EE-585 – Space Mission Design and Operations

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Ecole Polytechnique Fédérale de Lausanne

Week 09 – 14 Nov 2025

Today's outline

Planetary flybys and slingshots

Lunar trajectories I: getting to the Moon

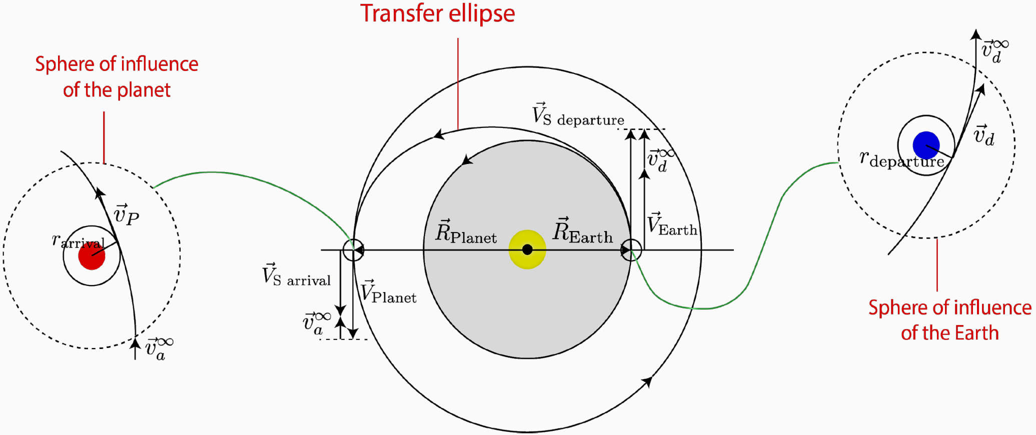
Lunar trajectories II: orbiting the Moon

Lunar trajectories III: returning to the Earth

Spacecraft propulsion

Planetary flybys and slingshots

Reminder: strategy for interplanetary transfer

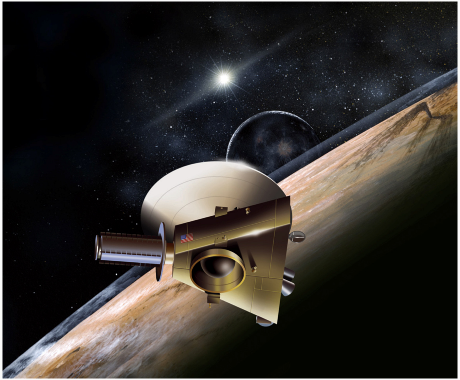


Gravity assist – a.k.a. slingshot – of New Horizons

Launched on 16 Jan 2006.
Gravity assist by Jupiter on 28 Feb 2007.
Pluto flyby on 14 July 2015.

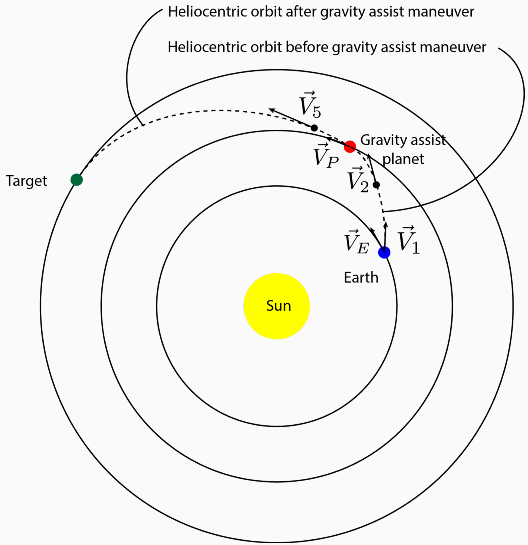
When a spacecraft on an planetocentric hyperbolic trajectory approaches a planet, it comes within its sphere of influence (SOI), passes its hyperbolic periapsis and exits the SOI, the gravity of that planet alters its heliocentric velocity $\vec{V}$ in amplitude $|\vec{V}|$ and direction $\vec{V}$.

