Smart Sensors for the IoT

4. Operational Amplifiers (OPAMPs)

Christian Enz

Integrated Circuits Lab (ICLAB), Institute of Microengineering (IMT), School of Engineering (STI)

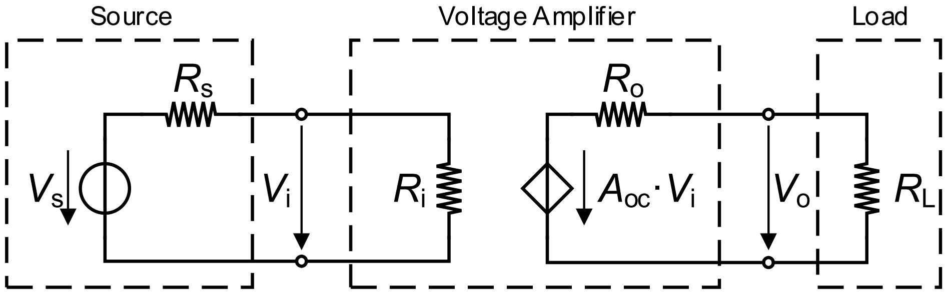
Swiss Federal Institute of Technology, Lausanne (EPFL), Switzerland

EPFL

Outline

- **OPAMPs fundamentals**
- Basic OPAMPs configurations
- OPAMPs non-idealities (bandwidth, CMRR, PSRR, offset, linearity, noise,…)
- OPAMPs macro-models
- Instrumentation amplifiers
- RC-active filters
- Operational transconductance amplifiers (OTAs)

Linear Voltage Amplifier



$$V_o = \frac{R_L}{R_o + R_L} \cdot A_{oc} \cdot V_i$$

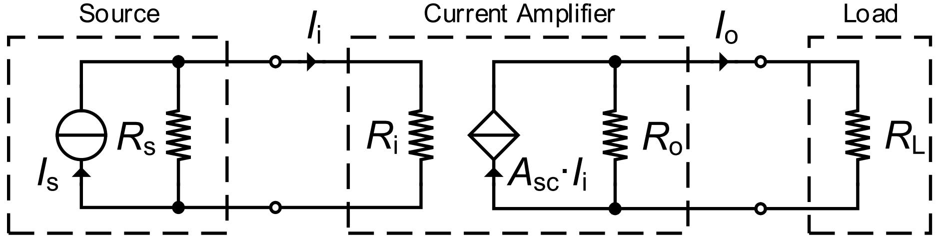
$$= \frac{R_L}{R_o + R_L} \cdot A_{oc} \cdot \frac{R_i}{R_s + R_i} \cdot V_s$$

$$A_v = \frac{V_o}{V_s} = \frac{R_i}{R_s + R_i} \cdot A_{oc} \cdot \frac{R_L}{R_o + R_L} = A_{oc}$$ for $R_i = \infty$ and $R_o = 0$

- A linear voltage amplifier is defined as a two-port device delivering an output voltage $V_o = A_{oc} \cdot V_i$ that is proportional to the input voltage V_i where A_{oc} is the **open-circuit voltage gain**
- It can be modelled by a voltage-controlled voltage source (VCVS) along with an **output series resistance** R_o
- The input port usually plays a purely passive role and is simply modeled by an **input resistance** R_i
- Loading is undesirable because it lowers the overall gain and makes it dependent on the particular input source and output load

S. Franco, *Design with Operational Amplifiers and Analog Integrated Circuits*, 4th ed., Mc Graw-Hill, 2015.

Linear Current Amplifier

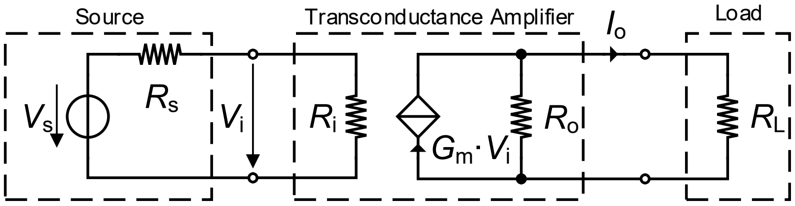


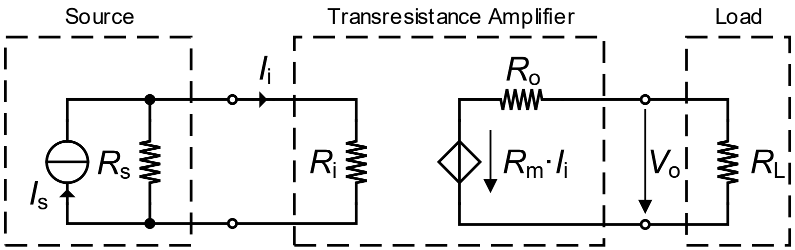
$$A_i = \frac{I_o}{I_s} = \frac{R_s}{R_s + R_i} \cdot A_{sc} \cdot \frac{R_o}{R_o + R_L} = A_{sc} \quad \text{for } R_i = 0 \text{ and } R_o = \infty$$

- A linear current amplifier is defined as a two-port device delivering an output current $I_o = A_{sc} \cdot I_i$ that is proportional to the input current I_i where A_{sc} is the **short-circuit current gain**
- It can be modelled by a current controlled current source (CCCS) along with an **output parallel resistance** R_o
- To eliminate loading, an ideal current amplifier has $R_i = 0$ and $R_o = \infty$, exactly the opposite of the ideal voltage amplifier

📖 S. Franco, *Design with Operational Amplifiers and Analog Integrated Circuits*, 4th ed., Mc Graw-Hill, 2015.

Linear Transconductance and Transresistance Amplifiers



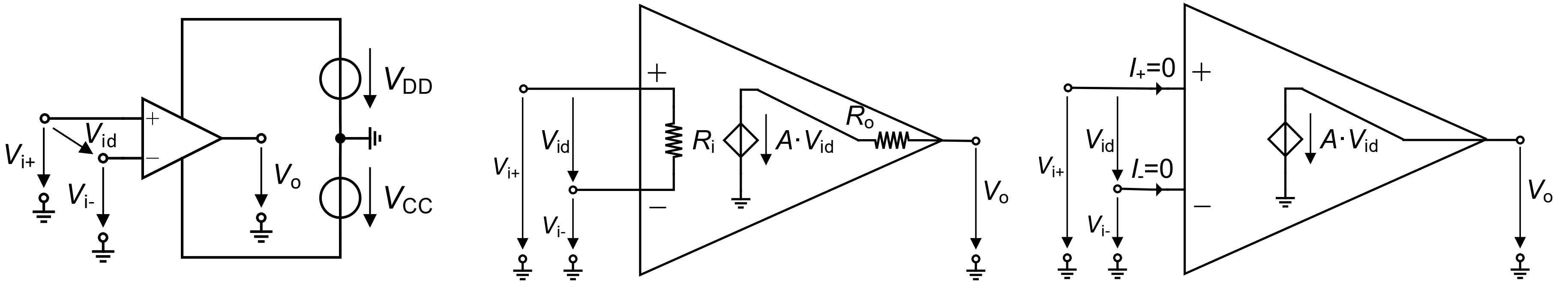


- An amplifier whose input is a voltage V_i and whose output is a current I_o is called a **transconductance amplifier**
- The dependent source is a voltage controlled current source (VCCS) of value G_m
- To avoid loading, an ideal transconductance amplifier has $R_i = \infty$ and $R_o = \infty$

- An amplifier whose input is a current I_i and whose output is a voltage V_o is called a **transresistance amplifier**
- It is modelled by a current-controlled voltage source (CCVS) of value R_m
- Ideally, such an amplifier has $R_i = 0$ and $R_o = 0$, the opposite of the transconductance amplifier

S. Franco, *Design with Operational Amplifiers and Analog Integrated Circuits*, 4th ed., Mc Graw-Hill, 2015.

The Ideal OPAMP



$$V_o = A \cdot (V_{i+} - V_{i-}) = A \cdot V_{id}$$

$$V_{id} = \frac{V_o}{A}$$

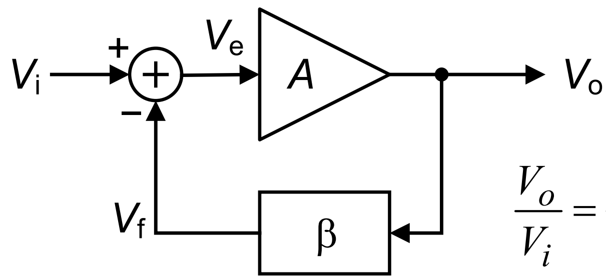
$$A \to \infty \quad R_i \to \infty \quad R_o = 0$$

$$I_+ = I_- = 0 \quad V_{id} \to 0$$

- The ideal OPAMP has infinite input impedance R_i, zero output impedance R_o and infinite open-loop gain A
- Since the gain is infinite, the differential input voltage V_{id} is zero whatever the value of the output voltage V_o
- Since the positive input is often connected to ground, the negative input is often called a **virtual ground**

S. Franco, *Design with Operational Amplifiers and Analog Integrated Circuits*, 4th ed., Mc Graw-Hill, 2015.

Basic Feedback Concept



$$V_f = \beta \cdot V_o$$
$$V_e = V_i - V_f = V_i - \beta \cdot V_o$$
$$V_o = A \cdot V_e = A \cdot (V_i - \beta \cdot V_o)$$

$$\frac{V_o}{V_i} = \frac{A}{1+\beta \cdot A} \cong \frac{1}{\beta} \quad \text{for } \beta \cdot A \gg 1$$

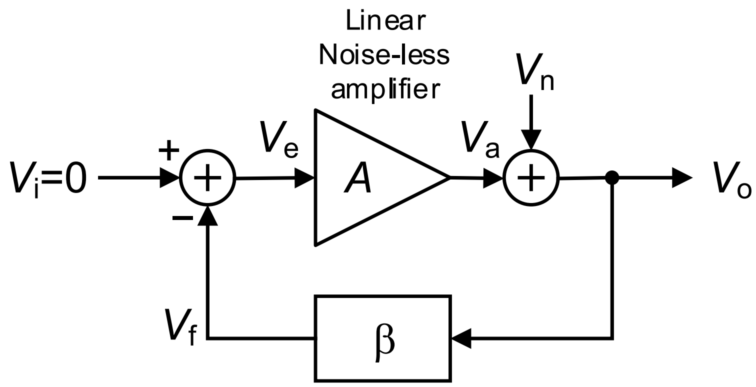
$$\frac{V_e}{V_i} = \frac{1}{1+\beta \cdot A} \cong \frac{1}{\beta \cdot A} \ll 1 \quad \text{for } \beta \cdot A \gg 1$$

- The **closed-loop gain** is given by

$$\frac{V_o}{V_i} = \frac{A}{1+\beta \cdot A} = \frac{1}{\beta} \cdot \frac{\beta \cdot A}{1+\beta \cdot A} = \frac{1}{\beta} \cdot \frac{T}{1+T} \cong \frac{1}{\beta} \quad \text{for } T = \beta \cdot A \gg 1$$

- where $T = \beta \cdot A$ is the **loop-gain** and $1 + T = 1 + \beta \cdot A$ the **feedback factor**
- Usually the feedback network is implemented by a passive network with a well-controlled **feedback gain β**
- On the other hand the forward gain A is not well-controlled. However if the loop-gain is much larger than 1, the closed-loop gain only depends on the well-controlled feedback gain β

Effect of Feedback on Noise



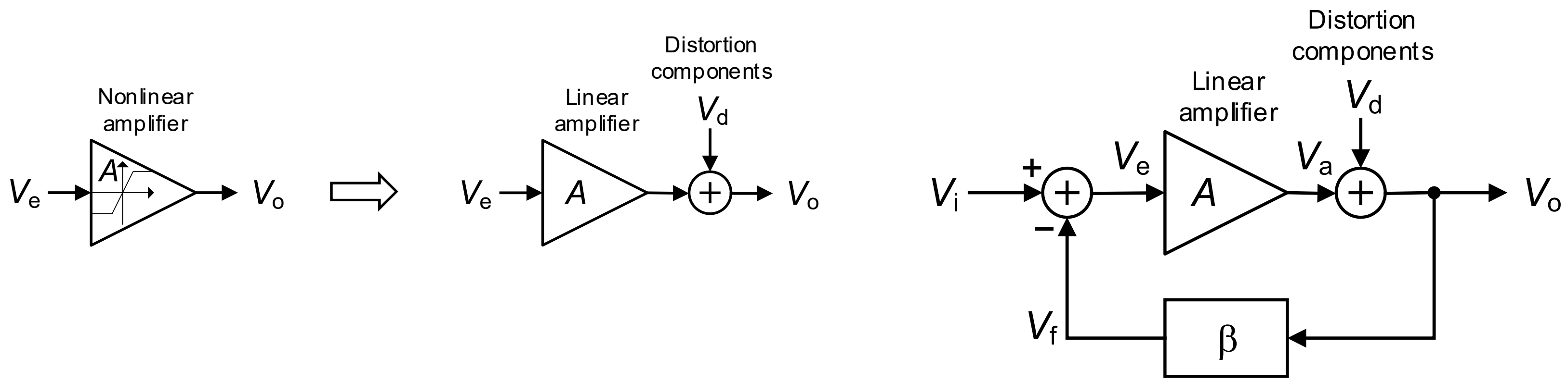
$$V_e = -V_f = -\beta \cdot V_o$$
$$V_a = A \cdot V_e = -A \cdot \beta \cdot V_o$$
$$V_o = V_a + V_n = V_n - A \cdot \beta \cdot V_o$$

- Assume there is no input ($V_i = 0$) and the noisy linear amplifier is modeled by a noiseless amplifier with additive noise V_n at the output
- The noise at the output is then given by

$$V_o = \frac{V_n}{1+\beta \cdot A} \cong \frac{V_n}{\beta \cdot A} \ll V_n \quad \text{for } \beta \cdot A \gg 1$$

- The amplifier output noise is **reduced by the feedback factor $1 + \beta \cdot A$** or the loop-gain $\beta \cdot A$ for $\beta \cdot A \gg 1$

Effect of Feedback on Distortion



- The **nonlinearity** of the amplifier (e.g. saturation characteristic) can be modelled to the first order by a linear amplifier to which the harmonics are added at the output
- The output voltage is then given by the sum of the **fundamental component** and the **harmonics** as

$$V_o = \underbrace{\frac{A}{1+\beta \cdot A} \cdot V_i}_{Fundamental} + \underbrace{\frac{1}{1+\beta \cdot A} \cdot V_d}_{harmonics}$$

