Smart Sensors for IoT
Exercise 6 (01.12.2021)

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Problem 1 Autozero SC Amplifiers

Referring to Fig. 1 and Fig. 2, and considering both OpAmps having infinite gain and an off-set voltage of 0.1 mV, what is the output residual off-set voltage in the two cases if:

- $C_1 = 10\,\text{pF}$ and $C_2 = 1\,\text{pF}$;
- $C_1 = 1\,\text{pF}$ and $C_2 = 1\,\text{pF}$.

Now, referring to Fig. 2, consider the OpAmp to have the same offset but limited gain A.

- Calculate the minimum A in order to have a 99.9% offset reduction for the two values of C_1 and C_2 above. Comment the results, focusing also on the impact of the offset on the A_v gain.

Optional problem [difficult]

Referring to Fig. 2 and Slide 20 in the lecture notes, derive the results for:

- V_{neq}
- $V_{os\text{-}res}$.

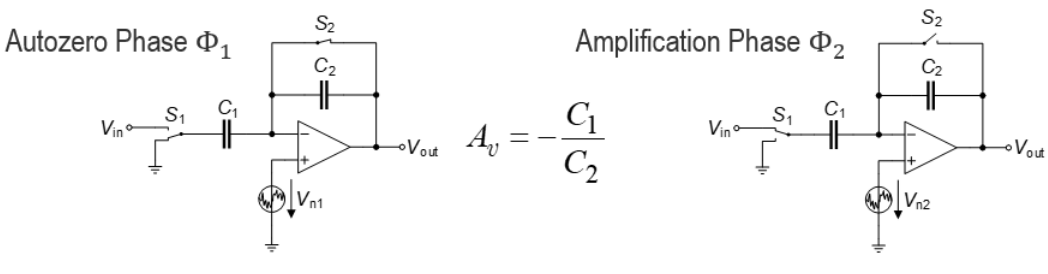

Figure 1: Autozero SC Amplifiers, first implementation.

Problem 2 The Chopper Stabilization (CS) technique

Consider the Chopper Stabilization (CHS) Principle chain shown in Fig. 3 and consider the amplifier DC gain $A_{dc} = 80\,\text{dB}$, the input signal amplitude $A_{in} = 10\,\text{mV}_{\text{0-P}}$, the flicker corner frequency $f_k = 10\,\text{kHz}$, the amplifier off-set voltage $V_{os} = 0.1\,\text{mV}$, $S_0 = 50\,\text{nV}/\sqrt{\text{Hz}}$

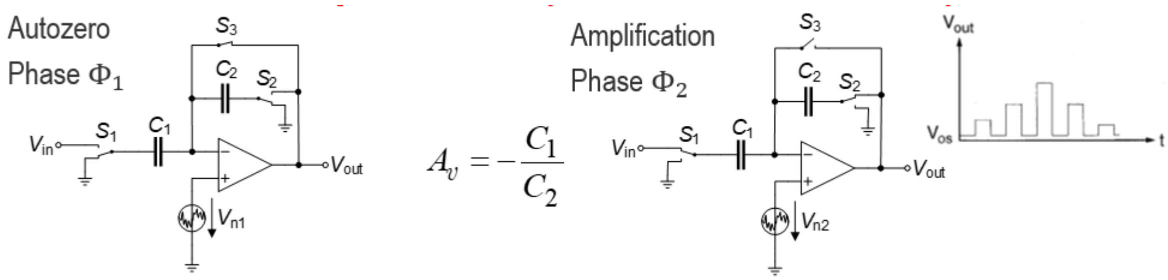

Figure 2: Autozero SC Amplifiers, second implementation.

- Calculate the equivalent DC gain.
- Chose a proper chopping period T so to have the total residual noise in the baseband around $92.5\,\mathrm{nV}/\sqrt{\mathrm{Hz}}$.
- Consider the residual input-referred offset due to the injection of the modulator switches. The input equivalent capacitance of the OpAmp is 500 fF, the relative tuning error between the selective amplifier center frequency and the chopper frequency is negligible and the worst case error charge Δq is 25 µC. Calculate the maximum switches on-resistance R_{on} resistance in order to have a residual input-referred offset smaller than 10 µV.

