# Smart Sensors for IoT
## Exercise 2 - 27.09.2023

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## Problem 1 Error comparison

A given sensor has a specified linearity error $\varepsilon_1$ of 1% of the reading plus 0.1% ($\varepsilon_{\text{FSO},1}$) of the full-scale output (FSO), which represents a systematic error. A second sensor having the same measurement range has a specified error $\varepsilon_2$ of 0.5% of the reading plus $\varepsilon_{\text{FSO},2} = 0.2\%$ FSO (systematic error).

- For what range of values is the first sensor more accurate than the second one?
- If the second sensor had a measurement range twice that of the first one, for what range of values would it be the more accurate?

## Problem 2 Systematic errors

In order to measure the drop in voltage across a resistor, we consider two alternative methods: (1) Use a voltmeter, whose accuracy is about 0.1% of the reading. (2) Use an ammeter, whose accuracy is also about 0.1% of the reading and apply Ohm's law.

- If the resistor has 0.1% tolerance, which method is more accurate?

## Recall about accuracy and precision

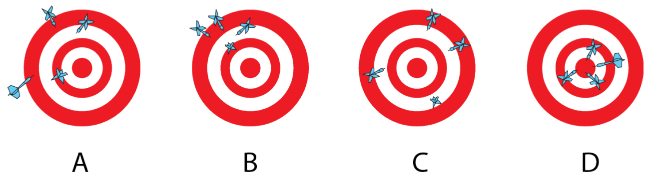


Figure 1: Dartboards showing different accuracy and precision scenarios.

A classic way of demonstrating the difference between precision and accuracy is with a dartboard. Think of the bulls-eye (center) of a dartboard as the true value. The closer darts land to the bulls-eye, the more accurate they are.

- If the darts are neither close to the bulls-eye, nor close to each other, there is neither accuracy, nor precision (Fig. 1A).

- If all of the darts land very close together, but far from the bulls-eye, there is precision, but not accuracy (Fig. 1B).
- If the darts are all about an equal distance from and spaced equally around the bulls-eye there is mathematical accuracy because the average of the darts is in the bulls-eye. This represents data that is accurate, but not precise (Fig. 1C). However, if you were actually playing darts this would not count as a bulls-eye!
- If the darts land close to the bulls-eye and close together, there is both accuracy and precision (Fig. 1D).

## Multiple Choice Questions

Select **one** answer for each of the questions below.

1. A student is asked to count the number of sensors in several arrays. On the specifications, it says that each array has 100 sensors. The student counts 100 of them on the first array; the other arrays either have too many or too few.

   How would you describe the array of sensors with 100 sensors inside?

   (a) Neither accurate nor precise.
   (b) Both accurate and precise.
   (c) Accurate, but the precision cannot be determined.
   (d) Precise, but not accurate.
   (e) Accurate, but not precise.

2. A student's scale measures the mass of objects as consistently 2 kg less than their actual mass. How would you describe the scale?

   (a) It is precise, but not accurate.
   (b) It is accurate, but not precise.
   (c) It is neither accurate nor precise.
   (d) Accuracy and precision are synonyms.
   (e) It is both accurate and precise.

3. An brand of thin copper wires claims that each wire has a mass of 25.5 g. After weighing three wires, a student observes the masses to be 25.5 g, 25.6 g, and 26.1 g.

   How can the student describe the accuracy and precision of the first wire he measured?

   (a) Accurate, but not precise.
   (b) Neither accurate nor precise.
   (c) There are insufficient data points to draw a conclusion.
   (d) Accurate and precise.
   (e) Precise, but not accurate.

4. Which of these is an example of high precision?

   (a) An archer hits the bulls-eye.
   (b) A student correctly calculates the acceleration due to gravity to be $9.8\,\mathrm{m/s^2}$
   (c) An archer hits the same spot on the target three times in a row.
   (d) A student tries to throw a pencil into the garbage can and makes it in.
   (e) A student correctly calculates the mass of an object to be 54 kg.

# Solutions to Exercise 3 (2021.10.06)

## Problem 1 Error comparison

A given sensor has a specified linearity error $\varepsilon_1$ of 1% of the reading plus 0.1% ($\varepsilon_{\text{FSO},1}$) of the full-scale output (FSO), which represents a systematic error. A second sensor having the same measurement range has a specified error $\varepsilon_2$ of 0.5% of the reading plus $\varepsilon_{\text{FSO},2} = 0.2\%$ FSO (systematic error).

- For what range of values is the first sensor more accurate than the second one?

Let us define the measured values as following:

$$\hat{x}_1 = x + \Delta x_1; \tag{1a}$$
$$\hat{x}_2 = x + \Delta x_2; \tag{1b}$$

where $\hat{x}_1$ and $\hat{x}_1$ are the measured values from the first and second sensors respectively, $x$ is the real value and $\Delta x_1$ and $\Delta x_2$ are the absolute errors for the two sensors, defined as:

$$\Delta x_1 = x \cdot \varepsilon_1 + \text{FSO} \cdot \varepsilon_{\text{FSO},1}; \tag{2a}$$
$$\Delta x_2 = x \cdot \varepsilon_2 + \text{FSO} \cdot \varepsilon_{\text{FSO},2}. \tag{2b}$$

In order for $\Delta x_1$ to be lower than $\Delta x_2$, by rearranging the previous equations, the following condition must be met:

$$x < \frac{\left(\varepsilon_{\text{FSO},1} - \varepsilon_{\text{FSO},2}\right)}{\varepsilon_1 - \varepsilon_2} \cdot \text{FSO} = 0.2 \cdot \text{FSO}. \tag{3}$$

Therefore the first sensor is more accurate than the second one if the measured values are less than the 20% of the full-scale input.

