C, 0.026 V corresponds to $0.026/0.1 = 0.26\,°\text{C}$. Therefore, in using the approximated expression, we cause, at 100 °C, an error of about one-quarter of a degree Celsius.

Problem 2 Wheatstone bridge linearization

The circuit in Fig. 2 is a linearized bridge. Assume the op amps to be ideal:

- show that the output voltage is directly proportional to the measured quantity.

Writing the current summation at nodes A and B (which are grounded), respectively:

$$\begin{cases} \dfrac{V_c}{R} + \dfrac{V_{ref}}{R} + \dfrac{V_0}{R_f} = 0; \\ \dfrac{V_c}{R + \delta R} + \dfrac{V_{ref}}{R} = 0; \end{cases} \tag{8}$$

leading to:

$$V_O = \frac{R_f \cdot \delta R}{R^2}V_{ref}. \tag{9}$$

Problem 3 Switched Capacitor circuit

Consider the Switched Capacitor circuit in Fig. 3, where the amplifier is ideal except it has a small offset voltage V_{os}.

- Evaluate the effect the offset voltage on the output.

Let us apply the charge conservation principle to the two phases (the principle is similar to the Kirchhoff's Current Law):

Phase 1

$$Q_1 = (V_{\text{in}} - V_{\text{os}})C_{ref} - V_{\text{os}}C_p - V_{\text{os}}C_x \tag{10}$$

Phase 2

$$Q_2 = -V_{\text{os}}C_{ref} + (V_{\text{in}} - V_{\text{os}})C_x + (V_{\text{out}} - V_{\text{os}})C_f - V_{\text{os}}C_p \tag{11}$$

By equaling (10) and (11):

$$Q_1 = Q_2 \rightarrow (V_{\text{in}} - V_{\text{os}})C_{ref} - V_{\text{os}}C_p - V_{\text{os}}C_x = -V_{\text{os}}C_{ref} + (V_{\text{in}} - V_{\text{os}})C_x + (V_{\text{out}} - V_{\text{os}})C_f - V_{\text{os}}C_p; \tag{12}$$

the following result is obtained:

$$V_{\text{out}} = V_{\text{os}} + \frac{C_{ref} - C_x}{C_f} V_{\text{in}}. \tag{13}$$