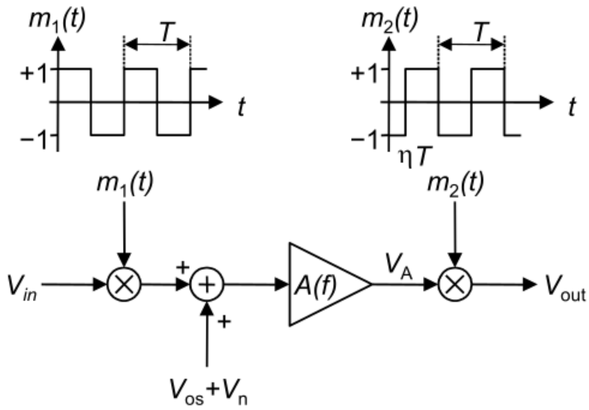

Figure 3: Chopper Stabilization (CHS) Principle chain.

Solutions to Exercise 6 (01.12.2021)

Problem 1 Autozero SC Amplifiers

Referring to Fig. 1 and Fig. 2, and considering both OpAmps having infinite gain and an off-set voltage of 0.1 mV, what is the output residual off-set voltage in the two cases if:

- $C_1 = 10\,\mathrm{pF}$ and $C_2 = 1\,\mathrm{pF}$;

In both implementations, $|A_v| = C_1/C_2 = 10$.

In the first implementation, $V_{os\text{-}res} = V_{os}/|A_v| = 0.1\,\mathrm{mv}/10 = 10\,\mathrm{\mu V}$.

In the second implementation, $V_{os\text{-}res} = 0\,\mathrm{V}$.

- $C_1 = 1\,\mathrm{pF}$ and $C_2 = 1\,\mathrm{pF}$.

In both implementations, $|A_v| = C_1/C_2 = 1$.

In the first implementation, $V_{os\text{-}res} = V_{os}/|A_v| = 0.1\,\mathrm{mv}/1 = 0.1\,\mathrm{mV}$.

In the second implementation, $V_{os\text{-}res} = 0\,\mathrm{V}$.

Note that the second implementation effectively cancels the off-set voltage.

Now, referring to Fig. 2, consider the OpAmp to have the same offset but limited gain A.

- Calculate the minimum A in order to have a 99% offset reduction for the two values of C_1 and C_2 above. Comment the results, focusing also on the impact of the offset on the A_v gain.

Ideally the residual offset for the implementation shown in Fig. 2. However due to the OpAmp non-ideality it can be larger. We want to have a residual offset due to the OpAmp finite gain which is 99.9% lower of V_{os}, that is $V_{os,res} < 0.1\,\mathrm{\mu V}$.

With the first pair of capacitor values, we can use the approximated formula given in the slides since $A_v \gg 1$:

$$V_{os,res} \approx -\frac{V_{os}}{A} \rightarrow A = -\frac{V_{os}}{V_{os,res}} = \frac{0.1\,\mathrm{mV}}{0.1\,\mathrm{\mu V}} = 1000\,\mathrm{V/V} = 60\,\mathrm{dB}. \tag{1}$$

Note that if we apply the not-approximated formula, we get $A = 1101\,\mathrm{V/V}$, which is close enough.

With the second pair of capacitor values, we cannot really use the approximated formula as $A_v = 1$:

$$V_{os,res} = -\frac{1 + 1/|A_{v,id}|}{1 + A} V_{os} \rightarrow A = -\frac{A_{v,id}(V_{os} + V_{os,res}) + V_{os}}{A_{v,id} V_{os,res}} = 2001\,\mathrm{V/V} = 66\,\mathrm{dB}. \tag{2}$$

Note that in both cases the finite gain impacts the $A_{v,id} = -C_2/C_1$ by reducing it by a factor $1 - \varepsilon$, with $\varepsilon = \frac{1+C_1/C_2}{A}$.