- The first term corresponds to the linear signal whereas the second term represents the distortion at the output
- The distortion components are **reduced by the feedback factor $1+\beta \cdot A$**

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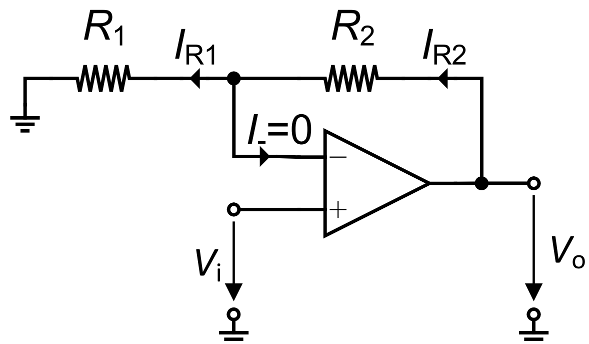
The Non-inverting Amplifier



- The output voltage is given by $V_o = A \cdot V_{id} = A \cdot (V_i - R_1 \cdot I_{R1})$
- Since for the ideal OPAMP the input current can be considered as null $I_{R1} = I_{R2}$
- The output voltage is also given by $V_o = (R_1 + R_2) \cdot I_{R1}$ from which we deduce $I_{R1} = V_o/(R_1 + R_2)$. The output voltage then writes

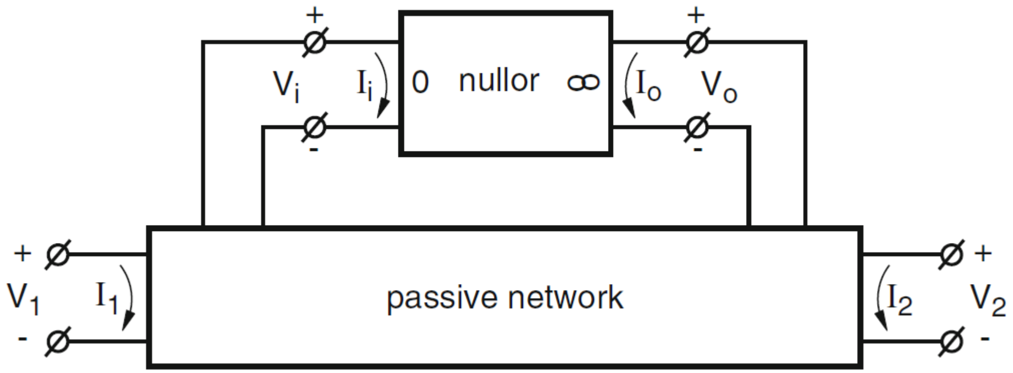
$$V_o = A \cdot \left(V_i - \frac{R_1}{R_1 + R_2} \cdot V_o\right) \Longrightarrow V_o \cdot \left(1 + \frac{A \cdot R_1}{R_1 + R_2}\right) = A \cdot V_i$$

- The voltage gain is then given by

$$A_v \triangleq \frac{V_o}{V_i} = \frac{1+R_2/R_1}{1+\frac{1+R_2/R_1}{A}} \cong 1 + \frac{R_2}{R_1} \text{ for } A \gg 1 + \frac{R_2}{R_1}$$

- The voltage gain is set by the ratio of resistances R_2 to R_1 and hence independent of the OPAMP gain

The Nullor – A Circuit Theory Equivalent Circuit



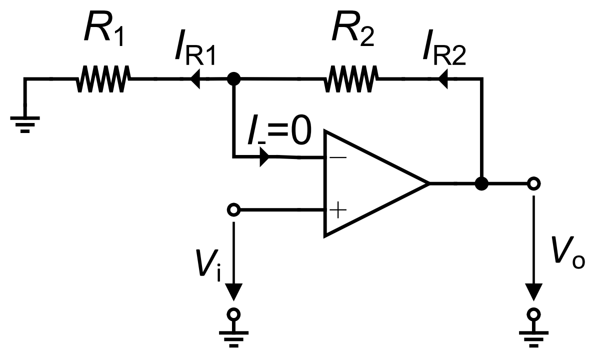
$$V_i = 0$$

$$I_i = 0$$

- In 1954, Tellegen introduced the concept of a universal active network element under the name of "ideal amplifier", but the name "nullor," generally accepted now, was given to it by Carlin in 1964
- The **nullor** is defined as a two-port network element whose ports are called input and output ports and whose input voltage V_i and input current I_i are both zero
- The output voltage V_o and the output current I_o will be determined by the passive network elements in such a way that the input requirements $V_i = 0$ and $I_i = 0$ are both satisfied

J. Huijsing, *Operational Amplifiers - Theory and Design*, 3rd ed., Springer, 2017.

The Non-inverting Amplifier – Ideal OPAMP

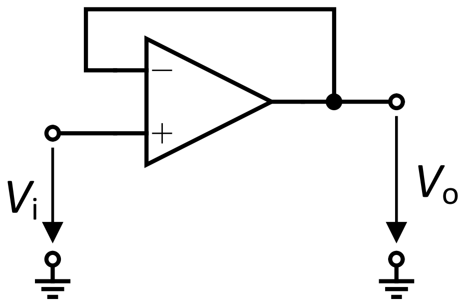


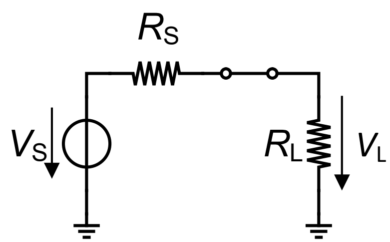
- Derivation is simplified when assuming an ideal OPAMP with infinite gain
- In this case $V_{id} = 0$ and $I_{-} = 0$ and we can write $V_{R1} = V_i = R_1 \cdot I_{R1}$ from which we get the current flowing in R_1 and R_2 $I_{R1} = V_i / R_1$
- The output voltage is given by $V_o = R_2 \cdot I_{R1} + V_i = (R_2/R_1 + 1) \cdot V_i$
- The voltage gain is then simply given by

$$A_v \triangleq \frac{V_o}{V_i} = 1 + \frac{R_2}{R_1}$$

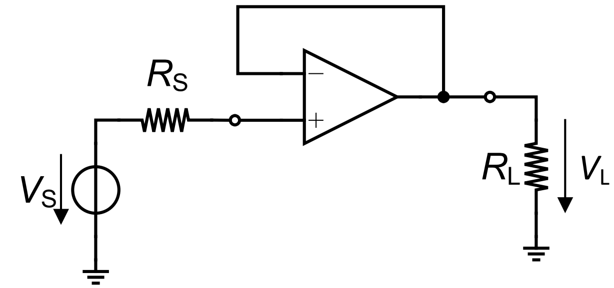
- which is identical to the previous result assuming $A \gg 1 + R_2/R_1$

The Voltage Follower





$$V_L = \frac{R_L}{R_S + R_L} \cdot V_S < V_S$$



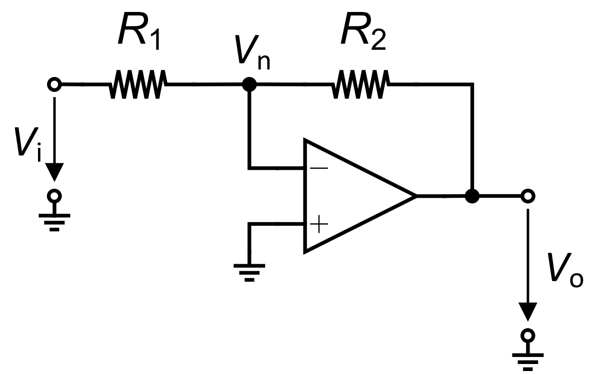
$$V_L = V_S$$

- Letting $R_1 \to \infty$ and $R_2 = 0$ in the noninverting amplifier turns it into the unity-gain amplifier, or voltage follower having the following closed-loop parameters

$$A_v = 1, R_i = \infty \text{ and } R_o = 0$$

- Acts as a resistance transformer, since looking into its input we see an open circuit, but looking into its output we see a short circuit to a source of value $V_o = V_i$
- No voltage loss because the voltage across the load resistance R_L is equal to the source voltage
- No current drawn from the source

The Inverting Amplifier

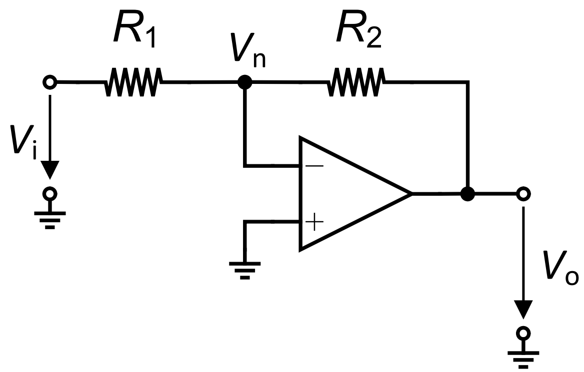


- The output voltage is given by $V_o = A \cdot V_{id} = -A \cdot V_n$
- Since for the ideal OPAMP the input current can be considered as null $I_{R1} = I_{R2}$
- The output voltage is also given by $V_o - V_i = (R_1 + R_2) \cdot I_{R1}$ from which we deduce $I_{R1} = (V_o - V_i)/(R_1 + R_2)$
- The virtual ground voltage is given by $V_n = R_1 \cdot I_{R1} + V_i = \frac{R_1}{R_1+R_2} \cdot (V_o - V_i) + V_i$
- Replacing in the above equation and solving for the voltage gain results in

$$A_v \triangleq \frac{V_o}{V_i} = \frac{-R_2/R_1}{1+\frac{1+R_2/R_1}{A}} \cong -\frac{R_2}{R_1} \text{ for } A \gg 1 + \frac{R_2}{R_1}$$

- Again, the voltage gain is set by the ratio of resistances R_2 to R_1 and hence independent of the OPAMP gain

The Inverting Amplifier – Ideal OPAMP



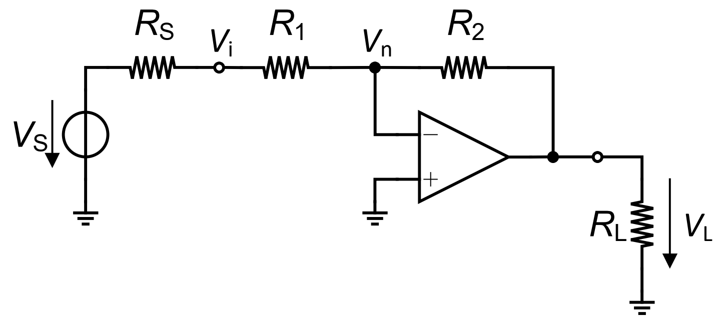
- The analysis is much simpler when considering an ideal OPAMP with infinite gain
- The voltage at the virtual ground is then null $V_n = 0$
- Since the current flowing into the negative input of the OPAMP is zero $I_- = 0$, the sum of the current at the virtual ground is given by

$$\frac{V_i}{R_1} + \frac{V_o}{R_2} = 0$$

- From which we get the voltage gain

$$A_v \triangleq \frac{V_o}{V_i} = -\frac{R_2}{R_1}$$

Effect of Source Loading



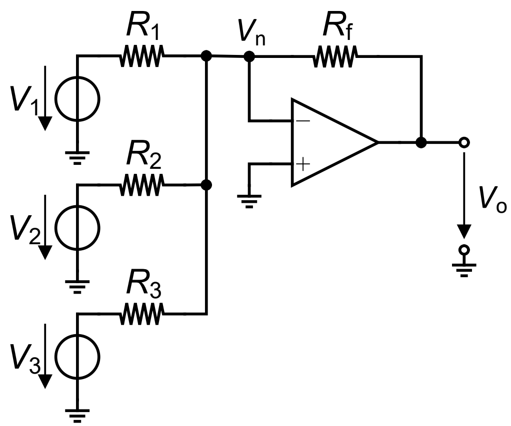
- Unlike its noninverting counterpart, the inverting amplifier will **load** the input source if the source is non ideal
- Assuming the OPAMP is ideal with infinite gain, the voltage at the virtual ground remains zero $V_n = 0$
- The sum of the currents at the virtual ground node is given by

$$\frac{V_S}{R_S + R_1} + \frac{V_L}{R_2} = 0$$

- From which we get the voltage gain including the effect of source loading

$$A_v \cong \frac{V_L}{V_S} = -\frac{R_2}{R_S + R_1}$$

The Summing Amplifier



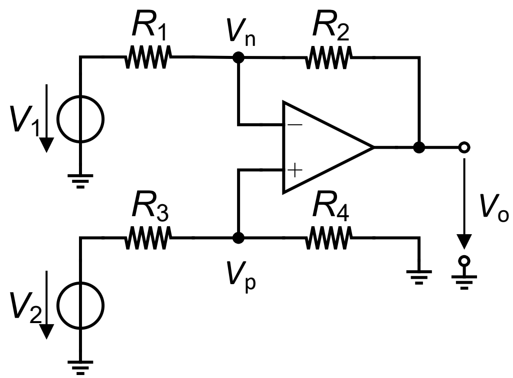
- Considering again an ideal OPAMP with infinite gain, the voltage at the virtual node is zero $V_n = 0$
- We can write the sum of the currents at the virtual ground node

$$\frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3} + \frac{V_o}{R_f} = 0$$

- The output voltage is then given by

$$V_o = -\frac{R_f}{R_1} \cdot V_1 - \frac{R_f}{R_2} \cdot V_2 - \frac{R_f}{R_3} \cdot V_3$$

The Difference Amplifier



- Setting $V_2 = 0$ makes $V_p = 0$ corresponding to inverting amplifier. The output voltage due to V_1 is then given by $V_{o1} = -R_2/R_1 \cdot V_1$
- Setting $V_1 = 0$ makes the amplifier operate like a non-inverting amplifier. The output voltage due to V_2 is then given by $V_{o2} = (1 + R_2/R_1) \cdot V_p = (1 + R_2/R_1) \cdot R_4/(R_3 + R_4) \cdot V_2$
- The output voltage is obtained by superposition as

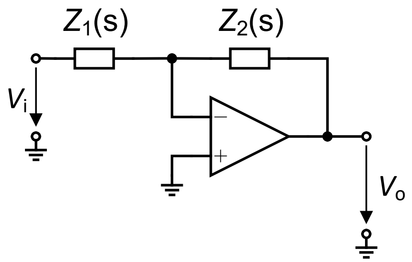
$$V_o = -\frac{R_2}{R_1} \cdot V_1 + \frac{1 + R_2/R_1}{1 + R_3/R_4} \cdot V_2$$

