Smart Sensors for the IoT

Sensors

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Outline

- **Definitions and sensors classifications**
- Sensor characteristics

Transducer and Sensor Definitions

- A **transducer** is a device that converts a signal from one physical form to a corresponding signal having a **different physical form** (energy converter)
- Transducer implies that input and output quantities are of **different form**
- There are six different kinds of signals - **mechanical**, **thermal**, **magnetic**, **electric**, **chemical**, and **radiation** (corpuscular and electromagnetic, including light) - any device converting signals of one kind to signals of a different kind is a transducer
- Devices converting any of these input signals into an **electric output** in form of voltage, current, or charge are called **sensors**
- The **stimulus** (or **measurand**) is the quantity, property, or condition that is received and converted into electrical signal which may be further described in terms of amplitude, polarity, frequency, phase, or digital code
- Examples of stimuli are light intensity and wavelength, sound, force, acceleration, distance, rate of motion, and chemical composition
- Any sensor is an energy converter: the process of sensing is a particular case of information transfer, and any transmission of information requires transmission of energy

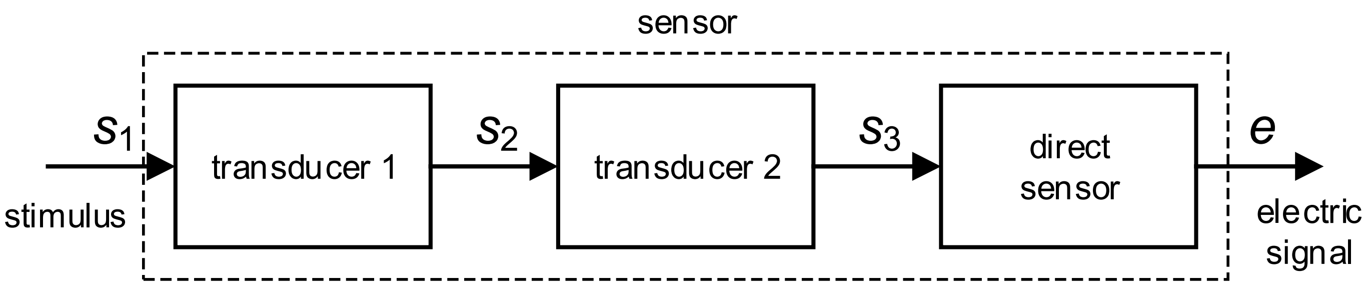
R. Pallas-Areny, J. G. Webster, *Sensors and Signal Conditioning*, 2nd ed., Wiley, 2001.

Electronic Measurement Systems

- Most measurement systems use **electric signals**, and hence rely on **sensors**
- Electronic measurement systems provide the following advantages:
  - Sensors can be designed for any nonelectric quantity, by selecting an appropriate material
  - Energy does not need to be drained from the process being measured because sensor output signals can be amplified
  - There is a variety of integrated circuits available for **electric signal conditioning** or modification
  - Many options exist for information display or recording by electronic means
  - Signal transmission is more versatile for electric signals

R. Pallas-Areny, J. G. Webster, *Sensors and Signal Conditioning*, 2nd ed., Wiley, 2001.

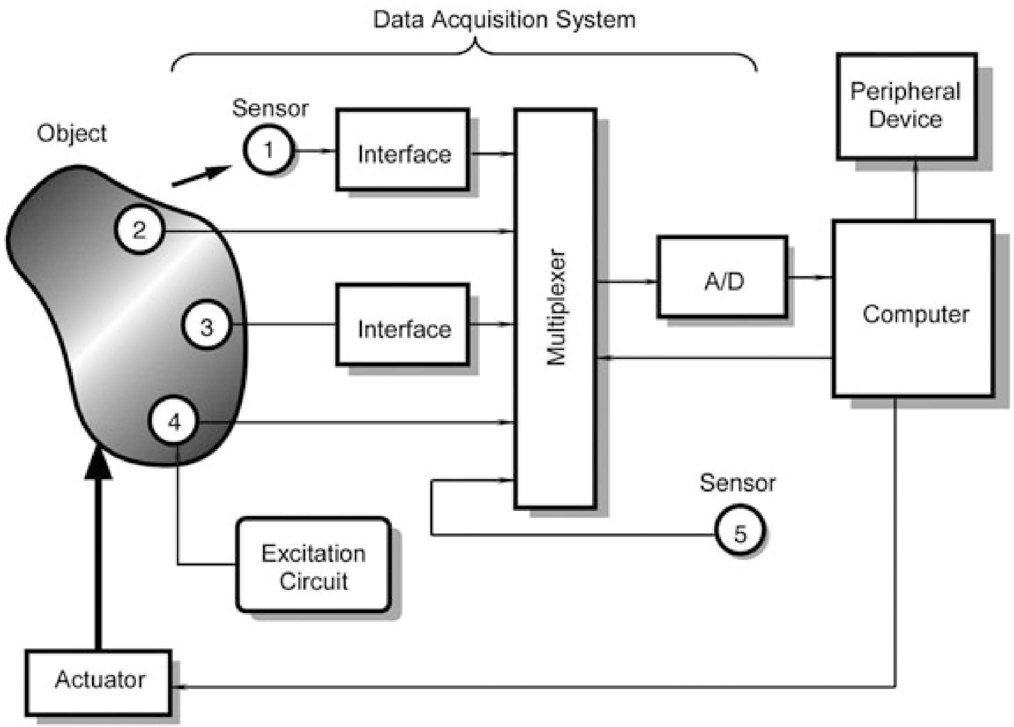
Direct and Hybrid Sensors



- A **direct sensor** converts a stimulus directly into an electrical signal or modifies an externally supplied electrical signal
- Examples of such physical effects are the photo detectors (imagers)
- A **hybrid sensor** (or simply sensor) in addition needs one or more transducers before a direct sensor can be employed to generate an electrical output
- For example, a chemical sensor may comprise two parts: the first part converts energy of an exothermal chemical reaction into heat (transducer) and another part, a thermopile, converts heat into an electrical output signal

J. Fraden, *Handbook of Modern Sensors – Physics, Designs, and Applications*, 5th edition, Springer, 2016.

Sensors in Data Acquisition System

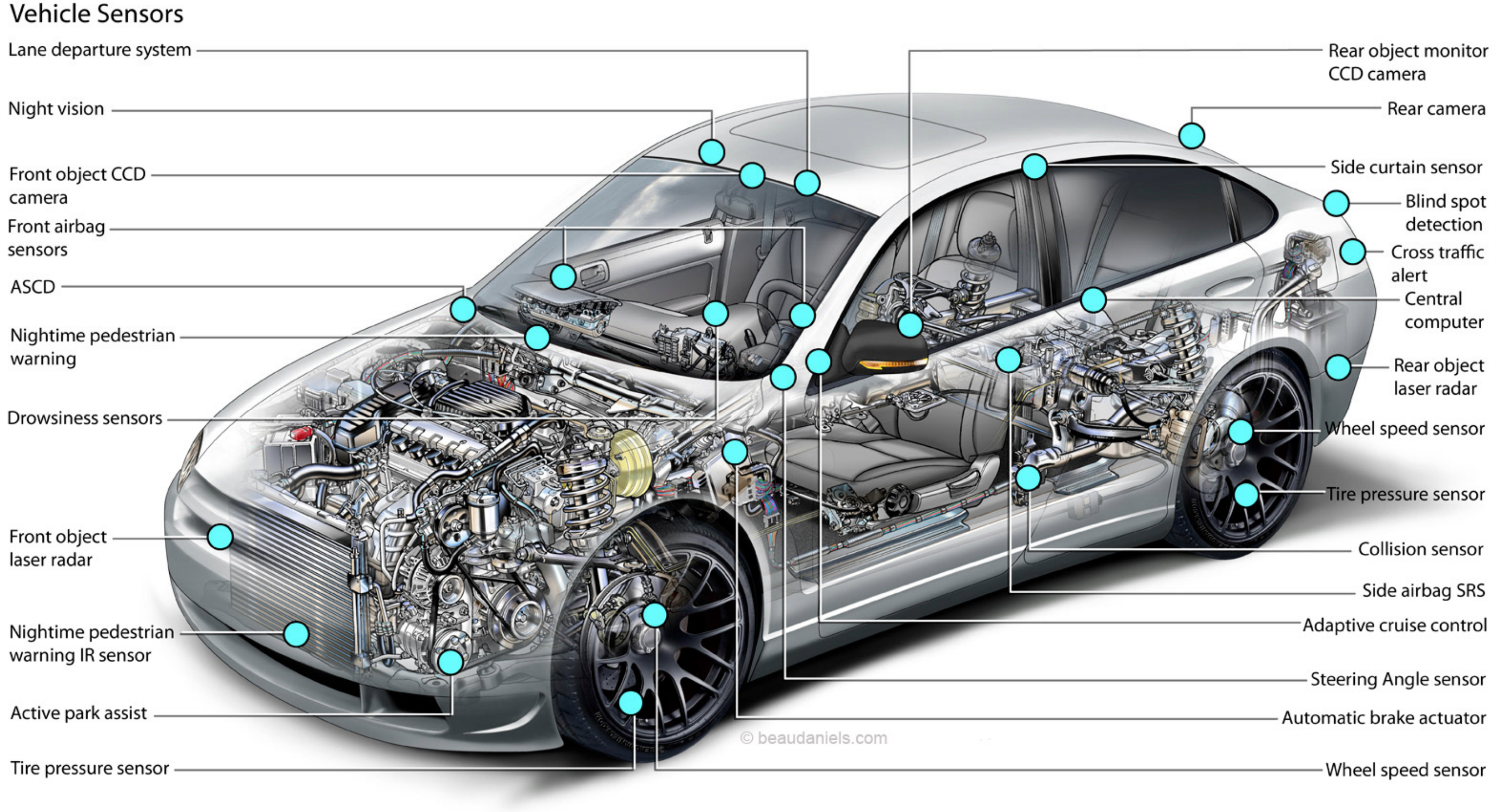


Sensor 1 is noncontact (for example a camera)
Sensors 2 and 3 are passive (for example strain gauge)
Sensor 4 is active (for example accelerometer)
Sensor 5 is internal to the data acquisition system (for example temperature sensor)