Optional problem [difficult]

Referring to Fig. 2 and Slide 16 in the lecture notes, demonstrate that:

- V_{neq}

In order to calculate the input equivalent noise V_{neq}, we set $V_{in} = 0\,\mathrm{V}$ and apply the charge conservation principles in the two phases.

During phase one Φ_1, V_{out} is at V_{n1} thanks to the virtual ground principle.

The total charge in this phase is:

$$Q_1 = V_{n1} \cdot C_1 + V_{n1} \cdot C_2. \tag{3}$$

During the second phase, the charge is given by:

$$Q_2 = V_{n2} \cdot C_1 - (V_{out} - V_{n2}) \cdot C_2. \tag{4}$$

By equating the two charges, i.e. setting $Q_1 = Q_2$, we get the noise at the output:

$$V_{n1} \cdot C_1 + V_{n1} \cdot C_2 = V_{n2} \cdot C_1 - (V_{out} - V_{n2}) \cdot C_2 \Rightarrow$$
$$\Rightarrow V_{out} = \left(1 + \frac{C_1}{C_2}\right) \cdot (V_{n1} - V_{n2}) = (1 + |A_v|) \cdot (V_{n1} - V_{n2}). \tag{5}$$

By referring (5) to the input, i.e by dividing V_{out} by the gain $|A_v| = C_1/C_2$, we get V_{neq}:

$$V_{neq} = \frac{V_{out}}{|A_v|} = \frac{(1 + |A_v|) \cdot (V_{n1} - V_{n2})}{|A_v|} = \left(1 + \frac{1}{|A_v|}\right) \cdot (V_{n1} - V_{n2}) \approx V_{n1} - V_{n2} \text{ for } |A_v| \gg 1. \tag{6}$$

- $V_{os\text{-}res}$.

Contrary to the previous case, if $V_{n1} = V_{n2} = V_{os}$, the residual offset voltage is nulled and only limited by the OpAmp dc gain $A_{dc} \triangleq G_m \cdot R_o$. In this scenario, the dc gain A_v needs to be recalculated in order to take into account the non-ideal OpAmp with the charge conservation principle and by considering ideal offset

During phase 1, the input voltage is at ground and there is no other voltage supply feeding the circuit, therefore:

$$Q_1 = 0. \tag{7}$$

During phase 2, $V_p \neq V_m$ and $V_{out} = (V_p - V_m) \cdot A_{dc}$, therefore:

$$V_m = -\frac{V_{\mathrm{out}}}{A_{dc}}. \tag{8}$$

The charge in phase 2 is given by:

$$Q_2 = (V_{in} - V_m) \cdot C_1 + (V_{out} - V_m) \cdot C_2 \Rightarrow \left(V_{in} - \left(-\frac{V_{out}}{A_{dc}}\right)\right) \cdot C_1 + \left(V_{out} - \left(-\frac{V_{out}}{A_{dc}}\right)\right) \cdot C_2. \tag{9}$$

By equating the two charges, i.e. setting $Q_1 = Q_2$, we get the noise at the output:

$$0 = (V_{in} - V_m) \cdot C_1 + (V_{out} - V_m) \cdot C_2 \Rightarrow \left(V_{in} - \left(-\frac{V_{out}}{A_{dc}}\right)\right) \cdot C_1 + \left(V_{out} - \left(-\frac{V_{out}}{A_{dc}}\right)\right) \cdot C_2; \tag{10}$$

from which:

$$A_{v,\ non\text{-}id} = \frac{V_{out}}{V_{in}} = -\frac{C_1}{C_2 + \dfrac{C_1 + C_2}{A_{dc}}}. \tag{11}$$

In order to calculate the effect of the off-set at the output, let us use the charge conservation principle by considering this non-ideality and again setting the input at ground.