- A particular case is obtained by setting $R_3/R_4 = R_1/R_2$, leading to

$$V_o = \frac{R_2}{R_1} \cdot (V_2 - V_1)$$

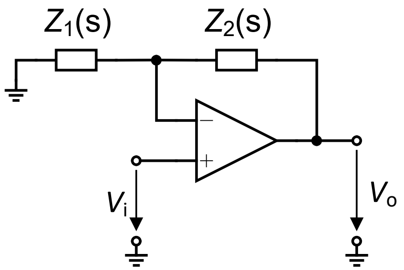
Basic Circuits Combining an OPAMP and Impedances

Inverting amplifier



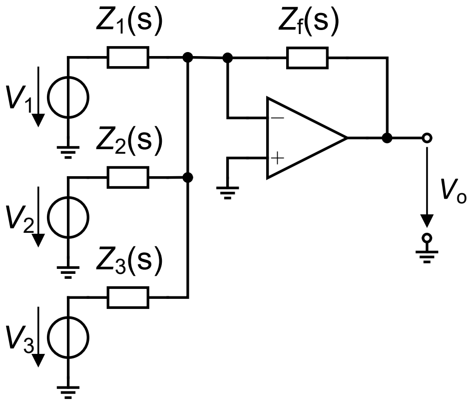
$$A_v(s) \triangleq \frac{V_o(s)}{V_i(s)} = -\frac{Z_2(s)}{Z_1(s)}$$

Non-inverting amplifier



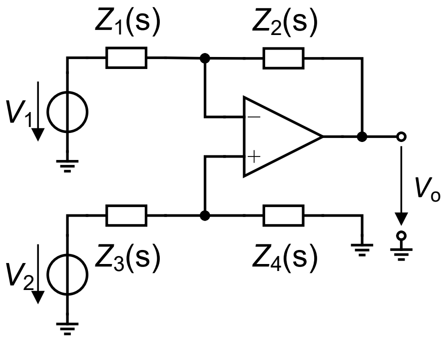
$$A_v(s) \triangleq \frac{V_o(s)}{V_i(s)} = 1 + \frac{Z_2(s)}{Z_1(s)}$$

Summing amplifier



$$V_o(s) = -\frac{Z_f(s)}{Z_1(s)}V_1(s) - \frac{Z_f(s)}{Z_2(s)}V_2(s) - \frac{Z_f(s)}{Z_3(s)}V_3(s)$$

Difference amplifier



$$V_o(s) = -\frac{Z_2(s)}{Z_1(s)}V_1(s) + \frac{1 + Z_2(s)/Z_1(s)}{1 + Z_3(s)/Z_4(s)}V_2(s)$$

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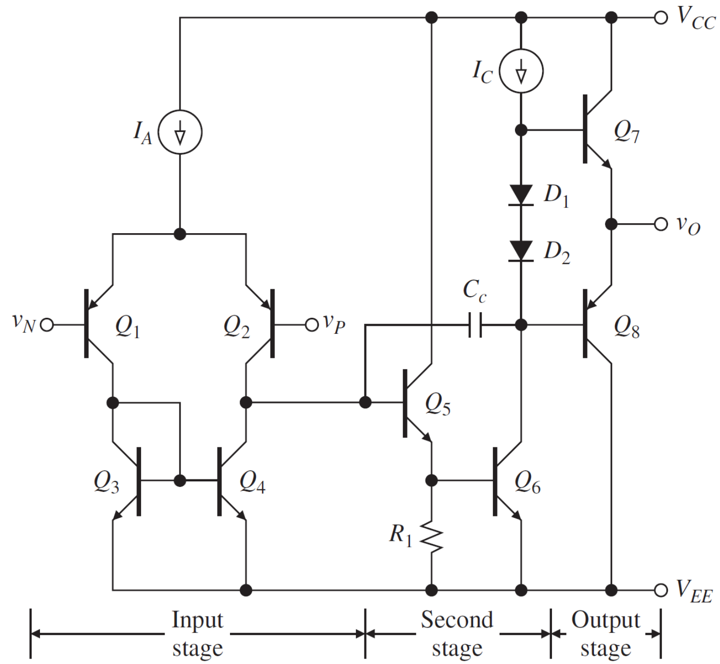
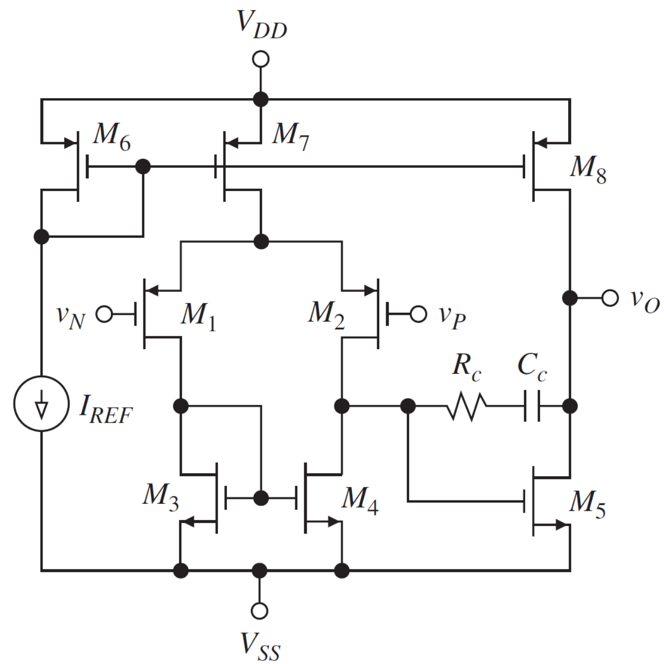
OPAMPs Simplified Schematics

Bipolar OPAMP



CMOS OPAMP

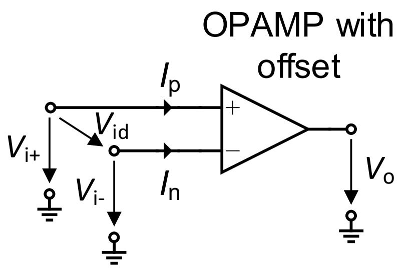


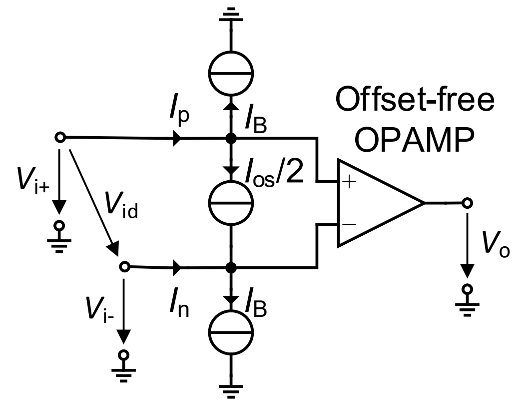
- High gain
- Input bias current
- Low output impedance
- Low flicker noise

- Medium gain
- No input current
- Medium output impedance
- Higher flicker noise

S. Franco, *Design with Operational Amplifiers and Analog Integrated Circuits*, 4th ed., Mc Graw-Hill, 2015.

Input Bias and Offset Currents



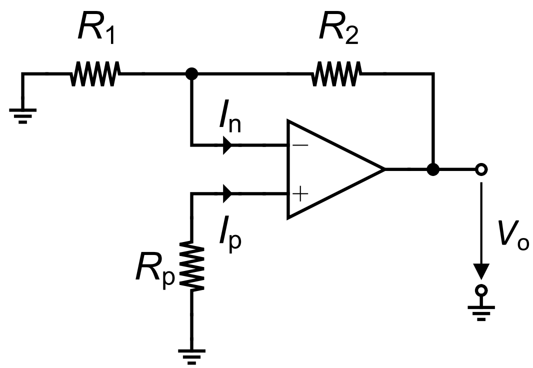


$$I_B \triangleq \frac{I_p + I_n}{2}$$

$$I_{os} \triangleq I_p - I_n$$

- Bipolar input stage suffer from non-zero input bias current
- Because of mismatch, I_p and I_n are usually not equal and suffer from an offset current $I_{os} \triangleq I_p - I_n$
- The non-ideal OPAMP can be modelled by an offset free OPAMP with 3 current sources, two current sources modeling the bias current $I_B \triangleq (I_p + I_n)/2$
- Usually the bias current I_B is much larger than the offset current I_{os} (typically one order of magnitude)

Effects of Input Bias and Offset Currents



- There are many circuits that, once their active inputs are set to zero, reduce to an equivalent circuit as shown above, including the inverting and noninverting amplifiers, the summing and difference amplifiers, I-V converters, and others
- The input bias currents will generate an output voltage given by

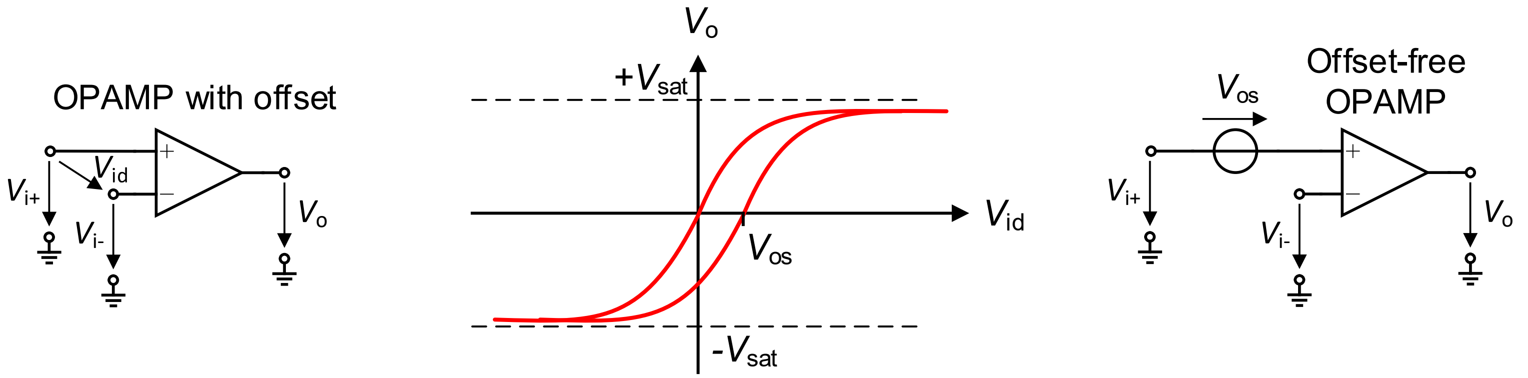
$$V_o = \left(1 + \frac{R_2}{R_1}\right) \cdot \left(R_{12} \cdot I_n - R_p \cdot I_p\right) \text{ where } R_{12} \triangleq R_1 \parallel R_2$$

- which can be rewritten in terms of I_B and I_{os}

$$V_o = \left(1 + \frac{R_2}{R_1}\right) \cdot \left[\left(R_{12} - R_p\right) \cdot I_B - \left(R_{12} + R_p\right) \cdot I_{os}/2\right]$$

- The effect of the bias current can be eliminated by choosing $R_p = R_{12}$ leaving only the effect of the input current mismatch I_{os}

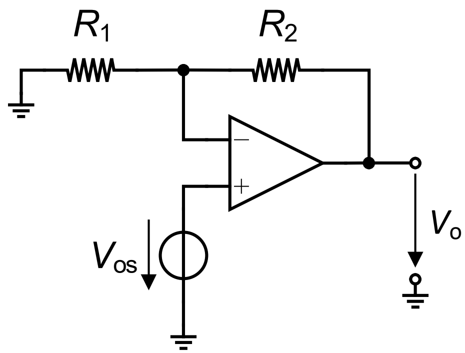
Offset Voltage



- Ideally, the output voltage should be equal to zero when setting the input differential voltage to zero
- However, because of the mismatch in the input stage (differential pair), the output voltage is not equal to zero
- The **offset voltage** is defined as the input differential voltage needed to bring the output voltage back to zero

$$V_o = A \cdot (V_{id} - V_{os}) = A \cdot (V_{i+} - V_{i-} - V_{os})$$

Effect of Input Offset Voltage

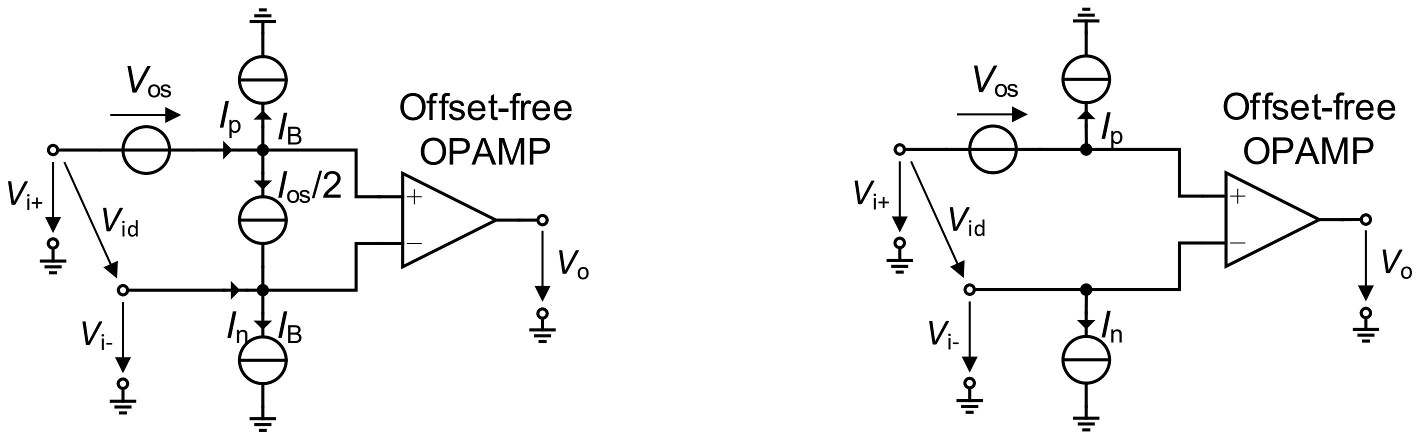


- The input-referred offset voltage is multiplied by the voltage gain to appear at the output as

$$V_o = \left(1 + \frac{R_2}{R_1}\right) \cdot V_{os}$$

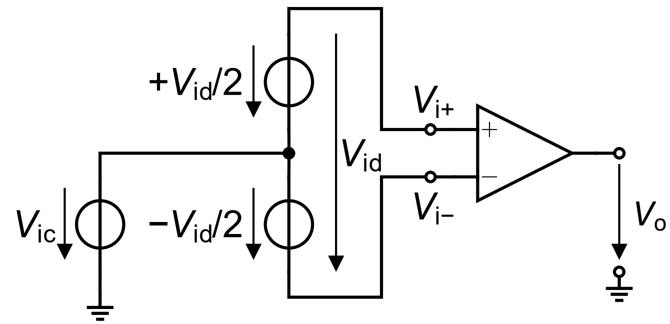
- Assume the offset is 10 mV, if the resistance ratio is set to 1000 the output voltage will be larger than 10 V, which may eventually drive the OPAMP into saturation!
- There are circuit techniques that will be discussed later that can cancel this offset voltage