- A sensor does not function by itself; it is always part of a larger system that may incorporate many other detectors, signal conditioners, processors, memory devices, data recorders, and actuators
- A sensor is always part of some kind of a data acquisition system which may be part of a larger control system that includes various feedback mechanisms

J. Fraden, *Handbook of Modern Sensors – Physics, Designs, and Applications*, 5th edition, Springer, 2016.

Example of Complex Sensor Systems



Source: beaudaniels.com

Sensor Classification – Active versus Passive

- A **passive sensor** does not need any additional energy source
  - It generates an electric signal in response to an external stimulus
  - The input stimulus energy is **converted** by the sensor into the output signal
  - Examples are a thermocouple, a photodiode, and a piezoelectric sensor
  - Many passive sensors are **direct sensors** as defined above
- **Active sensors** sometimes are called parametric because their own properties change in response to an external stimulus and these properties can be subsequently converted into electric signals
  - A sensor's parameter modulates the excitation signal and that modulation carries information of the measured value
  - An example of an active sensor is the **thermistor** which is a temperature-sensitive resistor. It does not generate any electric signal, but by passing electric current (excitation signal) through it, its resistance can be measured by detecting variations in current and/or voltage across the thermistor. These variations (presented in ohms) directly relate to temperature through a known transfer function
  - To measure the resistance of a sensor, electric current must be applied to it from an external power source

J. Fraden, *Handbook of Modern Sensors – Physics, Designs, and Applications*, 5th edition, Springer, 2016.

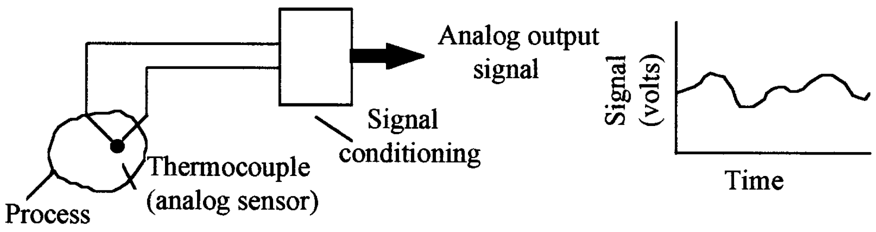
Sensor Classification – Absolute versus Relative

- Depending on the selected reference, sensors can be classified into **absolute** and **relative**
- An **absolute sensor** detects a stimulus in reference to an absolute physical scale that is independent on the measurement conditions
  - The **thermistor** mentioned above is an example of an absolute sensor. Its electrical resistance directly relates to the absolute temperature scale of Kelvin
- A **relative sensor** produces a signal that relates to some special case
  - The **thermocouple** is another very popular temperature sensor and is a relative sensor. It produces an electric voltage that is function of a temperature gradient across the thermocouple wires. Thus, a thermocouple output signal cannot be related to any particular temperature without referencing to a selected baseline
- Another example of absolute and relative sensors is a **pressure sensor**
  - An absolute pressure sensor produces signal in reference to vacuum—an absolute zero on a pressure scale
  - A relative pressure sensor produces signal with respect to a selected baseline that is not zero pressure—for example, to the atmospheric pressure

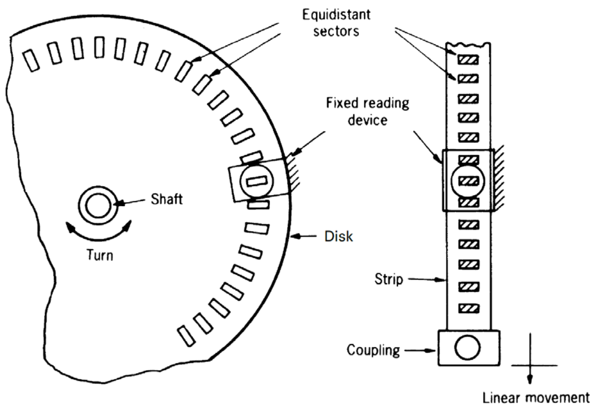
J. Fraden, *Handbook of Modern Sensors – Physics, Designs, and Applications*, 5th edition, Springer, 2016.

Analog and Digital Sensors

Example of Analog Sensor



Example of Digital Sensor



- **Analog sensors** provide a signal that is **continuous** in both its magnitude and its temporal (time) or spatial (space) content
- Most physical variables, such as current, temperature, displacement, acceleration, speed, pressure, light intensity, and strain, tend to be continuous in nature and are readily measured by an analog sensor and represented by an analog signal
- **Digital sensors** provide a signal that is a **direct digital representation** of the measurand
- Positioning sensor provide a quantized (digital) representation of the position

N. Norton, *Sensor and Analyzer Handbook*, Prentice-Hall, 1982.

Sensor Classification – Specs, Conversion phenomena

Sensor specifications

Sensitivity	Stimulus range (span)
Stability (short and long term)	Resolution
Accuracy	Selectivity
Speed of response	Environmental conditions
Overload characteristics	Linearity
Hysteresis	Dead band
Operating life	Output format
Cost, size, weight	Other

Conversion phenomena

Physical	Thermoelectric Photoelectric Photomagnetic Magnetoelectric Electromagnetic Thermoelastic Electroelastic Thermomagnetic Thermo-optic Photoelastic Other	Chemical	Chemical transformation Physical transformation Electrochemical process Spectroscopy Other
		Biological	Biochemical transformation Physical transformation Effect on test organism Spectroscopy Other

J. Fraden, *Handbook of Modern Sensors – Physics, Designs, and Applications*, 5th edition, Springer, 2016.

Sensor Classification – Stimulus

Stimulus	Stimulus	
Acoustic	*Mechanical*	Position (linear, angular)
Wave amplitude, phase		Acceleration
Spectrum polarization		Force
Wave velocity		Stress, pressure
Other		Strain
Biological		Mass, density
Biomass (types, concentration states)		Moment, torque
Other		Speed of flow, rate of
Chemical		mass transport
Components (identities, concentration, states)		Shape, roughness,
Other		orientation
Electric		Stiffness, compliance
Charge, current		Viscosity
Potential, voltage		Crystallinity, structural
Electric field (amplitude, phase, polarization,		integrity
spectrum)		Other
Conductivity	*Radiation*	Type
Permittivity		Energy
Other		Intensity
Magnetic		Other
Magnetic field (amplitude, phase, polarization,	*Thermal*	Temperature
spectrum)		Flux
Magnetic flux		Specific heat
Permeability		Thermal conductivity
Other		Other
Optical		
Wave amplitude, phase, polarization, spectrum		
Wave velocity		
Refractive index		
Emissivity, reflectivity, absorption		
Other		

J. Fraden, *Handbook of Modern Sensors – Physics, Designs, and Applications*, 5th edition, Springer, 2016.

Units of Measurements

- The base measurement system is known as SI or "Système International d'Unités"
- All other physical quantities are derivatives of these base units

Quantity	Name	Symbol	Defined by... (year established)
Length	meter	m	...the length of the path traveled by light in vacuum in 1/299,792,458 of a second... (1983)
Mass	kilogram	kg	...after a platinum-iridium prototype (1889)
Time	second	s	...the duration of 9,192,631,770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the cesium-133 atom (1967)
Electric current	ampere	A	force equal to 2×10^{-7} N/m of length exerted on two parallel conductors in vacuum when they carry the current (1946)
Thermodynamic temperature	kelvin	K	The fraction 1/273.16 of the thermodynamic temperature of the triple point of water (1967)
Amount of substance	mole	mol	...the amount of substance which contains as many elementary entities as there are atoms in 0.012 kg of carbon 12 (1971)
Luminous intensity	candela	cd	...intensity in the perpendicular direction of a surface of 1/600,000 m^2 of a blackbody at temperature of freezing Pt under pressure of 101,325 N/m^2 (1967)
Plane angle	radian	rad	(supplemental unit)
Solid angle	steradian	sr	(supplemental unit)