The amount by which the spacecraft speeds up or slows down is determined by the geometry of the approach, passing behind or in front of the planet.



Credits: Credits: NASA, Southwest Research Institute, Johns Hopkins University Applied Physics Laboratory

Definitions



where

$\vec{V}_P$: Heliocentric velocity of the gravity-assist-planet.

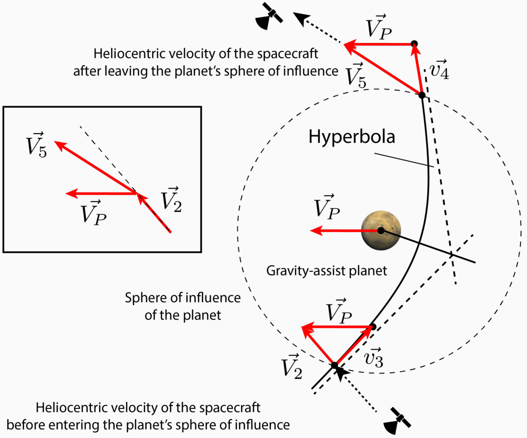
$\vec{V}_E = \vec{V}_\oplus$: Heliocentric velocity of Earth.

$\vec{V}_1$: Heliocentric velocity of the spacecraft after leaving Earth's sphere of influence.

$\vec{V}_2$: Heliocentric velocity of the spacecraft entering the gravity-assist planet's sphere of influence.

$\vec{V}_5$: Heliocentric velocity of the spacecraft leaving the gravity-assist planet's sphere of influence.

Slingshot manoeuvre profile



$\vec{V}_2$: Heliocentric velocity of the spacecraft entering the planet's sphere of influence (SOI).

$\vec{V}_P$: Heliocentric velocity of the planet.

$\vec{v}_3 = \vec{v}_a^{\infty}$: Planetocentric velocity of the spacecraft entering the planet's SOI.

$\vec{v}_4 = \vec{v}_d^{\infty}$: Planetocentric velocity of the spacecraft leaving the planet's SOI.

$\vec{V}_5$: Heliocentric velocity of the spacecraft leaving the gravity-assist planet's SOI.

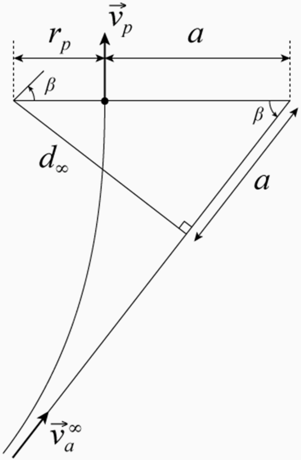
with

$$|\vec{v}_a^{\infty}| = |\vec{v}_3| = |\vec{v}_4| = |\vec{v}_d^{\infty}|$$

and, *in this example*,

$$|\vec{V}_5| = |\vec{V}_P + \vec{v}_4| > |\vec{V}_P + \vec{v}_3| = |\vec{V}_2|$$

Reminder: determination of important parameters



Conservation of total energy $\rightarrow v_p^2 = (v_a^\infty)^2 + v_{E r_p}$

As $a \cong \frac{\mu}{(v_a^\infty)^2}$

$a^2 + b^2 = c^2 = (a + r_p)^2$

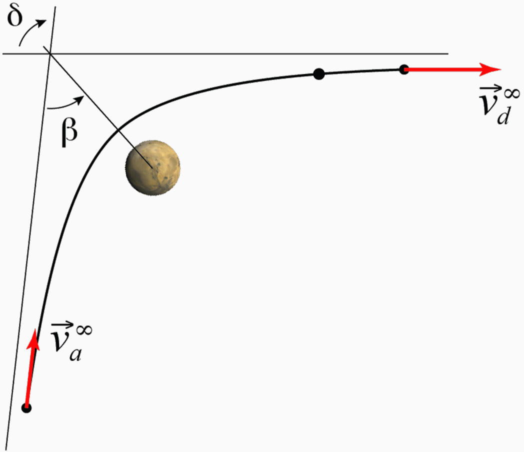
and $b = d_\infty$,

$$r_p = -\frac{\mu}{(v_a^\infty)^2} + \sqrt{\frac{\mu^2}{(v_a^\infty)^4} + d_\infty^2}$$