Combining Current Bias, Current Offset and Voltage Offset



- The bias and offset currents can be combined with the offset voltage by using one of the above circuits

Common Mode Rejection Ratio (CMRR)



- Ideally the amplifier should only amplify the differential voltage $V_{id} \triangleq V_{i+} - V_{i-}$
- However, because of internal mismatches, it will also amplify the common-mode voltage $V_{ic} \triangleq (V_{i+} + V_{i-})/2$. The output voltage is then given by

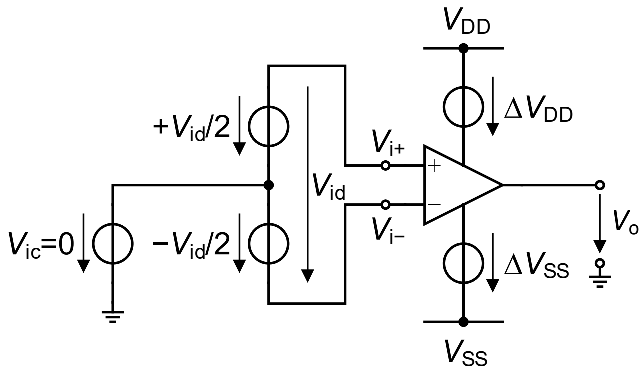
$$V_o = A_{dm} \cdot V_{id} + A_{cm} \cdot V_{ic} = A_{dm} \cdot \left(V_{id} + \frac{A_{cm}}{A_{dm}} \cdot V_{ic} \right) = A_{dm} \cdot \left(V_{id} + \frac{V_{ic}}{CMRR} \right)$$

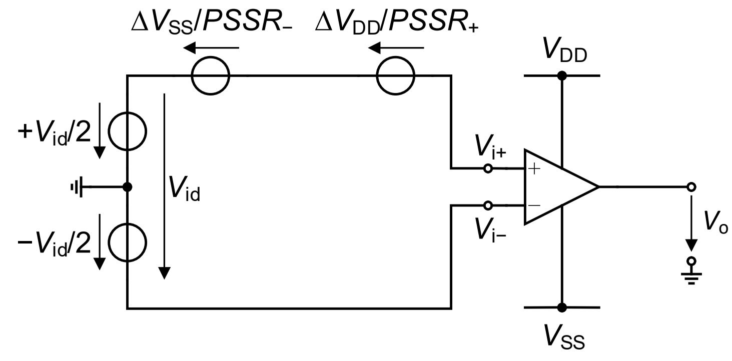
- Where A_{dm} is the **differential-mode gain** whereas A_{cm} is the **common-mode gain**
- The ability of the OPAMP to reject the common-voltage is defined as the **common-mode rejection ratio** (CMRR)

$$CMRR \triangleq \left| \frac{A_{dm}}{A_{cm}} \right|$$

- It represents the equivalent differential voltage that is needed to cancel the output voltage due to a certain input common-mode voltage

Power Supply Rejection Ratio (PSRR)





- Voltage variations on the positive and negative supplies ΔV_{DD} and ΔV_{SS} may also induce changes in the output voltage
- Assuming for simplicity that $V_{ic} = 0$, the output voltage change due to ΔV_{DD} and ΔV_{SS} is given as

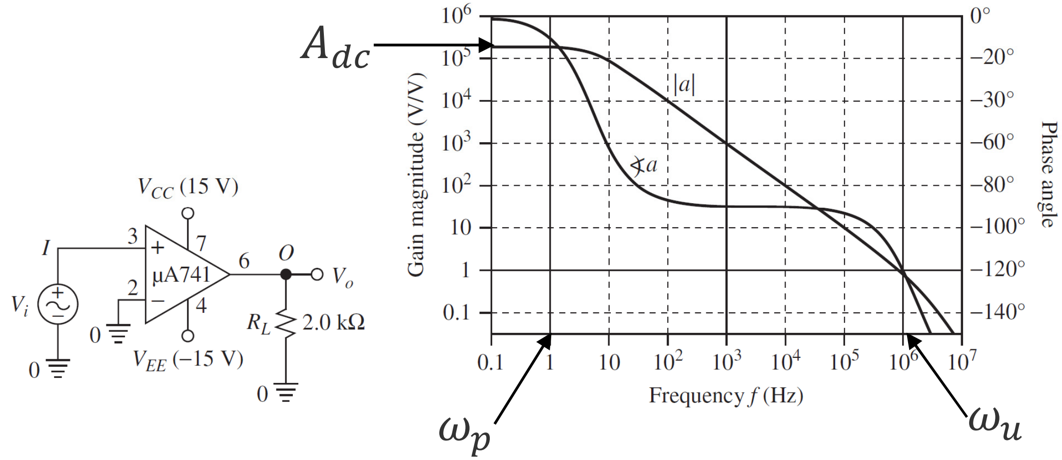
$$V_o = A_{dm} \cdot V_{id} + A_+ \cdot \Delta V_{DD} + A_- \cdot \Delta V_{SS} =$$

$$= A_{dm} \cdot \left(V_{id} + \frac{A_+}{A_{dm}} \cdot \Delta V_{DD} + \frac{A_-}{A_{dm}} \cdot \Delta V_{SS} \right) = A_{dm} \cdot \left(V_{id} + \frac{\Delta V_{DD}}{PSRR_+} + \frac{\Delta V_{SS}}{PSRR_-} \right)$$

- where A_+ and A_- are the small-signal gains from the positive and negative power-supplies to the output, respectively and

$$PSRR_+ \triangleq \left| \frac{A_{dm}}{A_+} \right| \text{ and } PSRR_- \triangleq \left| \frac{A_{dm}}{A_-} \right|$$

Frequency Response



- Practical OPAMPs provide high gain only from dc up to a given frequency, beyond which the gain decreases with frequency

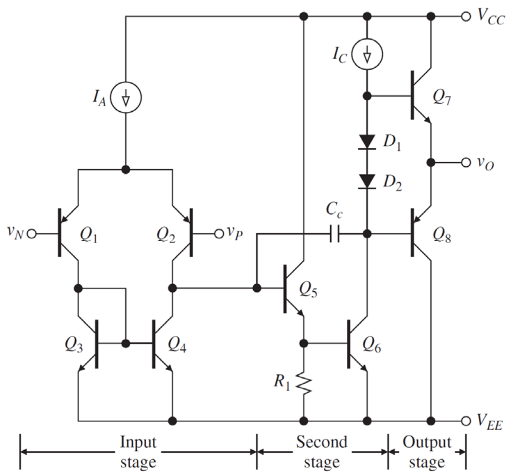
$$A(s) = \frac{A_{dc}}{1+s/\omega_p} = \frac{\omega_u}{s+\omega_p} \text{ with } \omega_u = A_{dc} \cdot \omega_p$$

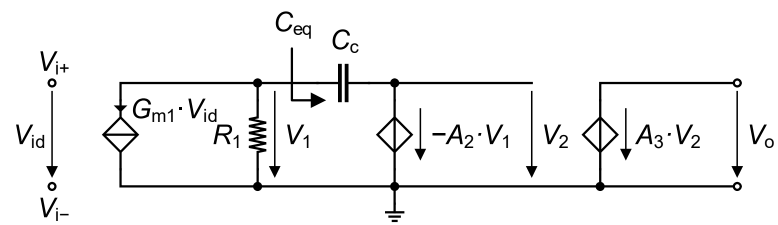
- Where A_{dc} is the **open-loop dc gain**, ω_p the **dominant pole** frequency and $\omega_u = A_{dc} \cdot \omega_p$ the **unity gain frequency** or **gain-bandwidth product**
- As shown above, in the frequency range from 10 Hz to 1 MHz, the OPAMP **behaves more like an integrator**

$$A(\omega) \cong \frac{\omega_u}{j\omega} \text{ for } \omega \gg \omega_p$$

📖 S. Franco, *Design with Operational Amplifiers and Analog Integrated Circuits*, 4th ed., Mc Graw-Hill, 2015.

Miller Compensation of Two-Stage OPAMP





- The transfer function is given by

$$A(s) = \frac{A_{dc}}{1 + s/\omega_p}$$

- With $A_{dc} = G_{m1} \cdot R_1 \cdot A_2 \cdot A_3 = G_{m1} \cdot R_1 \cdot A_2$ assuming that the gain of the output stage is unity ($A_3 = 1$)
- The dominant pole is given by $\omega_p = 1/(R_1 \cdot C_{eq})$ where C_{eq} is the Miller capacitance seen at the input of the 2nd stage $C_{eq} = (1 + A_2) \cdot C_c$ which takes advantage of the large gain of the 2nd-stage for achieving a large capacitance out of a small compensation capacitance C_c

S. Franco, *Design with Operational Amplifiers and Analog Integrated Circuits*, 4th ed., Mc Graw-Hill, 2015.

OPAMP Behaves More like an Integrator

- Practical OPAMPs provide high gain only from dc up to a given ω_p frequency, beyond which the gain decreases with frequency

$$A(s) = \frac{A_{dc}}{1 + s/\omega_p} = \frac{\omega_u}{s + \omega_p}$$

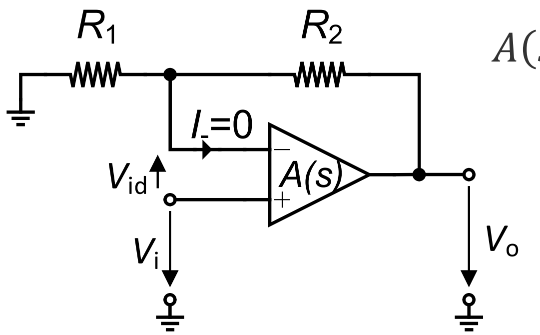
- where A_{dc} is the **dc open-loop gain**, ω_p the **dominant pole** frequency and $\omega_u = A_{dc} \cdot \omega_p$ is the **unity gain frequency** or **gain-bandwidth product (GBW)**
- As shown in the previous slide, in the frequency range from 10 Hz to 1 MHz, the OPAMP behaves more like an **integrator**

$$A(\omega) \cong \frac{\omega_u}{j\omega} \text{ for } \omega \gg \omega_p$$

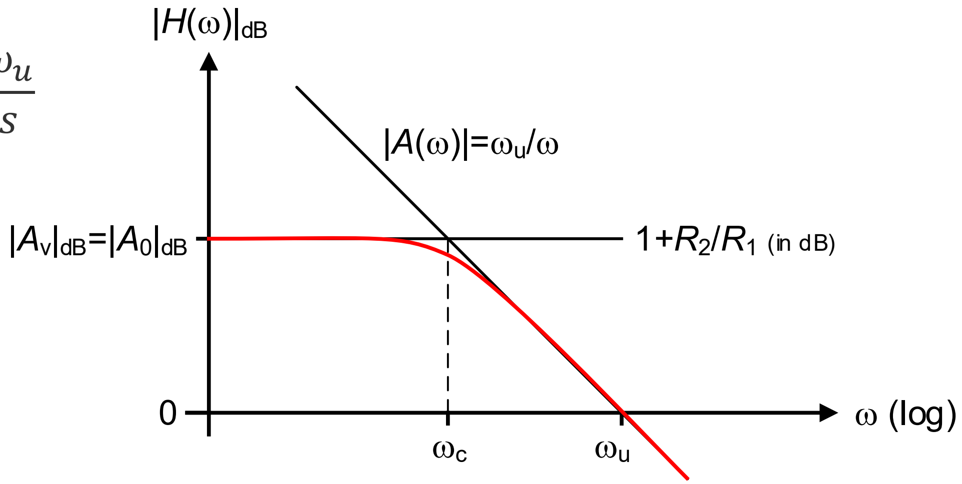
- The gain-bandwidth or unity gain frequency is a key parameter which should be large enough for the OPAMP to provide gain in the frequency band of interest

S. Franco, *Design with Operational Amplifiers and Analog Integrated Circuits*, 4th ed., Mc Graw-Hill, 2015.

Effect of Finite Gain-bandwidth Product on Noninverting Amplifier



$$A(s) = \frac{\omega_u}{s}$$

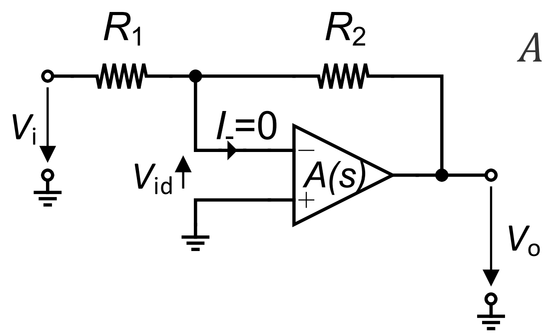


- We can reuse the previous result replacing the constant gain A by ω_u/s
- This results in an overall gain that becomes frequency dependent according to

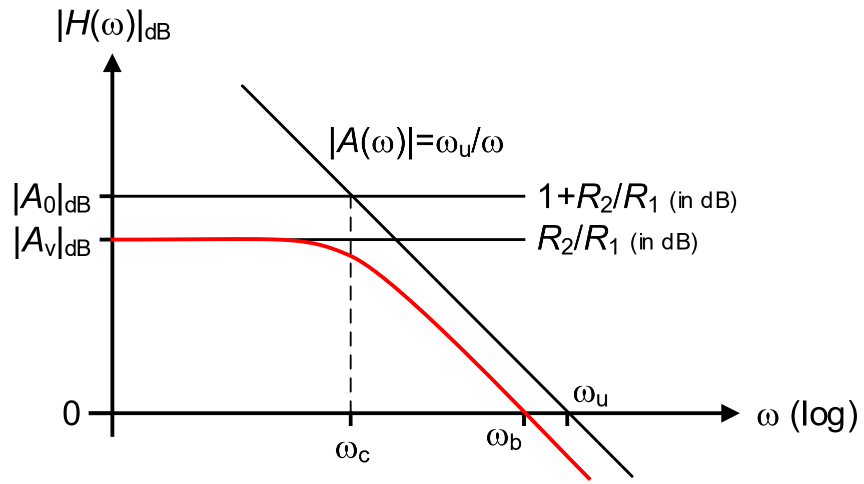
$$H(s) \triangleq \frac{V_o}{V_i} = \frac{A_v}{1+s/\omega_c} \text{ where } \omega_c = \frac{\omega_u}{A_0} \text{ and } A_v = A_0 = 1 + \frac{R_2}{R_1}$$