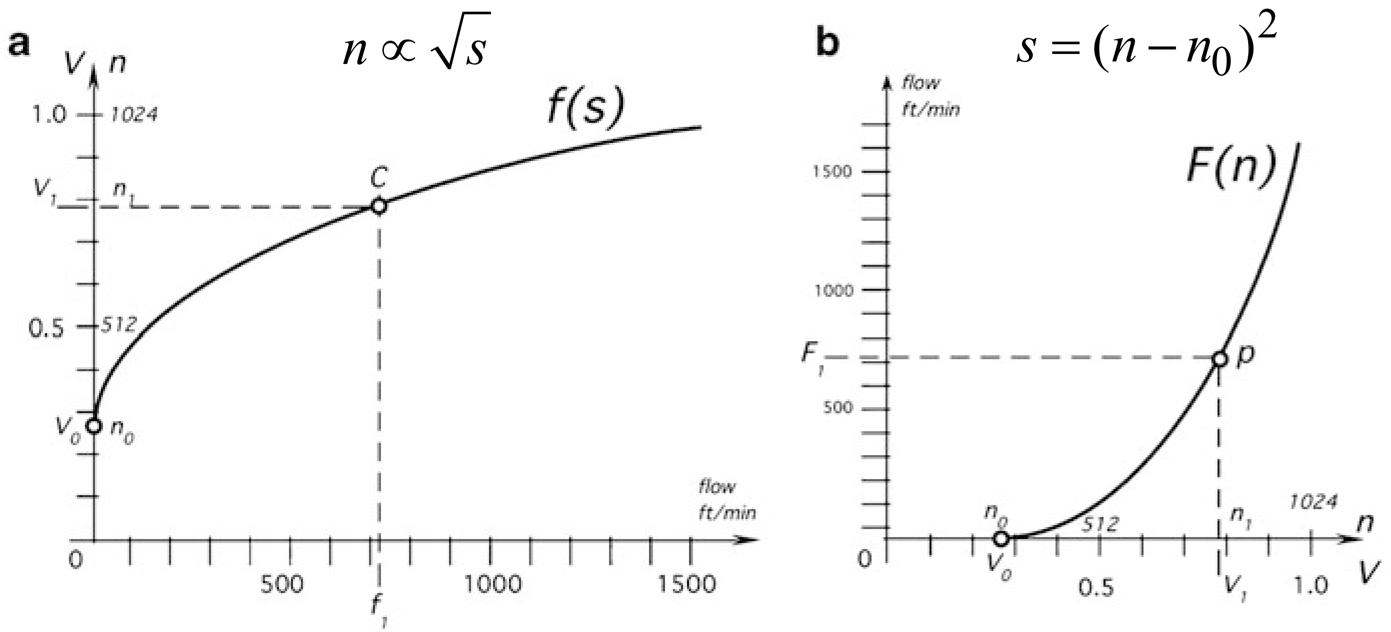
J. Fraden, *Handbook of Modern Sensors – Physics, Designs, and Applications*, 5th edition, Springer, 2016.

Outline

- Definitions and sensors classifications
- **Sensor characteristics**

Static Input-Output Transfer Function

Example of a thermo-anemometer



- The static transfer function $E = f(s)$ represents a relation between the input stimulus s and the electrical signal E produced by the sensor at its output
- Normally, the stimulus s is unknown while the output signal E is measured
- An **inverse** $s = F(E)$ of the transfer function is required to compute the stimulus from the sensor's measured response

J. Fraden, *Handbook of Modern Sensors – Physics, Designs, and Applications*, 5th edition, Springer, 2016.

Sensor Characteristics

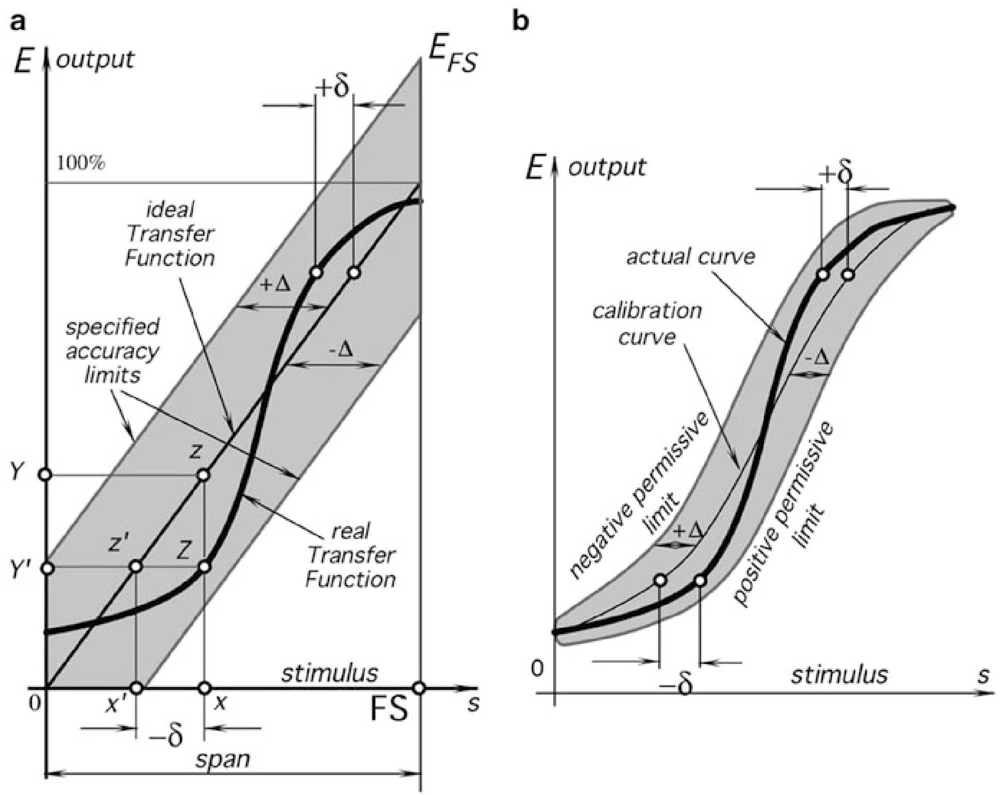
- **Static characteristics:**
  Properties of the system after all transient effects have settled to their final or steady state, including:
  - Accuracy
  - Discrimination
  - Precision
  - Errors
  - Drift
  - Sensitivity
  - Linearity
  - Hysteresis (backslash)
- **Dynamic characteristics:**
  Properties of the system transient response to an input:
  - Zero order systems
  - First order systems
  - Second order systems

Accuracy

- **Accuracy** is the quality that characterizes the capacity of a measuring instrument or a sensor for giving **results close to the true value** of the measured quantity
- The **true**, exact, or ideal value is the value that would be obtained by a perfect measurement and is, by nature, **indeterminate**
- The conventional true value of a quantity is the value attributed to a particular quantity and accepted, sometimes by convention, as having an uncertainty appropriate for a given purpose
- Sensor accuracy is determined through static calibration which consists of keeping constant all sensor inputs, except the one to be studied. This input is changed very slowly, thus taking successive constant values along the measurement range. The successive sensor output results are then recorded. Their plot against input values forms the **calibration curve**
- Obviously each value of the input quantity must be known. Measurement standards are such known quantities
- Their values should be at least ten times more accurate than that of the sensor being calibrated

R. Pallas-Areny, J. G. Webster, *Sensors and Signal Conditioning*, 2nd ed., Wiley, 2001.

Accuracy



- **Inaccuracy** is measured as a highest deviation of a value represented by the sensor from the ideal or true value of a stimulus at its input

J. Fraden, *Handbook of Modern Sensors – Physics, Designs, and Applications*, 5th edition, Springer, 2016.

Errors

- Any **discrepancy** between the true value for the measured quantity and the instrument reading is called an **error**
- The difference between the measurement result and the true value is called **absolute error**

$$absolute\ error = result - true\ value$$

- Sometimes it is given as a percentage of the maximal value that can be measured with the instrument (full-scale output, FSO) or with respect to the difference between the maximal and the minimal measurable values that is, the measurement range or span
- The common practice, however, is to specify the error as a quotient between the absolute error and the true value for the measured quantity defining the **relative error**

$$relative\ error = \frac{absolute\ error}{true\ value}$$

- Because true values are indeterminate, error calculations use a conventional true value
- Some sensors have a relative error specified only as a percentage of the FSO

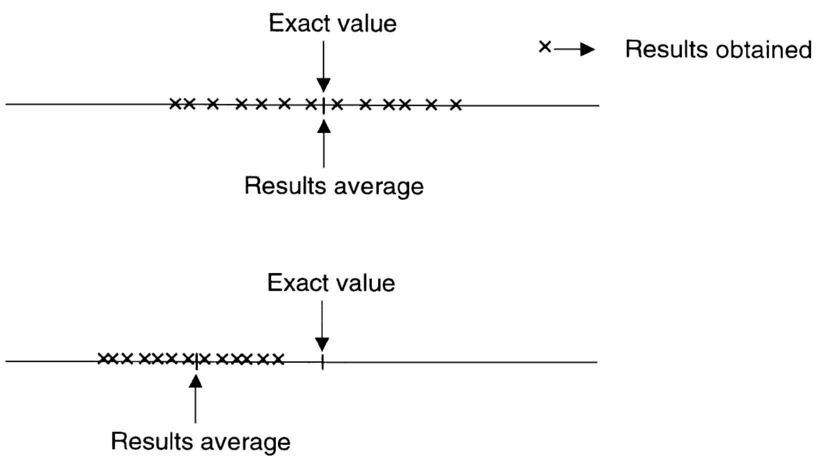
R. Pallas-Areny, J. G. Webster, *Sensors and Signal Conditioning*, 2nd ed., Wiley, 2001.

