SMART SENSORS FOR IOT

Exercise: THERMOELECTRIC GENERATOR MODULE

We want to recover waste heat from the exhaust gas of an industrial machine using a thermoelectric generator (TEG). The harvested energy is stored to power an array of wireless sensors monitoring the operation of the tool. The TEG module has n=100 thermocouples connected in series, each one consisting of p-type and n-type thermoelements as depicted in figure 1. The hot junction of the module is maintained in contact with the machine shield at 250°C whereas the temperature of the cold junction is approximately 50°C. As an approximation, we assume similar material properties for the p and n type thermoelements: $\alpha_p = -\alpha_n = 163\,\mu V/K$, $\rho_p = \rho_n = 1.46 \cdot 10^{-3}\,\Omega cm$ and $k_p = k_n = 1.2 \cdot 10^{-2}\,W/cmK$, being respectively the Seebek coefficients, the resistivity and the thermal conductivity of the materials. The cross-sectional area and length of the thermoelements are $A_n = A_p = 11\,mm^2$ and $L_n = L_p = 4\,mm$.

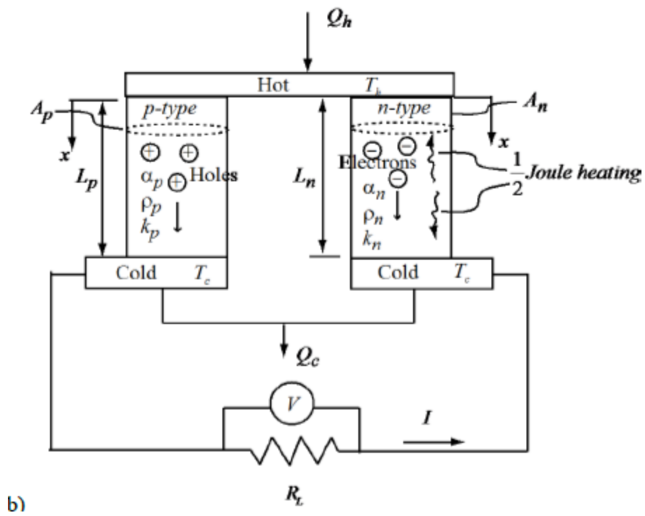

Figure 1: schematic of a single thermocouple included in the TEG module.

It can be demonstrated from thermodynamics considerations that the total output power of the entire module is expressed, in terms of single cell properties, as:

$$W_{module} = n[\alpha I(T_{Hot} - T_{Cold}) - I^2 R], \tag{1}$$

where $\alpha = \alpha_p - \alpha_n$ is the Seebeck coefficient of the thermocouple, I is the current flowing accross the load of each cell and R is the internal parasitic resistance of the thermoelements.

Thanks to the superposition of the electrical contribution from each thermocouple, the output power can also be expressed in terms of external load resistance R_L according to:

$$W_{module} = nI^2 R_L \tag{2}$$

Given the numerical data and the equations in the text, answer the following questions.

Questions:

1. Derive the equations for $V_{output}(I)$, $I(R_L, R)$) and $V_{output}(R_L, R)$
2. Derive the analytical expression of the output power for a given internal and load resistance (R, R_L)
3. Compute the internal resistance R and the thermal conductivity K of the thermocouple
4. For what value of R_L the output power is maximized? Compute the maximum output power
5. Compute the thermocouple figure of merit (reminder: zT factor, see the slides)

Solutions:

1) equating equations 1 and 2 with $W_{module} = IV_{output}$ we obtain the total voltage:

$$V_{output} = nIR_L = n[\alpha(T_{Hot} - T_{Cold}) - IR]. \tag{3}$$

From equation 3 the current is computed as:

$$I = \frac{\alpha(T_{Hot} - T_{Cold})}{R_L + R} \tag{4}$$

and combining the two equations we get

$$V_{output} = \frac{n\alpha(T_{Hot} - T_{Cold})}{R_L/R + 1}\frac{R_L}{R} \tag{5}$$

2) we can compute the output power combining equation 4 with equation 2:

$$W_{module} = \frac{n\alpha^2(T_{Hot} - T_{Cold})^2}{R}\frac{R_L/R}{(R_L/R + 1)^2} \tag{6}$$

3) since the two materials have similar coefficients, we have that

$$R = \rho L/A = 0.01\ \Omega,\ K = kA/L = 0.0066\ W/K \tag{7}$$

where $\rho = \rho_p + \rho_n$ and $k = k_p + k_n$.

4) The maximum power is attained by differentiating equation 5 with respect to the ratio of the resistances and setting the result to zero. However, we know from the course that the value is maximized when the load resistance is matched with the internal one (i.e., $R_L = R$). The maximum output power is therefore:

$$W_{max} = \frac{n\alpha^2(T_{Hot} - T_{Cold})^2}{4R} \approx 10\ W \tag{8}$$

5) The figure of merit is defined as zT, where:

$$Z = \frac{\alpha^2}{\rho k} = 1.5 \cdot 10^{-3}\ K^{-1} \tag{9}$$

and $T = T_{average} = (T_{Hot} + T_{Cold})/2 = 1\ K$. The result is FoM = zT = 0.64.