- At low frequency ($\omega \ll \omega_c = \omega_u/A_0$), the OPAMP gain A is larger than A_0 and the non-inverting amplifier gain $H(s)$ is equal to its ideal value $A_v = 1 + \frac{R_2}{R_1}$
- For frequency above ω_c, the OPAMP gain is not sufficient for its negative input to behave like a virtual ground and the amplifier gain is then limited by the OPAMP gain

Effect of Finite Gain-bandwidth Product on Inverting Amplifier



$$A(s) = \frac{\omega_u}{s}$$



- Similar analysis can be done for the inverting amplifier
- The input-output gain also becomes frequency dependent

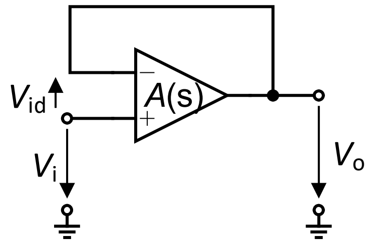
$$H(s) \triangleq \frac{V_o}{V_i} = \frac{A_v}{1+s/\omega_c} \text{ where } A_v = -\frac{R_2}{R_1},\ \omega_c = \frac{\omega_u}{A_0} \text{ and } A_0 = 1 + \frac{R_2}{R_1}$$

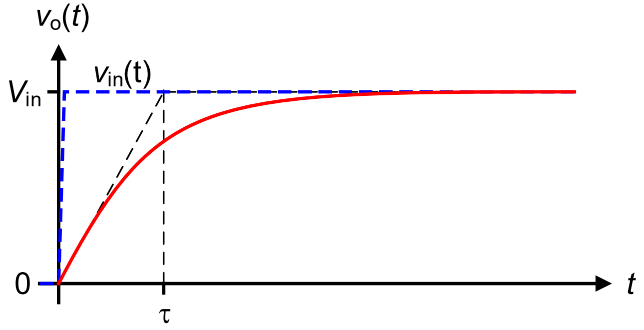
- At low frequency ($\omega \ll \omega_c = \omega_u/A_0$), the OPAMP gain A is larger than A_0 and the inverting amplifier gain $H(s)$ is equal to its ideal value $A_v = -R_2/R_1$
- For frequency above ω_c, the amplifier gain is then limited by the OPAMP open-loop gain

$$H(\omega) \cong \frac{\omega_u}{\omega}\frac{A_v}{A_0} \cong -\frac{\omega_u}{\omega} \text{ for } \frac{R_2}{R_1} \gg 1 \text{ (hence } A_0 \cong -A_v)$$

Voltage Follower Rise-time

$$A(s) = \frac{\omega_u}{s}$$





- Accounting for the **finite gain-bandwidth** product, the voltage follower gain becomes

$$H(s) \triangleq \frac{V_o}{V_i} = \frac{1}{1 + 1/A(s)} = \frac{1}{1 + s/\omega_u}$$

- The step response is then given by

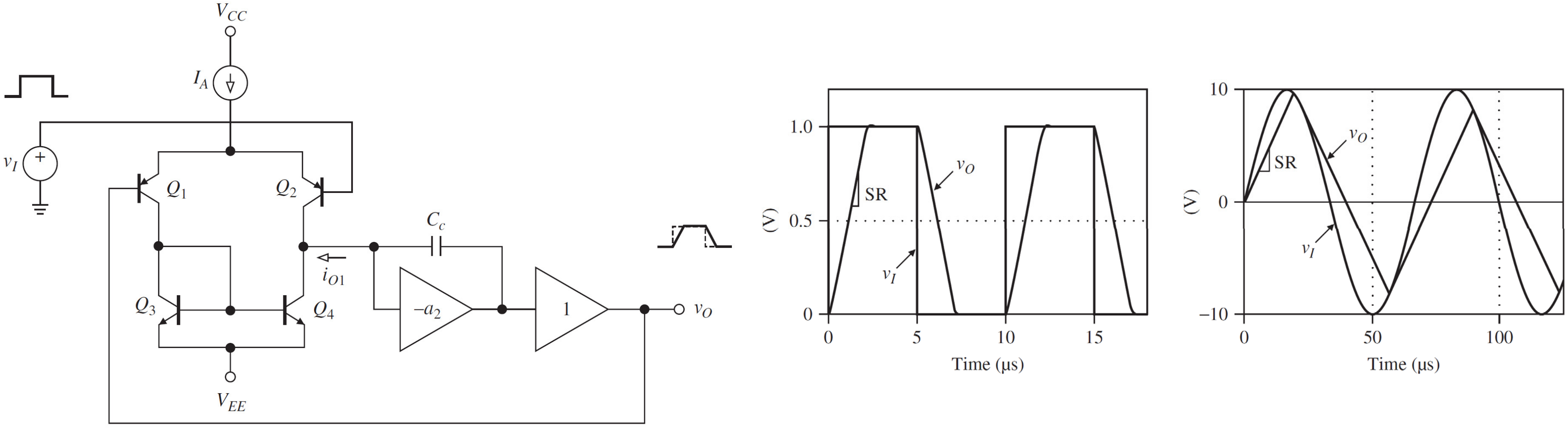
$$v_o(t) = V_{in}\left(1 - e^{-t/\tau}\right) \text{ where } \tau = 1/\omega_u$$

- The **rise-time** t_R is defined as the time it takes for v_o to swing from 10% to 90% of the final value V_{in}
- We easily find $t_{90\%} = -\tau \cdot ln(0.1)$ and $t_{10\%} = -\tau \cdot ln(0.9)$ and hence

$$t_R = \tau \cdot ln(9) = 2.2\,\tau = \frac{2.2}{\omega_u} = \frac{0.35}{f_u}$$

- which provides a link between the frequency-domain parameter f_u and the time domain parameter t_R

Slew-Rate



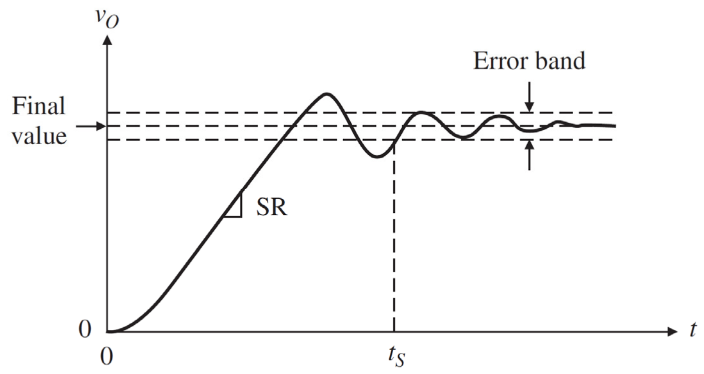
- For a **large input voltage** step, the differential pair will saturate and the maximum current available to charge the **compensation capacitance** C_c is equal to the constant bias current I_A
- The voltage can therefore not change faster then

$$SR \triangleq \left.\frac{dv_o}{dt}\right|_{max} = \frac{I_A}{C_c}$$

- which is limited by the bias current I_A

S. Franco, *Design with Operational Amplifiers and Analog Integrated Circuits*, 4th ed., Mc Graw-Hill, 2015.

Settling Time



- The **rise time t_R** and **slew rate SR** give an indication of how rapidly the output changes, respectively, under **small-signal** and **large-signal** conditions
- The **settling time t_S** is defined as the time it takes for the response to a **large input step** to settle and remain within a specified error band, symmetric about its final value
- Settling times are specified to accuracies of 0.1% and 0.01% of a 10-V input step
- The settling time t_S is comprised of an initial propagation delay due to higher-order poles, followed by an SR-limited transition to the vicinity of the final value, followed by a period to recover from the overload condition associated with the SR, and finally settle toward the final equilibrium value
- A fast t_R or a high SR does not necessarily guarantee a fast t_S

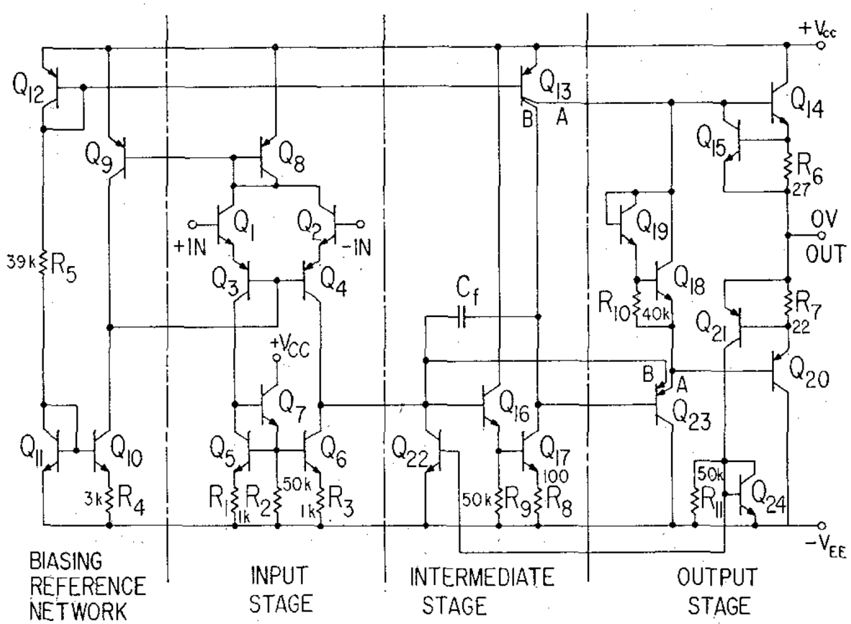
S. Franco, *Design with Operational Amplifiers and Analog Integrated Circuits*, 4th ed., Mc Graw-Hill, 2015.

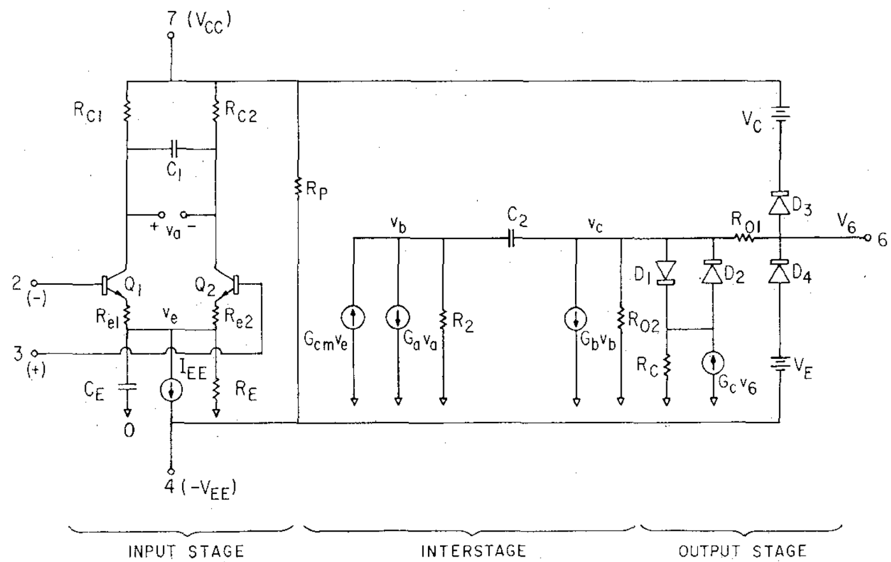
Outline

- OPAMPs fundamentals
- Basic OPAMPs configurations
- OPAMPs non-idealities (bandwidth, CMRR, PSRR, offset, linearity, noise,…)
- **OPAMPs macro-models**
- Instrumentation amplifiers
- RC-active filters
- Operational transconductance amplifiers (OTAs)

The Boyle Spice Macro-model

- The OPAMP models provided by vendors all use macro-models similar to the one below





G. R. Boyle, D. O. Pederson, B. M. Cohn and J. E. Solomon, JSSC, Dec. 1974.

Spice OPAMP Macromodels

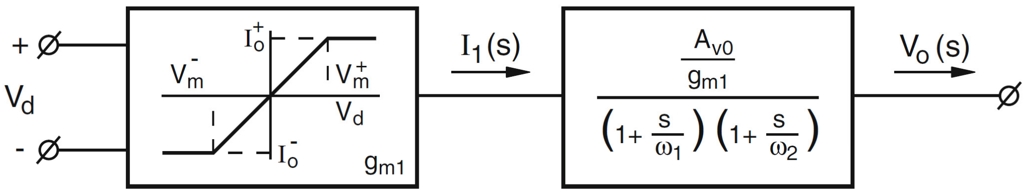


Fig. 2.5 A two-pole model taking into account the slew rate limitation. The maximum available currents of the input stage are given by I_0^+ and I_0^-. The transfer has the characteristic slope of g_{m1}

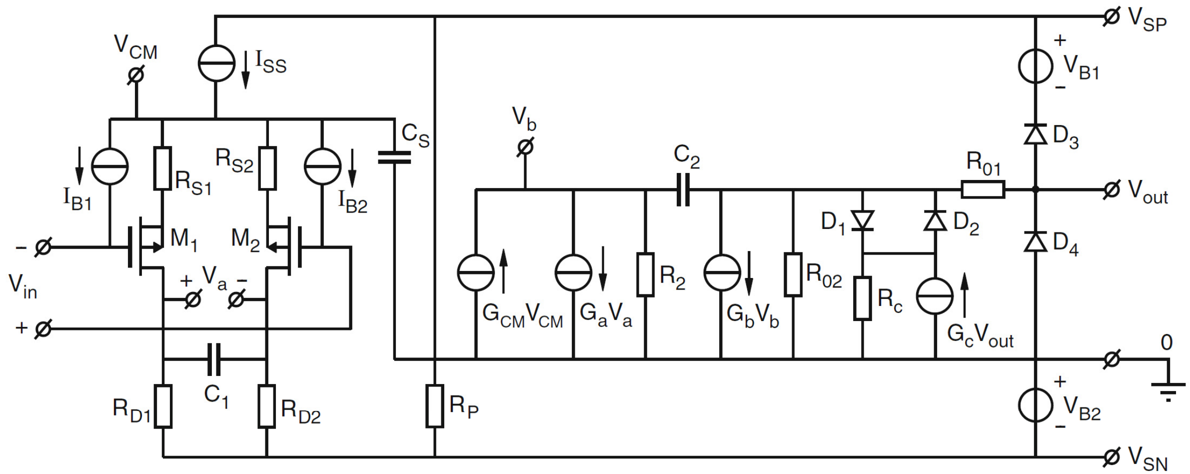


Fig. 2.6 SPICE macromodel of a Miller-compensated two-stage CMOS operational amplifier according to Boyle et al. The input stage is modeled by a differential transistor pair, while the output stage is built up by driven sources and saturation diodes