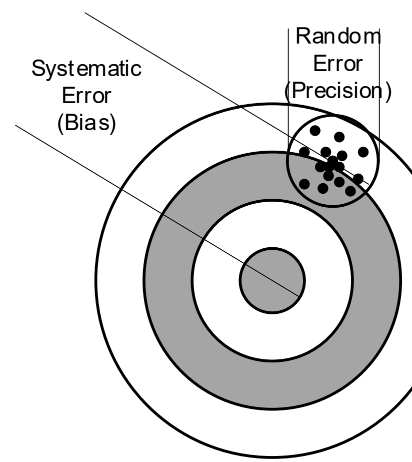
Precision

- **Precision** is the quality that characterizes the capability of a measuring instrument of giving the same reading when repetitively measuring the same quantity under the same prescribed conditions (environmental, operator, etc.), without regard for the coincidence or discrepancy between the result and the true value
- Precision implies an agreement between successive readings and a high number of significant figures in the result
- Therefore, it is a necessary but not sufficient condition for accuracy
- Clearly, accuracy without precision does not have any meaning and precision does not imply accuracy

High accuracy, but low repeatability and low precision

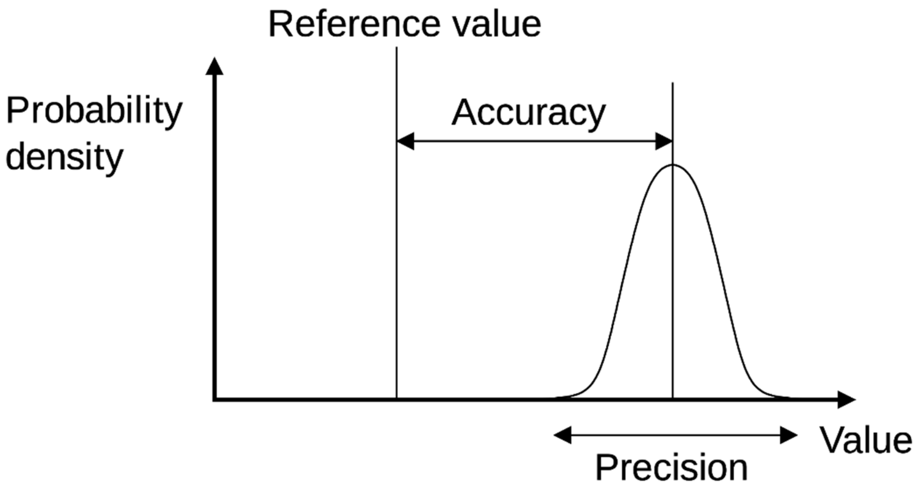
High repeatability and high precision, but low accuracy





R. Pallas-Areny, J. G. Webster, *Sensors and Signal Conditioning*, 2nd ed., Wiley, 2001.

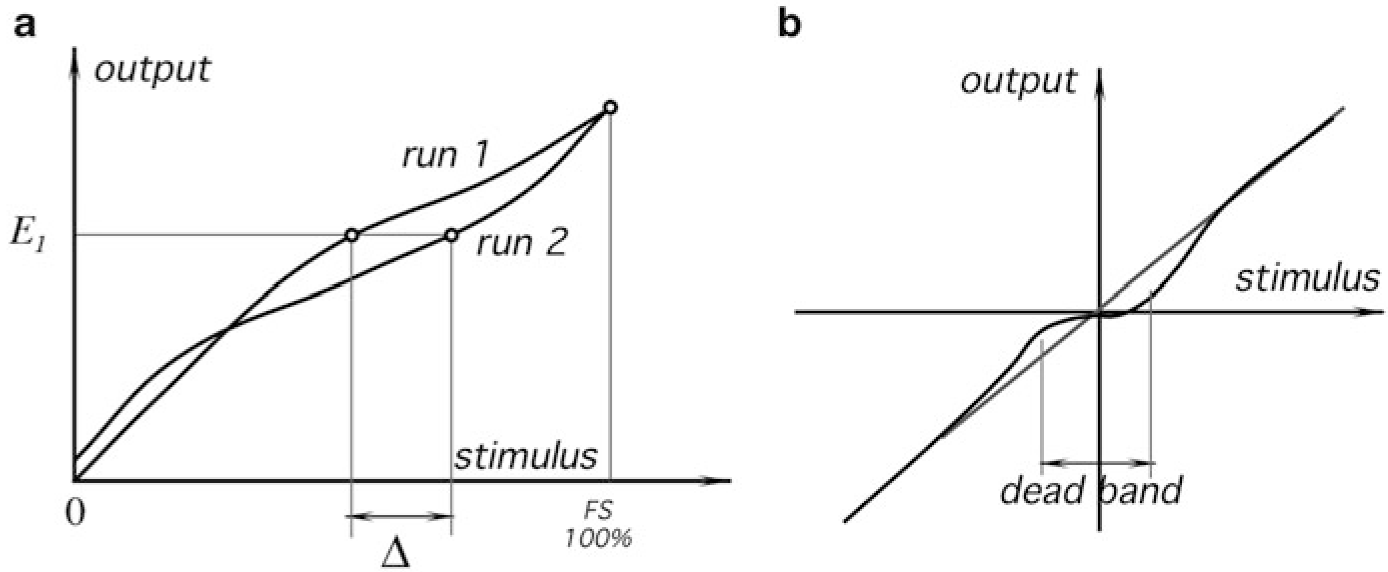
Difference between Accuracy and Precision



- **Accuracy** is the proximity of measurement results to the true value
- **Precision** is the degree to which repeated (or reproducible) measurements under unchanged conditions show the same results

www.wikipedia.org.

Repeatability



- The **repeatability** is the closeness of agreement between successive results obtained with the same method under the same conditions and in a **short time interval**
- Quantitatively, the repeatability is the minimum value that exceeds, with a specified probability, the absolute value of the difference between two successive readings obtained under the specified conditions
- If not stated, it is assumed that the probability level is 95 %

📖 J. Fraden, *Handbook of Modern Sensors – Physics, Designs, and Applications*, 5th edition, Springer, 2016.
📖 R. Pallas-Areny, J. G. Webster, *Sensors and Signal Conditioning*, 2nd ed., Wiley, 2001.

Reproducibility

- The **reproducibility** is also related to the degree of coincidence between successive readings when the same quantity is measured with a given method, but in this case with **a long-term set of measurements** or with measurements carried out by different people or performed with different instruments or in different laboratories
- Quantitatively, the **reproducibility** is the minimal value that exceeds, with a given probability, the absolute value of the difference between two single measurement results obtained under the above-mentioned conditions
- If not stated, it is assumed that the probability level is 95 %
- When a sensor output **changes with time** (for a constant input), it is sometimes said that there are instabilities and that the sensor **drifts**
- In particular, some sensors have zero and scale factor drifts specified
- The zero drift describes output variations when the input is zero
- Scale factor drift describes sensitivity changes

R. Pallas-Areny, J. G. Webster, *Sensors and Signal Conditioning*, 2nd ed., Wiley, 2001.

Sensitivity

- The **sensitivity** or scale factor is the slope of the calibration curve, whether it is constant or not along the measurement range
- For a sensor in which output y is related to the input x by $y = f(x)$, the sensitivity $S(x_a)$, at point x_a, is

$$S(x_a) \triangleq \left.\frac{df(x)}{dx}\right|_{x=x_a}$$

- It is desirable in sensors to have a high and, if possible, constant sensitivity
- For a sensor with response $y = k \cdot x + b$ the sensitivity is $S = k$ for the entire range of values for x where it applies
- For a sensor with response $y = k \cdot x^2 + b$ the sensitivity is $2kx$, and it changes from one point to another over the measurement range

R. Pallas-Areny, J. G. Webster, *Sensors and Signal Conditioning*, 2nd ed., Wiley, 2001.

Linearity

- The **linearity** describes the closeness between the calibration curve and a specified straight line. Depending on which straight line is considered, several definitions apply:
  - **Independent Linearity**: the straight line is defined by the least squares criterion. With this system the maximal positive error and the minimal negative error are equal. This is the method that usually gives the ”best” quality
  - **Zero-Based Linearity**: The straight line is also defined by the least squares criterion but with the additional restriction of passing through zero
  - **Terminal-Based Linearity**: The straight line is defined by the output corresponding to the lower input and the theoretical output when the higher input is applied
  - **End-Points Linearity**: The straight line is defined by the real output when the input is the minimum of the measurement range and the output when the input is the maximum (FSO)
  - **Theoretical Linearity**: The straight line is defined by the theoretical predictions when designing the sensor