- If the second sensor had a measurement range twice that of the first one, for what range of values would it be the more accurate?

By adjusting (2b) with the twice the FSO and replacing it into (3), it yields to the first sensor being more accurate if the measured values are less than the 60% of the full-scale output.

## Problem 2 Systematic errors

In order to measure the drop in voltage across a resistor, we consider two alternative methods: (1) Use a voltmeter, whose accuracy is about 0.1% of the reading. (2) Use an ammeter, whose accuracy is also about 0.1% of the reading and apply Ohm's law.

- If the resistor has 0.1% tolerance, which method is more accurate?

We first differentiate Ohm's law to obtain:

$$dV = RdI + IdR. \tag{4}$$

Dividing each term by $V$ yields:

$$\frac{dV}{V} = \frac{RdI + IdR}{V} = \frac{RdI + IdR}{IR} = \frac{dI}{I} + \frac{dR}{R}. \tag{5}$$

For small variations, we can approximate differentials by increments to obtain:

$$\frac{\Delta V}{V} = \frac{\Delta I}{I} + \frac{\Delta R}{R}. \tag{6}$$

The relative uncertainty for the current and resistance add together. Therefore, the uncertainty in the voltage when measuring current is:

$$\frac{\Delta V}{V} = \frac{0.1}{100} + \frac{0.1}{100} = 0.2\%. \tag{7}$$

The uncertainty when measuring voltage directly is 0.1%, hence lower.

## Multiple Choice Questions

Select **one** answer for each of the questions below.

1. A student is asked to count the number of sensors in several arrays. On the specifications, it says that each array has 100 sensors. The student counts 100 of them on the first array; the other arrays either have too many or too few.

   How would you describe the array of sensors with 100 sensors inside?

   (a) Neither accurate nor precise.

   (b) Both accurate and precise.

   (c) Accurate, but the precision cannot be determined.

   (d) Precise, but not accurate.

   (e) Accurate, but not precise.

   **Comment:** This array is accurate because it provided the correct number of sensors, however, the process is not precise as the results were clearly not repeatable.

   Accuracy deals with how close the measurement got to the accepted measurement. Precision deals with how consistent the measurement is. The array with 100 sensors inside matched the claim made on the array, meaning it was accurate. It was not precise because the other measurements show that the number of sensors is variable.

2. A student's scale measures the mass of objects as consistently 2 kg less than their actual mass. How would you describe the scale?

(a) It is precise, but not accurate.

(b) It is accurate, but not precise.

(c) It is neither accurate nor precise.

(d) Accuracy and precision are synonyms.

(e) It is both accurate and precise.

**Comment:** Precision measures is how consistently a device records the same answer. In this case, the student's scale is ALWAYS 2 kg short. Even though it displays the wrong value, it is consistent. That means it is precise. Measuring a 10 kg object will always display a mass of 8 kg; the results are easily reproduced.

Accuracy is how well a device measures something against its accepted value. In this case, the student's scale is not accurate because it is always off by 2 kg.

3. An brand of thin copper wires claims that each wire has a mass of 25.5 g. After weighing three wires, a student observes the masses to be 25.5 g, 25.6 g, and 26.1 g.

   How can the student describe the accuracy and precision of the first wire he measured?

   (a) Accurate, but not precise.

   (b) Neither accurate nor precise.

   (c) There are insufficient data points to draw a conclusion.

   (d) Accurate and precise.

   (e) Precise, but not accurate.

   **Comment:** The claim for the mass of the first wire is accurate; the brand says there should be 25.5 g for each wire and the first one was 25.5 g heavy.

   The claim on the first wire is not precise, as the results are not replicated universally throughout the experiment. The masses of the wires fluctuate, with the average of all three bags equal to 25.7 g.

4. Which of these is an example of high precision?

   (a) An archer hits the bulls-eye.

   (b) A student correctly calculates the acceleration due to gravity to be $9.8\,\mathrm{m/s^2}$

   (c) An archer hits the same spot on the target three times in a row.

   (d) A student tries to throw a pencil into the garbage can and makes it in.

   (e) A student correctly calculates the mass of an object to be 54 kg.

   **Comment:** Precision is a measure of reproducibility. If multiple trials produce the same result each time with minimal deviation, then the experiment has high precision. This is true even if the results are not true to the theoretical predictions; an experiment can have high precision with low accuracy.

   In contrast, accuracy is the measure of difference between a calculated value and the true value of a measurement. High accuracy demands that the experimental result be equal to the theoretical result.

   An archer hitting a bulls-eye is an example of high accuracy, while an archer hitting the same spot on the bulls-eye three times would be an example of high precision.