J. Huijsing, *Operational Amplifiers Theory and Design*, 3rd ed., Springer, 2017.

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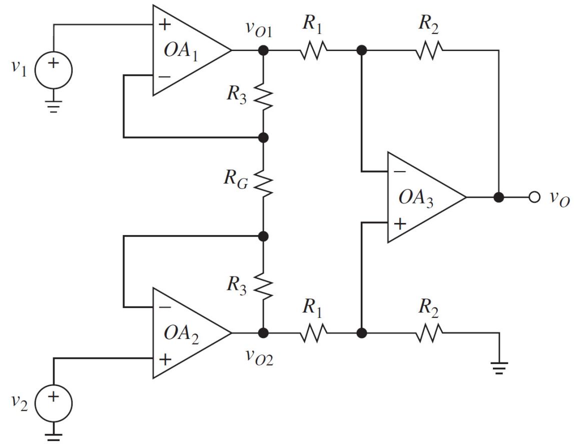
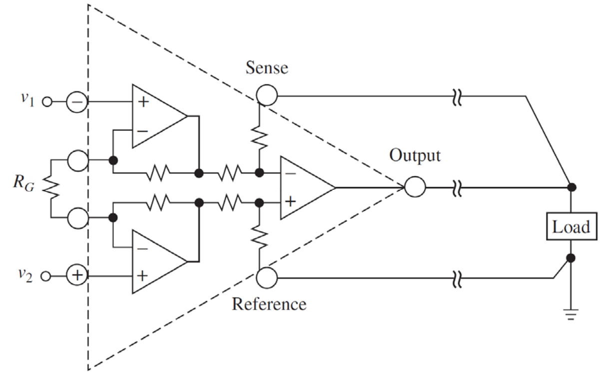
Instrumentation Amplifier (IA) Definition

- An instrumentation amplifier (IA) is a **difference amplifier** meeting the following specifications:
  1. Extremely **high** (ideally infinite) **common-mode** and **differential mode input impedances**
  2. Very **low** (ideally zero) **output impedance**
  3. **Accurate** and **stable gain**, typically in the range of 1 V/V to 103 V/V and
  4. Extremely high **common-mode rejection ratio** (CMRR) and **power supply rejection ratio** (PSRR)
- The IA is used to accurately amplify a low-level signal in the presence of a large common-mode component, such as a transducer output in process control and biomedicine
- For this reason, IAs find widespread application in test and measurement instrumentation, hence the name
- The difference amplifier presented above does not fulfil requirement 1
- It can be changed by adding voltage followers at both inputs

S. Franco, *Design with Operational Amplifiers and Analog Integrated Circuits*, 4th ed., Mc Graw-Hill, 2015.

The Three OPAMPs IA





- The differential voltage at the output of the first stage is given by

$$v_{o1} - v_{o2} = \left(1 + \frac{R_3}{R_G/2}\right) \cdot (v_1 - v_2)$$

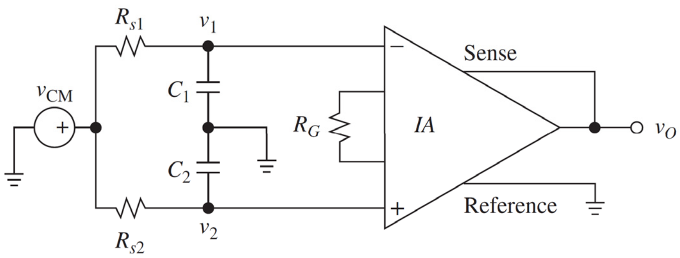
- The gain of the second stage being $-R_2/R_1$, the output voltage is given by

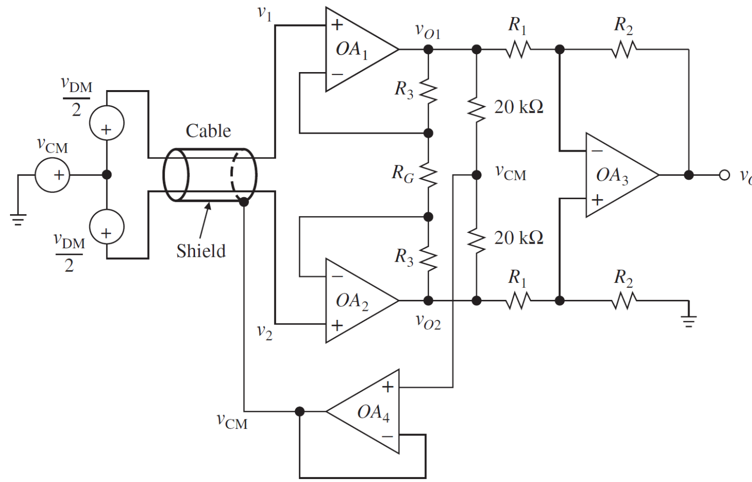
$$v_o = A \cdot (v_2 - v_1) \text{ with } A = \left(1 + 2\frac{R_3}{R_G}\right) \cdot \frac{R_2}{R_1}$$

- The **gain** is usually **set by resistor R_G** to avoid perturbing the bridge balance
- The sense and reference voltages are sensed right at the load terminals, so the effect of any signal losses in the long wires is eliminated by including these losses within the feedback loop

S. Franco, *Design with Operational Amplifiers and Analog Integrated Circuits*, 4th ed., Mc Graw-Hill, 2015.

IA with Active Guard Drive

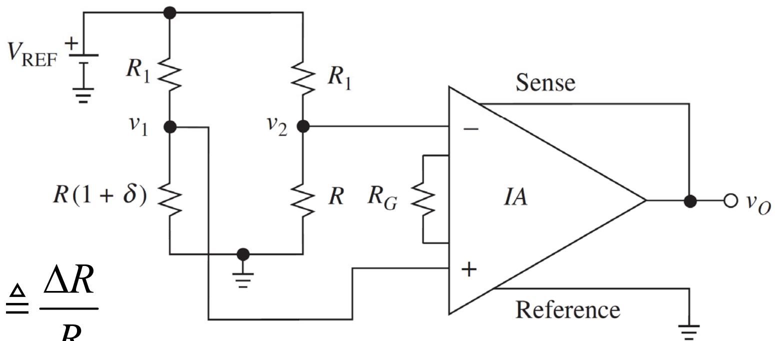




- The advantage of double-ended over single-ended transmission is that since the two wires tend to pick up identical noise, this noise will appear as a common-mode component and will thus be rejected by the IA
- Because unequal cable resistances and capacitances, any variation in v_{CM} will produce uneven signal variations downstream of the RC networks $v_1 \neq v_2$, resulting in a differential error signal
- This represents a degradation in the CMRR due to RC imbalance $CMRR \cong -20 log(2\pi f R_{dm} C_{cm})$ where $R_{dm} = |R_{s1} - R_{s2}|$ and $C_{cm} = (C_1 + C_2)/2$
- The effect of C_{cm} can be neutralized by driving the shield with the common-mode voltage itself so as to reduce the common-mode swing across C_{cm} to zero

S. Franco, *Design with Operational Amplifiers and Analog Integrated Circuits*, 4th ed., Mc Graw-Hill, 2015.

The Transducer Bridge Amplifiers



$$\delta \triangleq \frac{\Delta R}{R}$$

$$v_1 = \frac{R(1+\delta)}{R_1 + R(1+\delta)} \cdot V_{REF} = \frac{R}{R_1 + R} \cdot V_{REF} + \frac{\delta}{2 + \frac{R_1}{R} + \frac{R}{R_1} + \left(1 + \frac{R}{R_1}\right) \cdot \delta} \cdot V_{REF}$$

$$v_2 = \frac{R}{R_1 + R} \cdot V_{REF}$$

- Using the IA to amplify the difference $v_1 - v_2$ by a gain A

$$v_o = A \cdot \frac{\delta}{1 + \frac{R_1}{R} + \left(1 + \frac{R}{R_1}\right) \cdot (1 + \delta)} \cdot V_{REF}$$

- The output voltage is a nonlinear function of δ. However, δ can often be considered much smaller than unity, which leads to a linear relation wrt δ

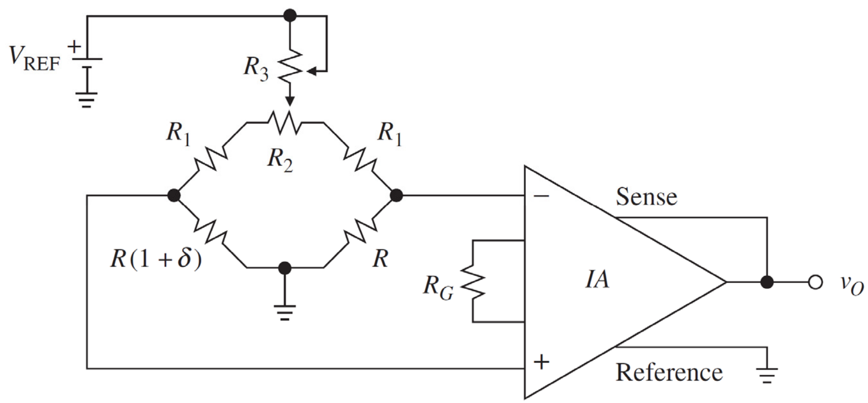
$$v_o \cong A \cdot \frac{\delta}{2 + \frac{R_1}{R} + \frac{R}{R_1}} \cdot V_{REF} \text{ for } \delta \ll 1$$

- Many bridges are designed with $R_1 = R$ which leads to

$$v_o = \frac{A \cdot V_{REF}}{4} \cdot \frac{\delta}{1 + \delta/2} \cong \frac{A \cdot V_{REF}}{4} \cdot \delta$$

S. Franco, *Design with Operational Amplifiers and Analog Integrated Circuits*, 4th ed., Mc Graw-Hill, 2015.

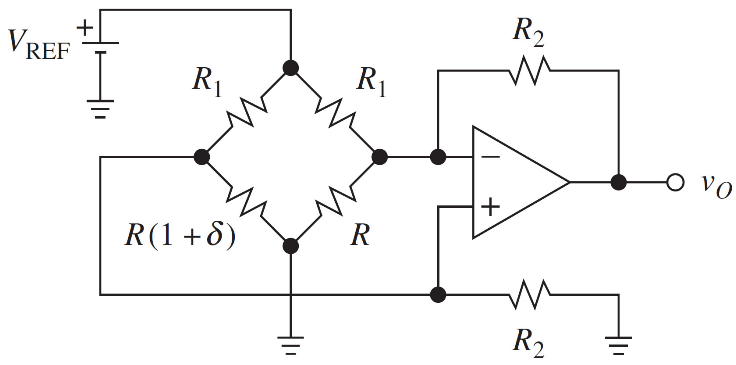
Bridge Calibration



- Because of resistance tolerances, including the tolerance of the transducer's reference value, the bridge is likely to be unbalanced and a trimmer should be included to balance it
- The tolerances in the values of the resistances and of V_{REF} will affect the bridge sensitivity, thus creating the need for adjustment of this parameter as well
- Varying R_2's wiper from its midway position will assign more resistance to one leg and less to the other thus allowing the compensation of their inherent **mismatches**
- Varying R_3 changes the bridge current and hence the magnitude of the voltage variation produced by the transducer, thus allowing the adjustment of the **sensitivity**

S. Franco, *Design with Operational Amplifiers and Analog Integrated Circuits*, 4th ed., Mc Graw-Hill, 2015.

Single-OPAMP Bridge Amplifier



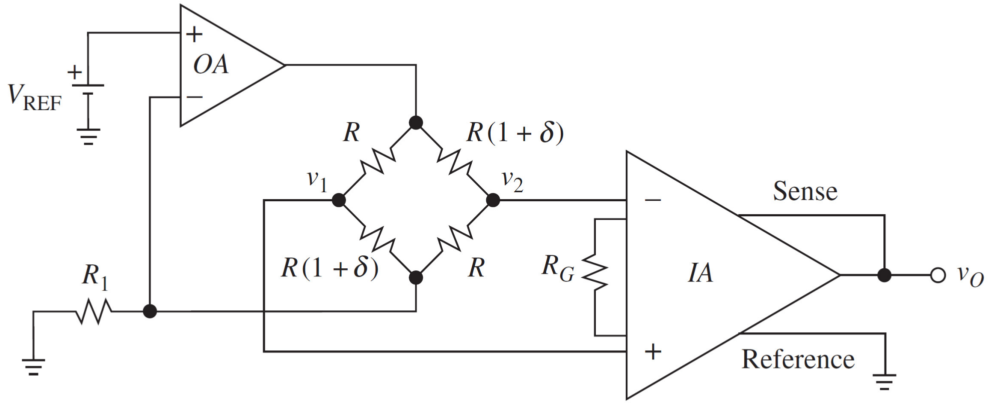
- For reasons of cost it is sometimes desirable to use a simpler amplifier than the full-fledged three OPAMPs IA
- It can be shown that the output voltage is given by

$$v_o = \frac{R_2}{R} \cdot \frac{\delta}{\frac{R_1}{R} + \left(1 + \frac{R_1}{R_2}\right) \cdot (1+\delta)} \cdot V_{REF} \cong \frac{R_2}{R} \cdot \frac{\delta}{1 + \frac{R_1}{R} + \frac{R_1}{R_2}} \cdot V_{REF} \text{ for } \delta \ll 1$$

- So, for $\delta \ll 1$, the output voltage depends linearly on δ
- To adjust the sensitivity and to null the effect of resistance mismatches, we can use the same scheme as above