R. Pallas-Areny, J. G. Webster, *Sensors and Signal Conditioning*, 2nd ed., Wiley, 2001.

Linearity

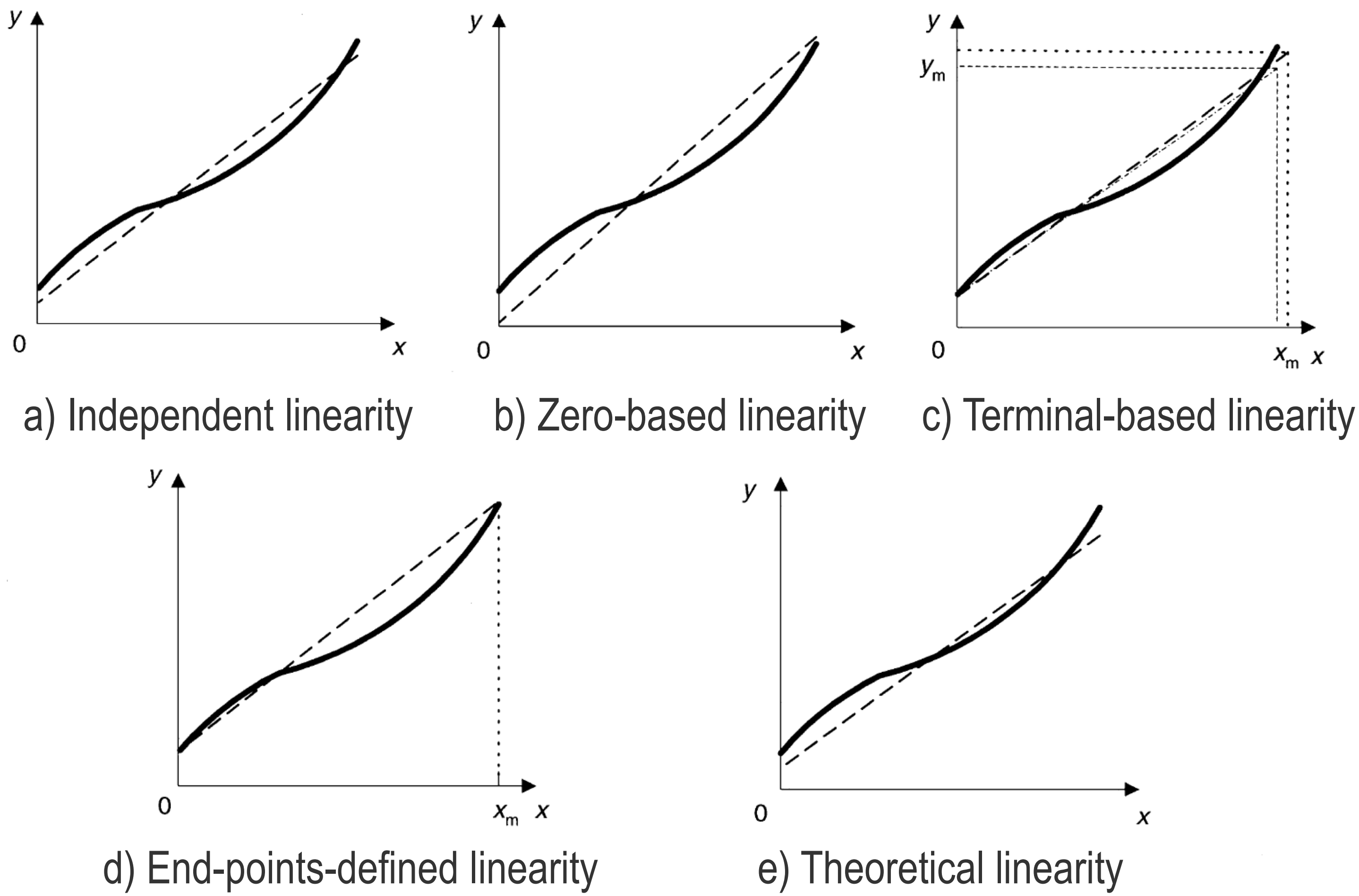


Figure 1.6 Different straight lines used as a reference to define linearity: (*a*) independent linearity (least squares method); (*b*) zero-based linearity (least squares adjusted to zero); (*c*) terminal-based linearity; (*d*) end-points-defined linearity; (*e*) theoretical linearity.

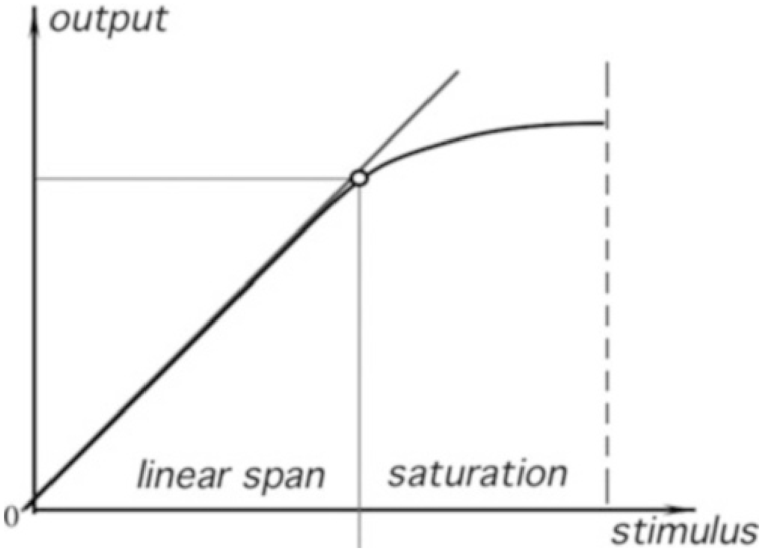
R. Pallas-Areny, J. G. Webster, *Sensors and Signal Conditioning*, 2nd ed., Wiley, 2001.

Linearity

- In summary, the linearity of the calibration curve indicates to what extent a sensor's **sensitivity is constant**
- However, for a sensor to be acceptable, it does not need to have a high linearity
- The interest of linearity is that when sensitivity is constant we only need to divide the reading by a constant value (the sensitivity) in order to determine the input
- In linear instruments the nonlinearity equals the inaccuracy
- Current measurement systems incorporate microprocessors so that there is more interest in repeatability than in linearity, because we can produce a lookup table giving input values corresponding to measured values
- By using interpolation, it is possible to reduce the size of that table to a reasonable dimension

R. Pallas-Areny, J. G. Webster, *Sensors and Signal Conditioning*, 2nd ed., Wiley, 2001.

Saturation



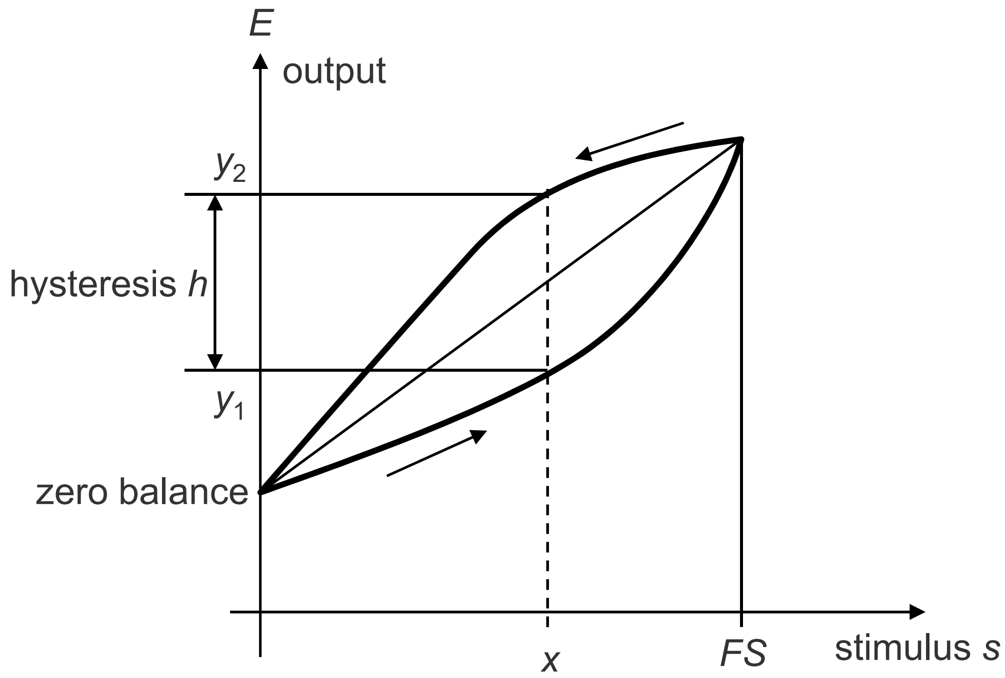
- Every sensor has its operating limits
- Even if it is considered linear, at some levels of the input stimuli, its output signal no longer will be responsive
- Further increase in stimulus does not produce a desirable output
- It is said that the sensor exhibits a span-end nonlinearity or **saturation**

Resolution and Threshold

- The main factors that influence linearity are **resolution**, **threshold**, and **hysteresis**
- The **resolution** (or discrimination) is the minimal change of the input necessary to produce a detectable change at the output
- When the input increment is from zero, then it is called the **threshold**
- When the input signal can display fast changes, the **noise floor** of the sensor determines the resolution
- Noise is a **random fluctuation** of the sensor output unrelated to the measured quantity

R. Pallas-Areny, J. G. Webster, *Sensors and Signal Conditioning*, 2nd ed., Wiley, 2001.

Hysteresis



- The **hysteresis** refers to the difference between two output values that correspond to the same input, depending on the direction (increasing or decreasing) of successive input values