During phase 1, $V_p \neq V_m$, and $V_{\text{out}} = (V_p - V_m) \cdot A_{dc}$. But $V_{\text{out}} = V_m$, therefore:

$$V_m = \frac{1}{1 + 1/A_{dc}} \cdot V_{os}. \tag{12}$$

The charge in phase 1 is given by:

$$Q_1 = V_m \cdot C_1 + V_m \cdot C_2 = \frac{C_1 + C_2}{1 + 1/A_{dc}} \cdot V_{os} \tag{13}$$

During phase 2, again $V_p \neq V_m$ and $V_{\text{out}} = (V_p - V_m) \cdot A_{dc}$, therefore:

$$V_m = -\frac{V_{\text{out}}}{A_{dc}} + V_{os}. \tag{14}$$

The charge in phase 2 is given by:

$$\begin{aligned} Q_2 = V_m \cdot C_1 - (V_{out} - V_m) \cdot C_2 = \left(-\frac{V_{\text{out}}}{A_{dc}} + V_{os}\right) C_1 + \left(V_{out} - \left(-\frac{V_{\text{out}}}{A_{dc}} + V_{os}\right)\right) \cdot C_2 = \\ = V_{os}(C_1 + C_2) - V_{out} \cdot C_2 - V_{out} \cdot \frac{C_1 + C_2}{A_{dc}} \end{aligned} \tag{15}$$

By equating the two charges, i.e. setting $Q_1 = Q_2$, we get the off-set transferred at the output:

$$\frac{C_1 + C_2}{1 + 1/A_{dc}} \cdot V_{os} = V_{os}(C_1 + C_2) - V_{out} \cdot C_2 - V_{out} \cdot \frac{C_1 + C_2}{A_{dc}}; \tag{16}$$

which leads to:

$$V_{out} = \frac{C_1 + C_2}{\left(1 + \frac{1}{A_{dc}}\right) \cdot (A_{dc} \cdot C_2 + C_1 + C_2)} \cdot V_{os}. \tag{17}$$

Finally, in order to get the input referred residual off-set voltage, we dived by $|A_{v,non\text{-}id}|$:

$$V_{os\text{-}res} = \frac{V_{out}}{|A_{v,non\text{-}id}|} = \frac{\frac{C_1+C_2}{\left(1+\frac{1}{A_{dc}}\right)\cdot(A_{dc}\cdot C_2+C_1+C_2)}\cdot V_{os}}{\frac{C_2}{C_1+\frac{C_1+C_2}{A_{dc}}}} = \frac{C_1+C_2}{C_1\cdot(A_{dc}+1)}\cdot V_{os} = \frac{1}{A_{dc}+1}\cdot\left(1+\frac{1}{|A_v|}\right)\cdot V_{os} \quad (18)$$

Problem 2 The Chopper Stabilization (CS) technique

Consider the Chopper Stabilization (CHS) Principle chain shown in Fig. 3 and consider the amplifier DC gain $A_{dc} = 80\,\mathrm{dB}$, the input signal amplitude $A_{in} = 10\,\mathrm{mV_{0\text{-}P}}$, the flicker corner frequency $f_k = 10\,\mathrm{kHz}$, the amplifier off-set voltage $V_{os} = 0.1\,\mathrm{mV}$, $S_0 = 50\,\mathrm{nV}/\sqrt{\mathrm{Hz}}$

- Calculate the equivalent DC gain;

The gain is defined in dB. In convert it into linear, we do $A_{lin} = 10^{80/20} = 10^4 = 1000$.

$A_{eq} = \frac{8}{\pi^2}\cdot A_{lin} \approx 0.8\cdot A_{lin} = 0.8\cdot 10000 = 8000$.

- Chose a proper chopping period f_{chop} so to have the total residual noise in the baseband around $92.5\,\mathrm{nV}/\sqrt{\mathrm{Hz}}$.

Since $S_{SC} \approx S_0(1+0.85 f_k/f_{chop})$ then it results $f_{chop} \approx f_k = 10\,\mathrm{kHz}$.