S. Franco, *Design with Operational Amplifiers and Analog Integrated Circuits*, 4th ed., Mc Graw-Hill, 2015.

Bridge Linearization

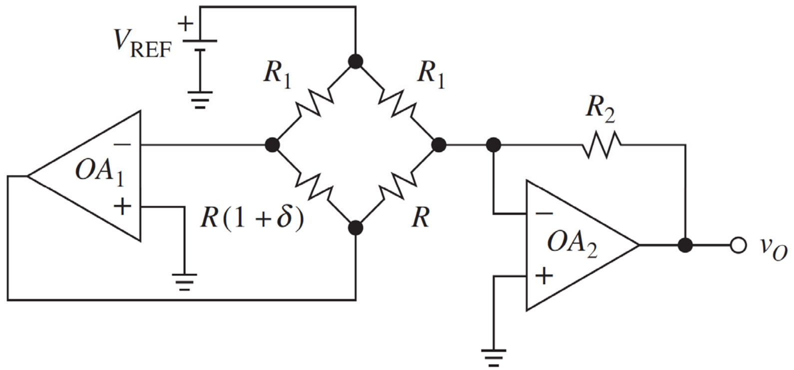


- All bridge circuits discussed so far suffer from a nonlinear response to δ
- The above circuit linearizes the bridge by driving it with a constant current by placing the entire bridge within the feedback loop of a floating-load V-I converter setting the bridge current to $I_B = V_{REF}/R_1$
- By using a transducer pair as shown, I_B will split equally between the two legs
- Since OA keeps the bottom node of the bridge at V_{REF}, we have $v_1 = V_{REF} + R(1+\delta)\,I_B/2$ and $v_2 = V_{REF} + R\,I_B/2$ so

$$v_o = \frac{A \cdot R \cdot V_{REF}}{2R_1} \cdot \delta$$

S. Franco, *Design with Operational Amplifiers and Analog Integrated Circuits*, 4th ed., Mc Graw-Hill, 2015.

Bridge Linearization



- The above schematic uses a single-transducer element and two inverting-type OPAMPs
- The response is again linearized by placing the bridge within the feedback loop of the V-I converter OA1 leading to

$$v_o = \frac{R_2 \cdot V_{REF}}{R_1} \cdot \delta$$

S. Franco, *Design with Operational Amplifiers and Analog Integrated Circuits*, 4th ed., Mc Graw-Hill, 2015.

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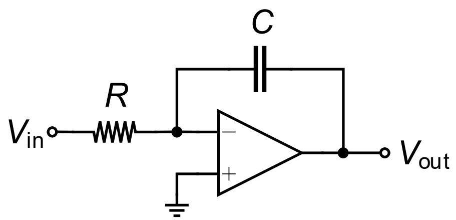
- OPAMPs macro-models
- Instrumentation amplifiers
- **RC-active filters**
- Operational transconductance amplifiers (OTAs)

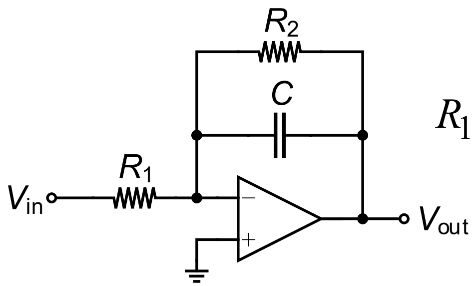
RC-active Filters

Integrator



$$H(s) \triangleq \frac{V_{out}}{V_{in}} = \frac{1}{s \cdot \tau} \quad \text{with} \quad \tau = RC$$

1st-order low-pass filter

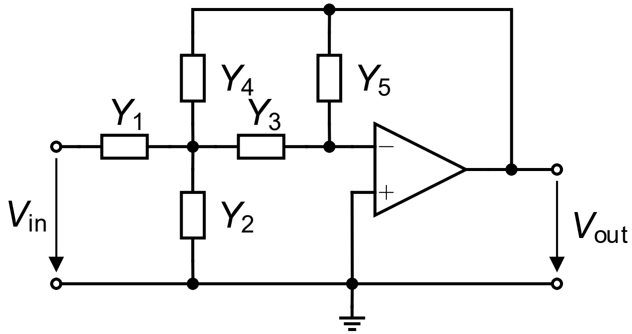


$$R_1 = R_2 = R$$

$$H(s) \triangleq \frac{V_{out}}{V_{in}} = -\frac{1}{1 + \frac{s}{\omega_c}} \quad \text{with} \quad \omega_c \triangleq \frac{1}{R \cdot C} = \frac{1}{\tau}$$

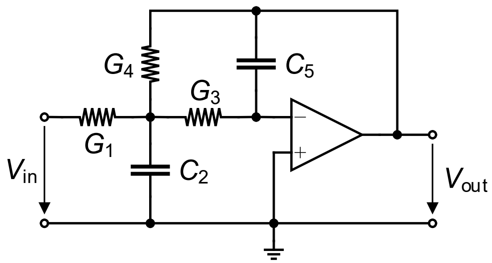
- Assuming the OPAMP gain is large enough, RC-active filters offer
  - good linearity
  - and are insensitive to the parasitic input capacitance (however the cut-off frequency may depend on the load capacitance)
- On the other hand they must drive low impedance loads which leads to higher power consumption

Higher Order Single OPAMP RC-active Filers



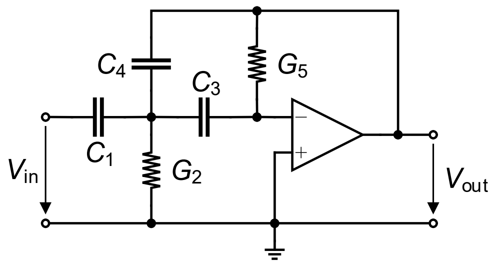
$$H(s) \triangleq \frac{V_{out}}{V_{in}} = \frac{-Y_1 Y_3}{Y_5\left(Y_1+Y_2+Y_3+Y_4\right)+Y_3 Y_4}$$

Low-pass



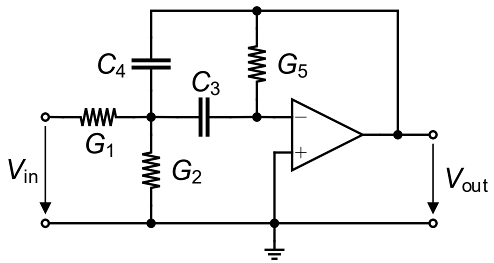
$$\frac{-G_1 G_3}{C_2 C_5 \cdot s^2 + C_5\left(G_1+G_3+G_4\right)\cdot s + G_3 G_4}$$

High-pass



$$\frac{-C_1 C_3 \cdot s^2}{C_3 C_4 \cdot s^2 + G_5\left(C_1+C_3+C_4\right)\cdot s + G_2 G_5}$$

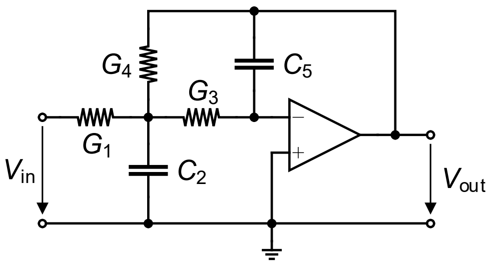
Band-pass



$$\frac{-G_1 C_3 \cdot s}{C_3 C_4 \cdot s^2 + G_5\left(C_3+C_4\right)\cdot s + G_5\left(G_1+G_2\right)}$$

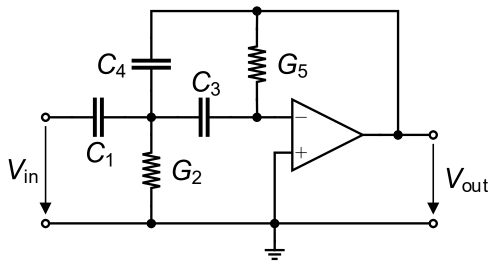
Higher Order Single OPAMP RC-active Filers

Low-pass



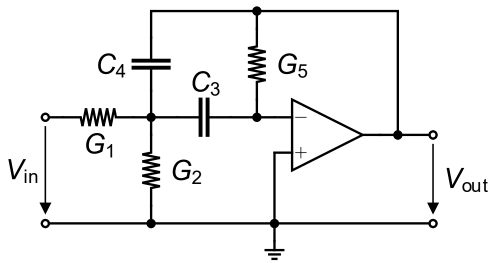
$$H(s) = \frac{K}{s^2 + \frac{\omega_p}{Q_p} \cdot s + \omega_p^2}$$

High-pass



$$H(s) = \frac{K \cdot s^2}{s^2 + \frac{\omega_p}{Q_p} \cdot s + \omega_p^2}$$

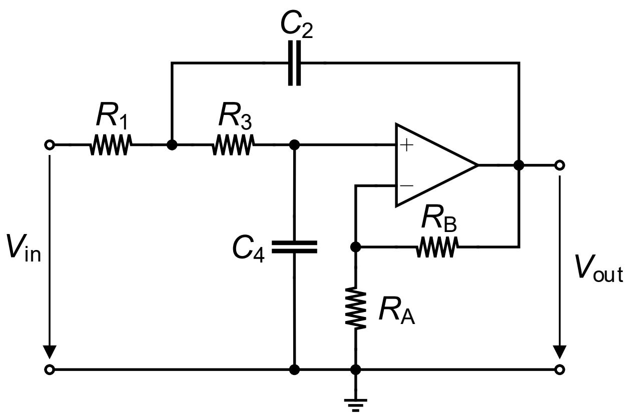
Band-pass



$$H(s) = \frac{K \cdot s}{s^2 + \frac{\omega_p}{Q_p} \cdot s + \omega_p^2}$$

Elements	Low-pass	High-pass	Band-pass
Y_1	$G_1 = K/\omega_p$	$C_1 = K$	$G_1 = K$
Y_2	$C_2 = \frac{Q_p\left(2\omega_p^2 + K\right)}{\omega_p^2}$	$G_2 = \omega_p(2+K)Q_p$	$G_2 = 2\omega_p Q_p - K$
Y_3	$G_3 = \omega_p$	$C_3 = 1$	$C_3 = 1$
Y_4	$G_4 = G_3$	$C_4 = C_3$	$C_4 = C_3$
Y_5	$C_5 = \frac{\omega_p^2}{Q_p\left(2\omega_p^2 + K\right)}$	$G_5 = \frac{\omega_p}{Q_p(2+K)}$	$G_5 = \frac{\omega_p}{2Q_p}$

The Sallen and Key Filter



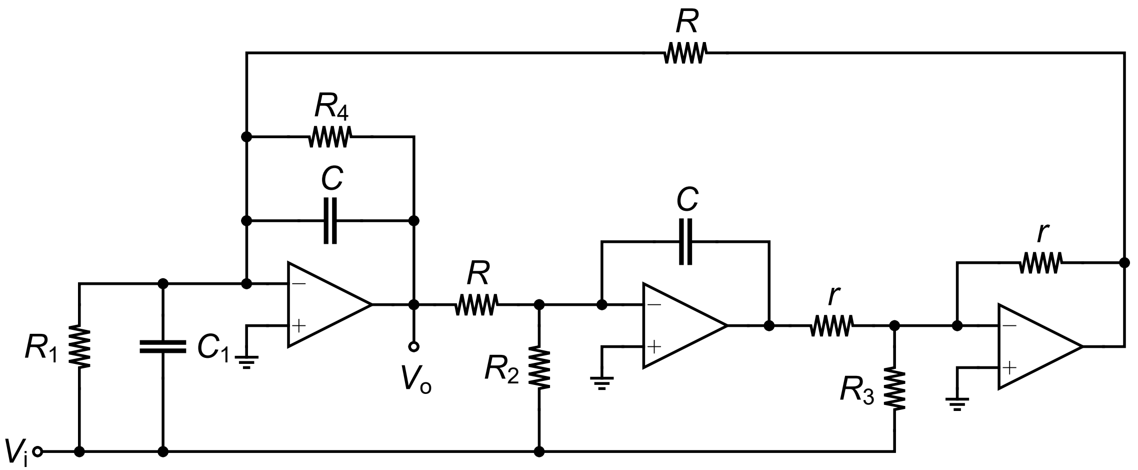
$$H(s) \triangleq \frac{V_{out}}{V_{in}} = \frac{\dfrac{K}{R_1 R_3 C_2 C_4}}{s^2 + \left(\dfrac{1}{R_3 C_4} + \dfrac{1}{R_1 C_2} + \dfrac{1}{R_3 C_2} - \dfrac{K}{R_3 C_4}\right) \cdot s + \dfrac{1}{R_1 R_3 C_2 C_4}}$$

$$= \frac{K \cdot \omega_p^2}{s^2 + \dfrac{\omega_p}{Q_p} \cdot s + \omega_p^2}$$

$$K = 1 + \frac{R_B}{R_A} \quad \omega_p = \frac{1}{\sqrt{R_1 R_3 C_2 C_4}} \quad \frac{1}{Q_p} = \sqrt{\frac{R_3 C_4}{R_1 C_2}} + \sqrt{\frac{R_1 C_4}{R_3 C_2}} + (1-K)\sqrt{\frac{R_1 C_2}{R_3 C_4}}$$

- If $R_1 = R_3 = R$ and $C_2 = C_4 = C$ then $\omega_p = \frac{1}{RC}$ and $Q_p = \frac{1}{3-K}$
- To have a unity dc gain $K = 1$, the amplifier can be configured as a voltage follower $R_B = 0$ and $R_A = \infty$

The Generalized Tow-Thomas Biquad



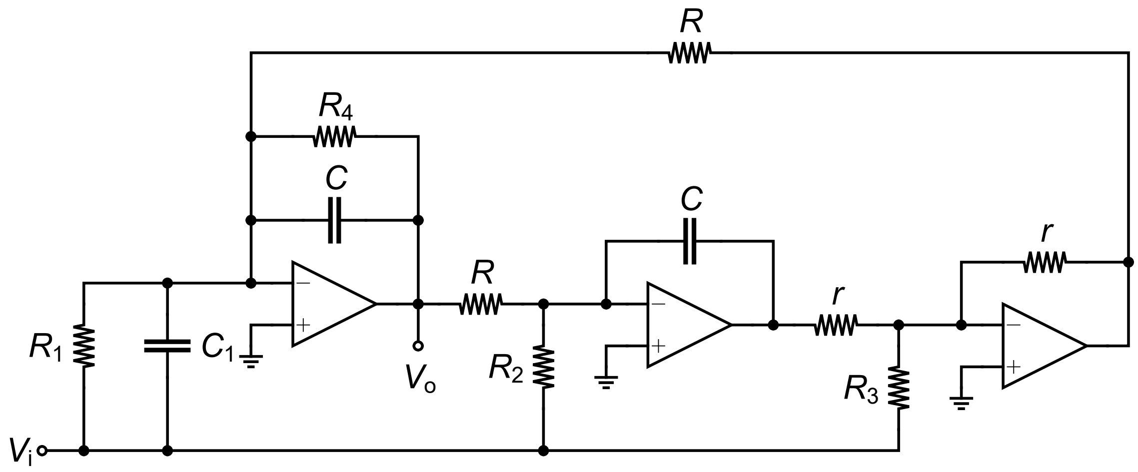
- A more versatile 3 OPAMPs biquad is the generalized Tow-Thomas shown above
- The transfer function is given by

$$T(s) \triangleq \frac{V_o}{V_i} = -\frac{\frac{C_1}{C}s^2 + \frac{1}{RC}\left(\frac{R}{R_1} - \frac{r}{R_3}\right)s + \frac{1}{RR_2C^2}}{s^2 + \frac{s}{R_4C} + \frac{1}{(RC)^2}}$$