J. Fraden, *Handbook of Modern Sensors – Physics, Designs, and Applications*, 5th edition, Springer, 2016.

Systematic Errors

- The static calibration of a sensor allows to detect and correct the so-called systematic errors
- An error is said to be **systematic** when in the course of measuring the same value of a given quantity under the same conditions, it remains constant in absolute value and sign or varies according to a definite law when measurement conditions change
- Because time is also a measurement condition, the measurements must be made in a short time interval
- Systematic errors yield **measurement bias**
- Such errors are caused not only by the instrument, but also by the method, the user (in some cases), and a series of factors (climatic, mechanical, electrical, etc.) that never are ideal - that is, constant and known
- The presence of systematic errors can therefore be discovered by measuring the same quantity with two different devices, by using two different methods, by using the readings of two different operators, or by changing measurement conditions in a controlled way and observing their influence on results
- To determine the consistency of the different results it is necessary to use statistical methods

R. Pallas-Areny, J. G. Webster, *Sensors and Signal Conditioning*, 2nd ed., Wiley, 2001.

Random Errors

- **Random errors** are those that remain after eliminating the causes of **systematic errors**
- They appear when the same value of the same quantity is measured repeatedly, using the same instrument and the same method
- They have the following properties:
  - Positive and negative random errors with the same absolute value have the same occurrence probability
  - Random errors are less probable as the absolute value increases
  - When the number of measurements increases, the arithmetic mean of random errors in a sample (set of measurements) approaches zero
  - For a given measurement method, random errors do not exceed a fixed value. Readings exceeding that value should be repeated and, if necessary, studied separately
- The absence of changes from one reading to another when measuring the same value of the same quantity several times does not necessarily imply an absence of random errors

R. Pallas-Areny, J. G. Webster, *Sensors and Signal Conditioning*, 2nd ed., Wiley, 2001.

Dynamic Characteristics

- The sensor response to variable input signals differs from that exhibited when input signals are constant, which is described by the static characteristics
- The reason is the presence of **energy-storing elements**, such as inertial elements (mass, inductance, etc.) and capacitance (electric, thermal, fluid, etc.)
- The dynamic characteristics are the **dynamic error** and **speed of response** (time constant, delay)
- They describe the behavior of a sensor with applied variable input signals
- The **dynamic error** is the difference between the indicated value and the true value for the measured quantity, when the static error is zero
- It describes the difference between a sensor's response to the same input magnitude, depending on whether the input is constant or variable with time
- The **speed of response** indicates how fast the measurement system reacts to changes in the input variable

R. Pallas-Areny, J. G. Webster, *Sensors and Signal Conditioning*, 2nd ed., Wiley, 2001.

Zero-Order Measurement Systems

- The output of a **zero-order** sensor is related to its input according to

$$y(t) = k \cdot x(t)$$

- Its behavior is characterized by its **static sensitivity** k and remains constant regardless of input frequency
- Hence, its **dynamic error** and its **delay** are both **zero**
- An input-output relationship such as above requires that the sensor does not include any energy-storing element

R. Pallas-Areny, J. G. Webster, *Sensors and Signal Conditioning*, 2nd ed., Wiley, 2001.

First-order Measurement Systems

- In a **first-order sensor** there is an element that stores energy and another one that dissipates it
- The relationship between the input $x(t)$ and the output $y(t)$ is described by a differential equation with the form

$$a_1 \cdot \frac{dy(t)}{dt} + a_0 \cdot y(t) = x(t)$$

- The corresponding transfer function is

$$H(s) \triangleq \frac{Y(s)}{X(s)} = \frac{k}{s \cdot \tau + 1}$$

- where $k = 1/a_0$ is the **static sensitivity** and $\tau = a_1/a_0$ is the **system's time constant**. The system's corner (angular) frequency is $\omega_c = 1/\tau$
- Two parameters are necessary to characterize the system: k for the **static response** and ω_c or τ for the **dynamic response**

R. Pallas-Areny, J. G. Webster, *Sensors and Signal Conditioning*, 2nd ed., Wiley, 2001.

First-order Measurement Systems Response

TABLE 1.4 Dynamic Error and Delay for a First-Order Measurement System for Different Common Test Inputs

Input	Dynamic Error	Delay
Step $u(t)$	0	τ
Ramp Rt	$R[t + k(\tau - t)]$ or $R\tau$	τ
Sinusoid A, ω	$1 - \dfrac{1}{\sqrt{1 + \omega^2\tau^2}}$	$\dfrac{\arctan \omega\tau}{\omega}$

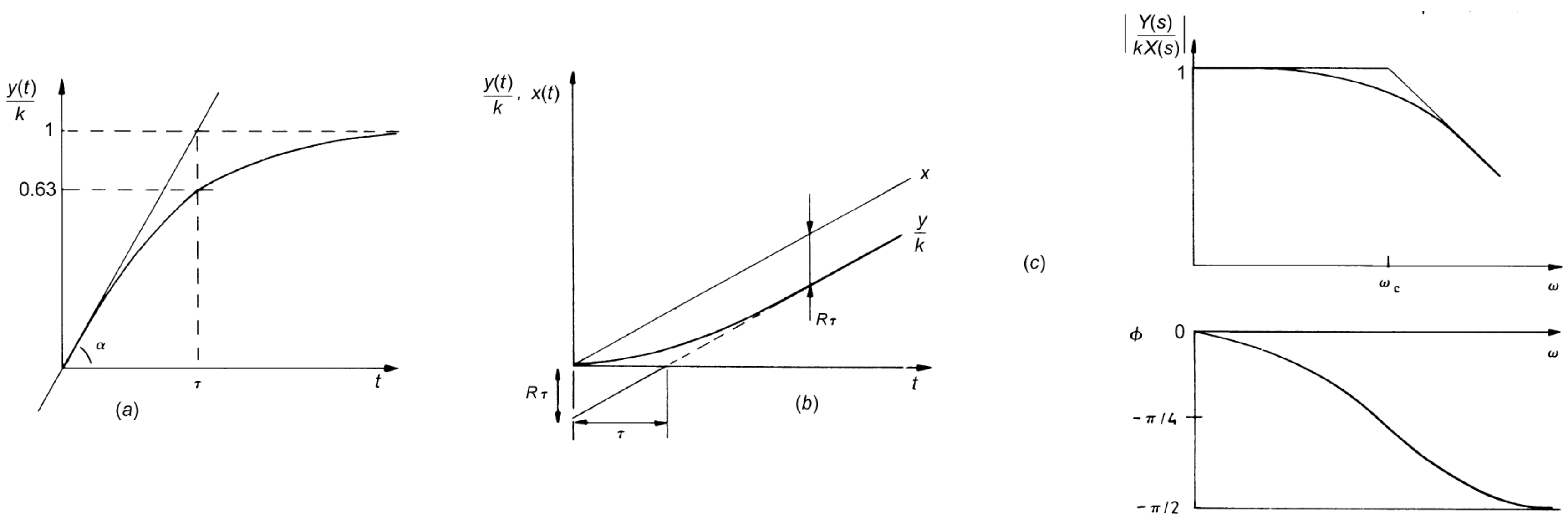


Figure 1.8 First-order system response to (*a*) a unit step input, (*b*) a ramp input, and (*c*) a sinusoidal input (amplitude modulus and phase).

R. Pallas-Areny, J. G. Webster, *Sensors and Signal Conditioning*, 2nd ed., Wiley, 2001.

2^{nd}-order Measurement Systems

- A **second-order sensor** contains two energy-storing elements and one energy dissipating element.
- Its input $x(t)$ and output $y(t)$ are related by a 2^{nd}-order linear differential equation

$$a_2 \cdot \frac{d^2 y}{dt^2} + a_1 \cdot \frac{dy(t)}{dt} + a_0 \cdot y(t) = x(t)$$

- The corresponding transfer function is

$$H(s) \triangleq \frac{Y(s)}{X(s)} = \frac{k \cdot \omega_n^2}{s^2 + 2\zeta \cdot \omega_n \cdot s + \omega_n^2}$$

- where k is the **static sensitivity**, ζ is the **damping ratio**, and ω_n is the **natural undamped angular frequency** for the sensor
- Two coefficients determine the dynamic behavior, namely $\omega_n^2 = \frac{a_0}{a_2}$ and $\zeta = \frac{a_1}{2\sqrt{a_0 \cdot a_2}}$, while a single one determines the static behavior $k = \frac{1}{a_0}$
- Notice that these three parameters are related and that a modification in one of them may change another one. Only a_0, a_1 and a_2 are independent

📖 R. Pallas-Areny, J. G. Webster, *Sensors and Signal Conditioning*, 2nd ed., Wiley, 2001.

2nd-order Measurement Systems – Response

TABLE 1.5 Outputs of a Second-Order Measuring System for Different Common Test Inputs

Input	Output	
Unit step $u(t)$		
$0 < \zeta < 1$	$1 - \dfrac{e^{-\delta t}}{\sqrt{1-\zeta^2}} \sin(\omega_d t + \phi)$	$\delta = \zeta\omega_n$ $\omega_d = \omega_n\sqrt{1-\zeta^2}$ $\phi = \arcsin\dfrac{\omega_d}{\omega_n}$
$\zeta = 1$	$1 - e^{-\delta t}(1 + \omega_n t)$	
$\zeta > 1$	$1 + \dfrac{\omega_n}{2\sqrt{\zeta^2-1}}\left(\dfrac{e^{-at}}{a} - \dfrac{e^{-bt}}{b}\right)$	$a = \omega_n(\zeta + \sqrt{\zeta^2-1})$ $b = \omega_n(\zeta - \sqrt{\zeta^2-1})$
Ramp Rt		
$0 < \zeta < 1$	$R\left\{t - \dfrac{2\zeta}{\omega_n}\left[1 - \dfrac{e^{-\zeta\omega_n t}}{2\zeta\sqrt{1-\zeta^2}}\sin(\sqrt{1-\zeta^2}\omega_n t + \phi)\right]\right\}$	$\phi = \arctan\left(\dfrac{2\zeta\sqrt{1-\zeta^2}}{2\zeta^2-1}\right)$
$\zeta = 1$	$R\left\{t - \dfrac{2\zeta}{\omega_n}\left[1 - \left(1 + \dfrac{\omega_n \tau}{2}\right)e^{-\omega_n t}\right]\right\}$	
$\zeta > 1$	$R\left\{t - \dfrac{2\zeta}{\omega_n}\left[1 + \dfrac{2\zeta(-\zeta - \sqrt{\zeta^2-1}+1)}{4\zeta\sqrt{\zeta^2-1}}e^{-at} + \dfrac{2\zeta(-\zeta - \sqrt{\zeta^2-1}-1)}{4\zeta\sqrt{\zeta^2-1}}e^{-bt}\right]\right\}$	
Sinusoid A, ω	$\dfrac{kA}{\sqrt{\left(1 - \dfrac{\omega^2}{\omega_n^2}\right)^2 + \left(\dfrac{2\zeta\omega}{\omega_n}\right)^2}} \sin(\omega t - \phi)$	$\phi = \arctan\dfrac{2\zeta\omega/\omega_n}{1 - \left(\dfrac{\omega}{\omega_n}\right)^2}$

R. Pallas-Areny, J. G. Webster, *Sensors and Signal Conditioning*, 2nd ed., Wiley, 2001.

2nd-order Measurement Systems – Response

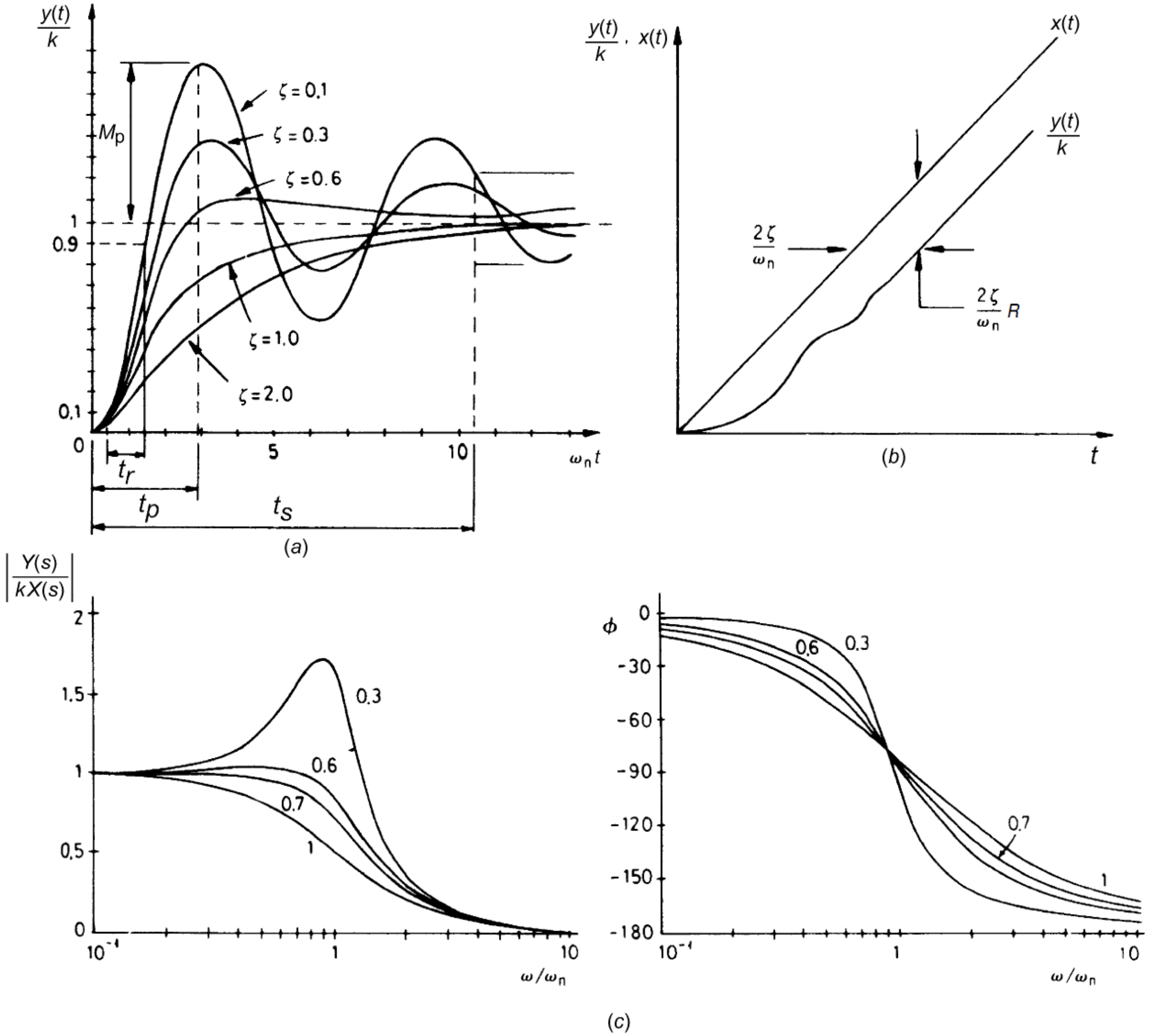


Figure 1.9 Second-order system response to (*a*) a unit step input, (*b*) a ramp input, and (*c*) a sinusoidal input (amplitude modulus and phase), for different damping ratios.

R. Pallas-Areny, J. G. Webster, *Sensors and Signal Conditioning*, 2nd ed., Wiley, 2001.

2^{nd}-order Measurement Systems – Rise-time and Overshoot

- When the input is a unit step, if the system is overdamped ($\zeta > 1$) or is critically damped ($\zeta = 1$) , there is neither overshoot nor steady-state dynamic error
- In an underdamped system, ($\zeta < 1$) , the steady-state dynamic error is zero, but the speed and the overshoot in the transient response are related
- In general, the faster the speed, the larger the overshoot
- The **rise time** t_r is the time spent to rise from 10% to 90% of the final output value, and it is given by

$$t_r = \frac{arctan(-\omega_d/\delta)}{\omega_d}$$

- where $\delta = \zeta \cdot \omega_n$ is the **attenuation** and $\omega_d = \omega_n \cdot \sqrt{1 - \zeta^2}$ is the **natural damped angular frequency**
- The time elapsed to the first peak t_p is

$$t_p = \frac{\pi}{\omega_d}$$

- and the maximum **overshoot** M_p is

$$M_p = e^{-(\delta/\omega_d)\pi}$$

R. Pallas-Areny, J. G. Webster, *Sensors and Signal Conditioning*, 2nd ed., Wiley, 2001.

2nd-order Measurement Systems – Settling Time

- The time for the output to settle within a defined band around the final value t_s, or **settling time**, depends on the width of that band
- For $0 < \zeta < 0.9$, for a $\pm 2\%$ band, $t_s \cong 4/\delta$, and it is minimal when $\zeta = 0.76$
- For a $\pm 5\%$ band, $t_s \cong 3/\delta$, and it is minimal when $\zeta = 0.68$
- The speed of response is optimal for $0.5 < \zeta < 0.8$
- When the input is a ramp with slope R, the steady-state dynamic error is

$$e_d = \frac{2\zeta \cdot R}{\omega_n}$$

- and the delay is $2\zeta/\omega_n$

R. Pallas-Areny, J. G. Webster, *Sensors and Signal Conditioning*, 2nd ed., Wiley, 2001.

2nd-order Measurement Systems – Frequency Response

- To describe the frequency response of a second-order system where $0 < \zeta < 0.707$, we note that the frequency of resonance is almost the same as the natural damped frequency

$$\omega_d = \omega_n \cdot \sqrt{1 - \zeta^2}$$

- and the magnitude of that resonance at $\omega = \omega_d$ is

$$M_r = \frac{1}{2\zeta\sqrt{1 - \zeta^2}}$$

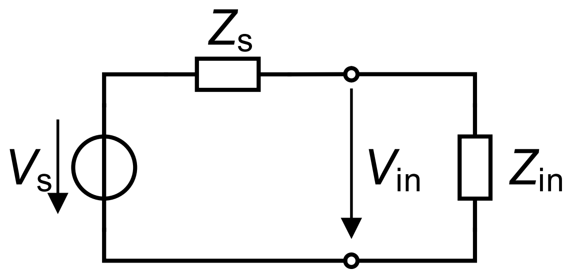
R. Pallas-Areny, J. G. Webster, *Sensors and Signal Conditioning*, 2nd ed., Wiley, 2001.