- Clearly, it is possible to obtain any kind of second-order filter function by a proper choice of the component values
- The choice is summarized in the table on the next slide

T. Deliyannis, Y. Sun and J. K. Fidler, *Continuous-Time Active Filter Design*, CRC Press, 1999.

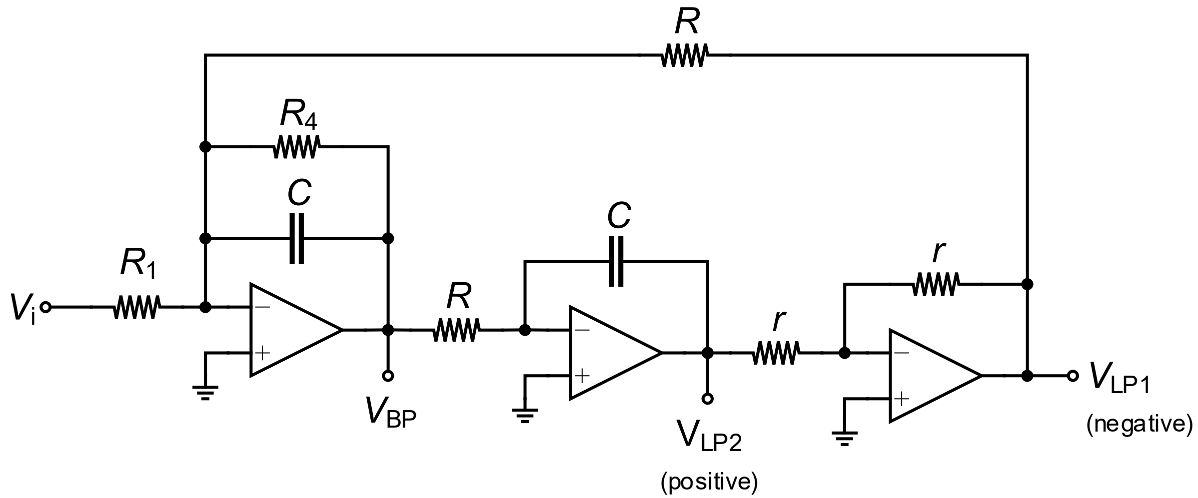
The Generalized Tow-Thomas Biquad



Type	Conditions		Comments
LP	$C_1 = 0$	$R_1 = R_3 = \infty$	
BP	$C_1 = 0$	$R_1 = R_2 = \infty$	Positive sign
BP	$C_1 = 0$	$R_2 = R_3 = \infty$	Negative sign
HP	$C_1 = C$	$R_1 = R_2 = R_3 = \infty$	
Notch	$C_1 = C$	$R_1 = R_3 = \infty$	
Allpass	$C_1 = C$	$R_1 = \infty, r = R_3/Q$	

T. Deliyannis, Y. Sun and J. K. Fidler, *Continuous-Time Active Filter Design*, CRC Press, 1999.

The LP and BP Tow-Thomas Biquad



- The particular **LP** and **BP** cases of the Tow-Thomas biquad are shown above

$$T_{BP}(s) \triangleq \frac{V_{BP}}{V_i} = K \frac{\omega_0 s}{s^2+\frac{\omega_0}{Q}s+\omega_0^2} \text{ and } T_{LP}(s) \triangleq \frac{V_{LP1}}{V_i} = \frac{K}{s^2+\frac{\omega_0}{Q}s+\omega_0^2}$$

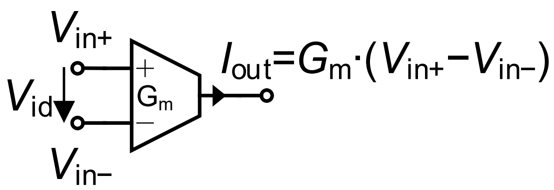
- Where $\omega_0 = \frac{1}{RC}$, $Q = \frac{R_4}{R}$ and $K = -\frac{R}{R_1}$
- We see that R_4 sets the Q-factor **independently** of the center frequency ω_0 and R_1 sets the gain in the passband

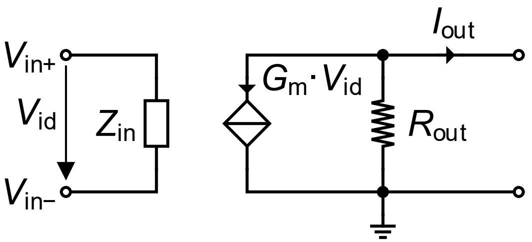
T. Deliyannis, Y. Sun and J. K. Fidler, *Continuous-Time Active Filter Design*, CRC Press, 1999.

Outline

- OPAMPs fundamentals
- Basic OPAMPs configurations
- OPAMPs non-idealities (bandwidth, CMRR, PSRR, offset, linearity, noise,…)
- OPAMPs macro-models
- Instrumentation amplifiers
- RC-active filters
- **Operational transconductance amplifiers (OTAs)**

Operational Transconductance Amplifiers (OTAs)

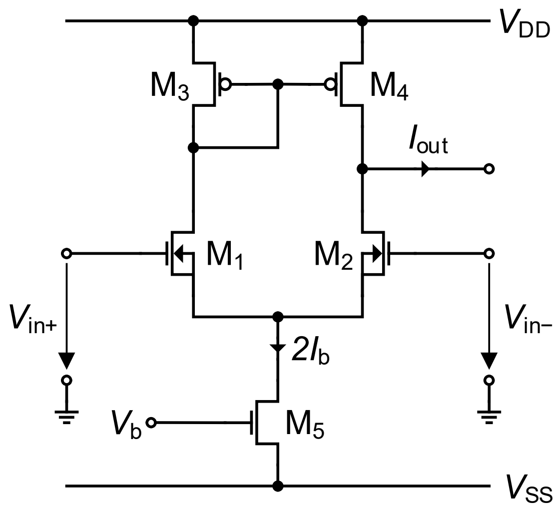




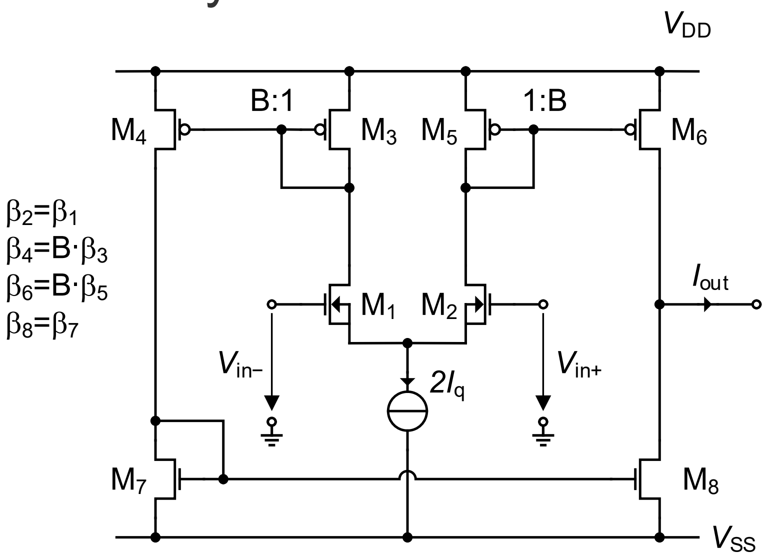
- The operational transconductance amplifier (OTA) is a voltage controlled current source (VCCS) controlled by a differential voltage
- It has a high output impedance R_{out} and the input impedance Z_{in} is usually capacitive
- The main difference between an OTA and an OPAMP is that the dominant pole of the OTA is set by the external load capacitance C_L whereas for an OPAMP it is set by the internal compensation capacitance C_c
- The OTA therefore does not need any compensation capacitance C_c which usually leeds to lower power consumption

Basic OTA Schematics

Miller OTA



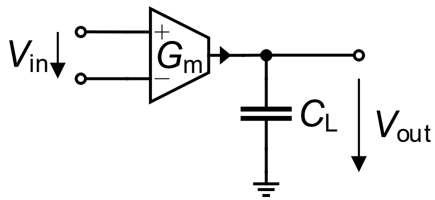
Symmetrical OTA



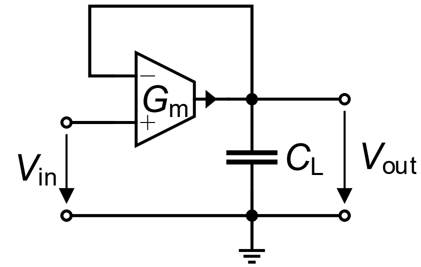
- The above schematics show the simplest OTA schematics
- At low-voltage, they may provide limited dc gain which can be improved by adding cascade stages

OTA-based Integrator

Integrator

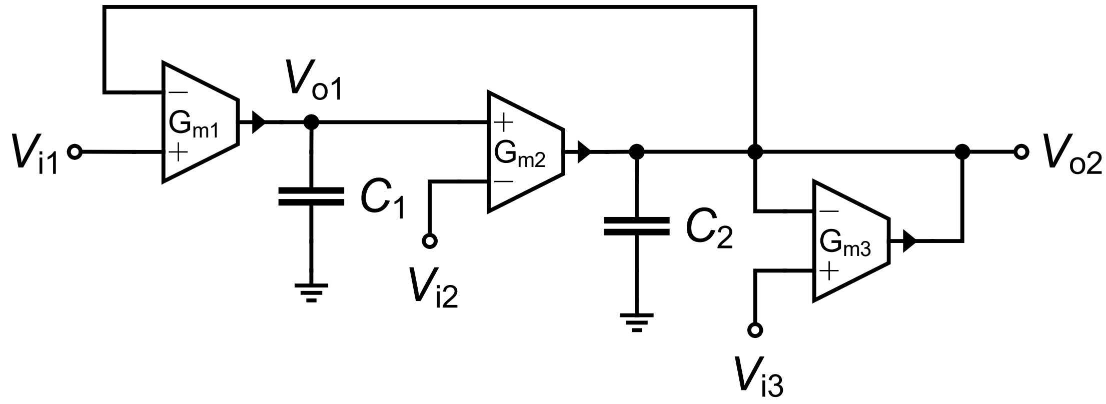


1st-order low-pass filter



- The OTA can be used to build an integrator which are used for building higher order filters
- Contrary to RC-active filters, the linearity of the integrator or filter directly depends on the linearity of the OTA

The Tow-Thomas OTA-C Biquad



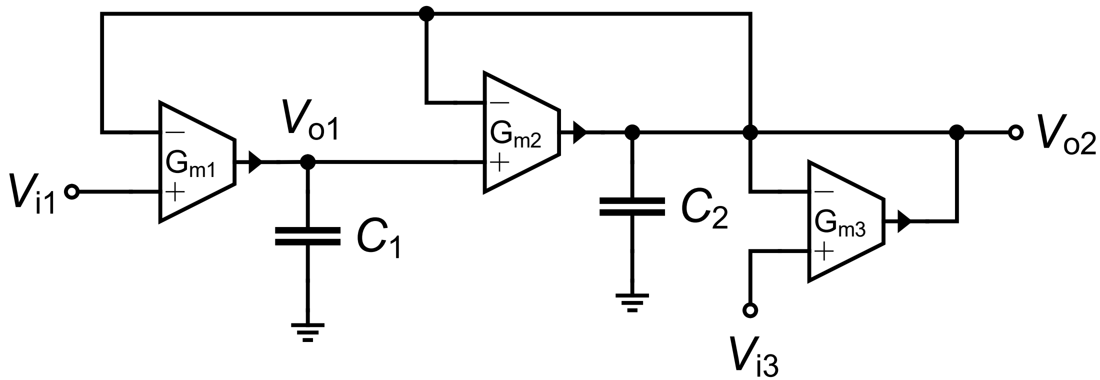
- The **Tow-Thomas** filter can also be realized with **OTAs** using lossy integrators as shown above
- The various transfer functions are given by

$$V_{o1} = \frac{(s\tau_2 + k_{22})V_{i1} + V_{i2} - k_{22}V_{i3}}{s^2\tau_1\tau_2 + k_{22}s\tau_1 + 1} \text{ and } V_{o2} = \frac{V_{i1} - s\tau_1 V_{i2} + k_{22}s\tau_1 V_{i3}}{s^2\tau_1\tau_2 + k_{22}s\tau_1 + 1}$$

- with $k_{22} = \frac{G_{m3}}{G_{m2}}$, $\tau_1 = \frac{C_1}{G_{m1}}$ and $\tau_2 = \frac{C_2}{G_{m2}}$
- This circuit is simple and has very **low sensitivity** and **low parasitic effects**

T. Deliyannis, Y. Sun and J. K. Fidler, *Continuous-Time Active Filter Design*, CRC Press, 1999.

The Feedback Lossy Integrator OTA-C Biquad



- This is a variant of the previous Tow-Thomas lossy integrators OTA biquad where the input of G_{m2} has been connected to its output (making it lossy)
- The output is then given by

$$V_{o1} = \frac{(s\tau_2 + k_{22})V_{i1} - (k_{22}-1)V_{i3}}{s^2\tau_1\tau_2 + k_{22}s\tau_1 + 1} \text{ and } V_{o2} = \frac{V_{i1} + (k_{22}-1)s\tau_1 V_{i3}}{s^2\tau_1\tau_2 + k_{22}s\tau_1 + 1}$$

- with $k_{22} = 1 + \frac{G_{m3}}{G_{m2}}$, $\tau_1 = \frac{C_1}{G_{m1}}$ and $\tau_2 = \frac{C_2}{G_{m2}}$

T. Deliyannis, Y. Sun and J. K. Fidler, *Continuous-Time Active Filter Design*, CRC Press, 1999.