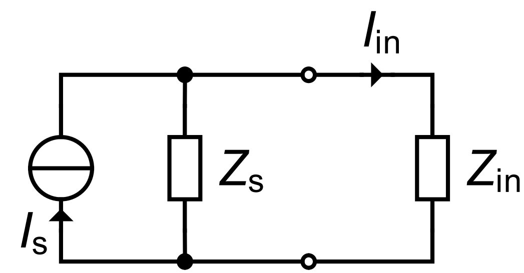
Dynamic Range

- The **dynamic range** for a measurand is the quotient between the measurement range and the desired resolution
- Any stage for processing the signal from a sensor must have a dynamic range equal to or larger than that of the measurand
- For example, to measure a temperature from 0 to 100 °C with 0.1 °C resolution, we need a dynamic range of at least (100 – 0)/0.1 = 1000 (60 dB). Hence a 10-bit ADC should be appropriate to digitize the signal because 2^{10} = 1024. Let us assume we have a 10-bit ADC whose input range is 0 to 10 V; its resolution will be 10 V/1024 = 9.8 mV. If the sensor sensitivity is 10 mV/°C and we connect it to the ADC, the 9.8 mV resolution for the ADC will result in a 9.8 mV/(10 mV/ ° C) = 0.98 ° C resolution! In spite of having the suitable dynamic range, we do not achieve the desired resolution in temperature because the output range of our sensor (0 to 1 V) does not match the input range for the ADC (0 to 10 V).
- The basic function of voltage amplifiers is to amplify the input signal so that its output extends across the input range of the subsequent stage

J. G. Webster, *The Measurement, Instrumentation and Sensors Handbook*, CRC Press, 1999.

Sensor Impedance and Loading Error



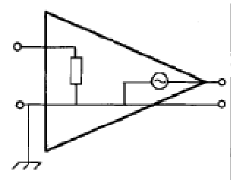
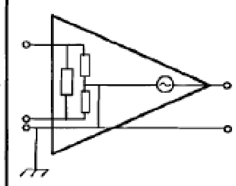
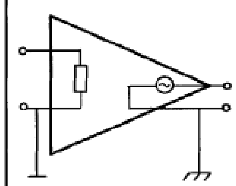
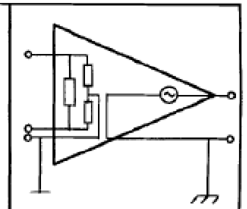
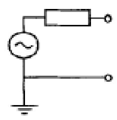
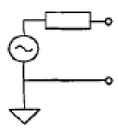
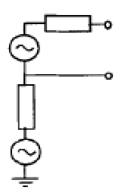


$$V_{in} = \frac{Z_{in}}{Z_{in} + Z_s} \cdot V_s \cong V_s \quad \text{for} \quad Z_{in} \gg Z_s$$

$$I_{in} = \frac{Z_s}{Z_{in} + Z_s} \cdot I_s \cong I_s \quad \text{for} \quad Z_{in} \ll Z_s$$

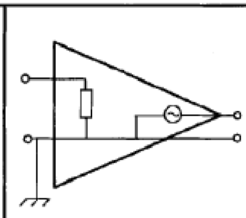
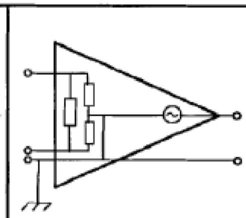
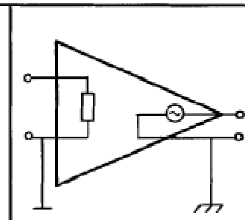
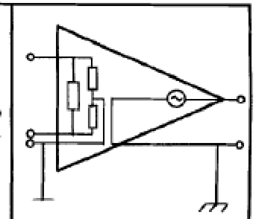
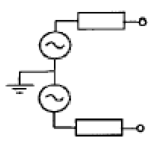
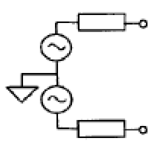
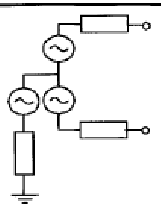
- Ideally, the measurement should not alter the value of the measured signal and any such alteration is a **loading error**
- The sensor output impedance Z_s determines the requirements of the input impedance Z_{in} of the signal conditioner
- Z_{in} also controls the power that is drawn from the sensor
- The voltage detected will equal the signal voltage only when $Z_{in} \gg Z_s$ otherwise $V_{in} \neq V_s$ and there will be a **loading effect** or **loading error**
- Signals with very high output impedance are better modeled as current sources
- In order for $I_{in} = I_s$, it is required that $Z_{in} \ll Z_s$
- If Z_{in} is not low enough, then there is a **shunting effect**

Compatibility between Signal Sources and Conditioners

Conditioner input / Signal source				
	Incompatible unless grounds are very close	Compatible if CMRR is large	Compatible	Compatible
	Compatible	Compatible	Compatible	Compatible
	Incompatible unless grounds are very close	Compatible if CMRR is large	Compatible for large Z_i	Compatible

J. G. Webster, *The Measurement, Instrumentation and Sensors Handbook*, CRC Press, 1999.

Compatibility between Signal Sources and Conditioners

Conditioner input / Signal source				
	Incompatible	Compatible	Compatible for large Z_i	Compatible
	Compatible	Compatible	Compatible	Compatible
	Incompatible	Compatible if CMRR is large	Compatible for large Z_i	Compatible

Note: When grounded, signals sources and amplifiers are assumed to be grounded at different points. Isolation impedance is assumed to be very high for floating signal sources but finite (Z_i) for conditioners.

J. G. Webster, *The Measurement, Instrumentation and Sensors Handbook*, CRC Press, 1999.

Sensor Model

- A mathematical model of a sensor is a powerful tool in assessing its performance and best interfacing it with the electronics
- The model may address both static and dynamic responses and usually includes the sensor's transfer function
- The model can be derived from the sensor physical operation by applying physical laws to some simple lumped parameter sensor elements
- For the analysis, a sensor is divided into simple elements and each element is considered separately
- However, once the equations describing the elements have been formulated, individual elements can be recombined to yield the model of the original sensor

J. Fraden, *Handbook of Modern Sensors – Physics, Designs, and Applications*, 5th edition, Springer, 2016.

Dynamic Models of Sensor Elements

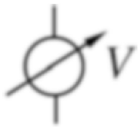
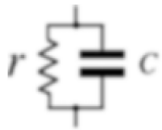
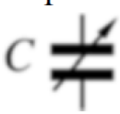
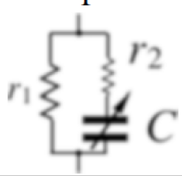
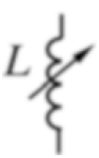
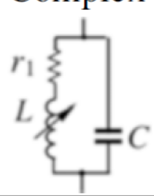
- The various elements and their governing equations for mechanical, thermal and electrical systems are summarized in the table below

Mechanical, thermal, and electrical analogies

MECHANICAL	THERMAL	ELECTRICAL	
MASS M $F=M\frac{d(v)}{dt}$	CAPACITANCE C $Q=C\frac{dT}{dt}$	INDUCTOR L $V=L\frac{di}{dt}$	CAPACITOR $i=C\frac{dV}{dt}$
SPRING k $F=k\int vdt$	CAPACITANCE C $T=\frac{1}{C}\int Qdt$	CAPACITOR C $V=\frac{1}{C}\int idt$	INDUCTOR L $i=\frac{1}{L}\int Vdt$
DAMPER b $F=bv$	RESISTANCE R $Q=\frac{1}{R}(T_2-T_1)$	RESISTOR R $V=Ri$	RESISTOR R $i=\frac{1}{R}V$

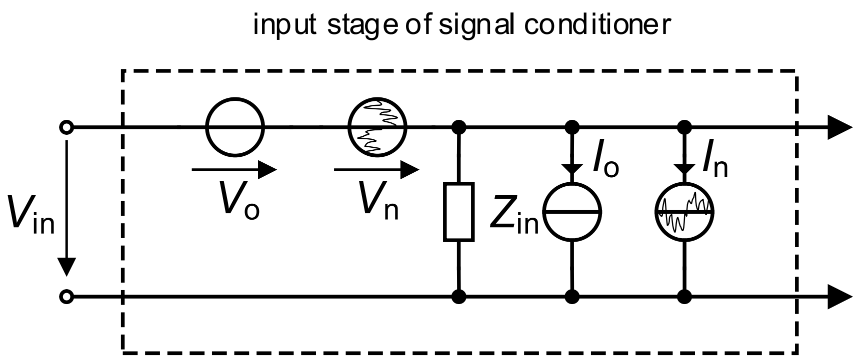
J. Fraden, *Handbook of Modern Sensors – Physics, Designs, and Applications*, 5th edition, Springer, 2016.

Signal Conditioners

Sensor type	Sensor impedance	Signal conditioner front stage
Voltage out 	Very low resistive 	High input resistance amplifier ("voltmeter")
Current out 	Very high complex 	High input impedance amplifier or low input resistance circuit ("amperemeter")
Resistive 	Resistive 	Resistance-to-voltage converter ("Ohm-meter")
Capacitive 	Complex 	Capacitance-to-voltage converter ("capacitance meter")
Inductive 	Complex 	Inductance meter

J. Fraden, *Handbook of Modern Sensors – Physics, Designs, and Applications*, 5th edition, Springer, 2016.

Equivalent Circuit of Signal Conditioner Input Stage



- Z_{in} is the interface circuit (i.e. amplifier) **input impedance**
- V_0 is the interface circuit **offset voltage**
- I_0 represents the input stage **bias current** and/or the external **leakage current**
- V_n is the equivalent input **noise** voltage of the interface circuit
- I_n is the equivalent input **shunt noise** of the interface circuit