and

$$\cos\beta = \frac{a}{a + r_p} = \frac{a}{c}$$

Slingshot manoeuvre profile



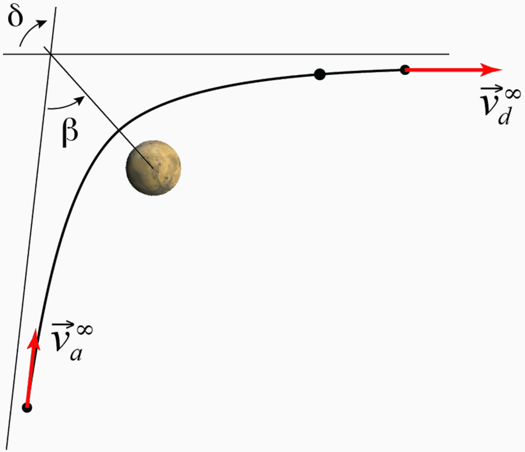
In the vicinity of a planet, the trajectory of a spacecraft is hyperbolic with a periapsis at distance r_p to the center of the planet.

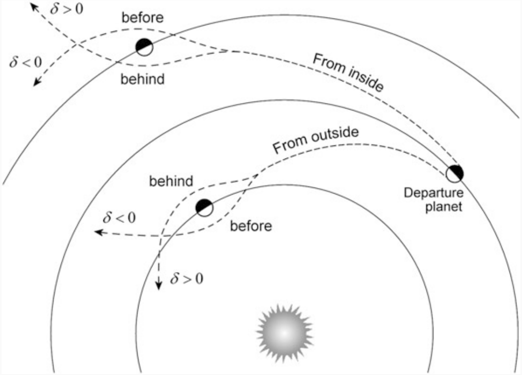
δ is the angle between the directions of $\vec{v}_a^{\infty}$ and $\vec{v}_d^{\infty}$.

$$\cos\left(\frac{\delta}{2}\right) = \cos\beta = \frac{a}{a + r_p}$$

$\rightarrow$ A slingshot manoeuvre – aka gravity assit – can be designed to increase or decrease the spacecraft's heliocentric velocity. You have to adjust the arrival angle β (which depends on d_∞, the aiming radius) and the angle between $\vec{V}_P$ and $\vec{v}_a^{\infty}$ to design the manoeuvre.

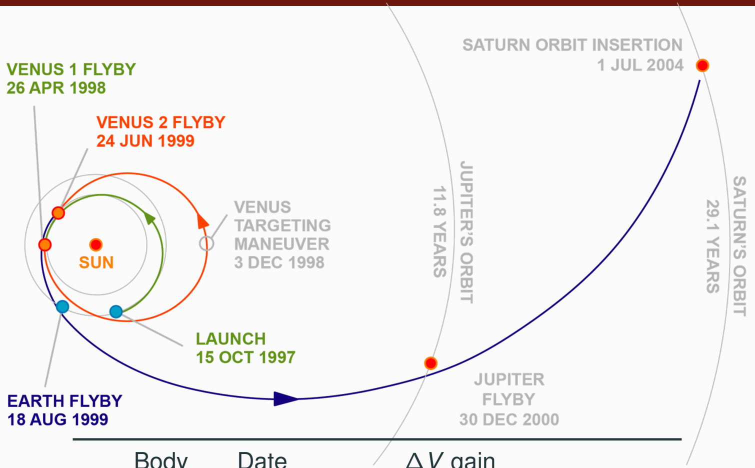
ΔV depends on the angle δ





Credits: U. Walter, *Astronautics, The Physics of Space Flight*, 3rd Ed.

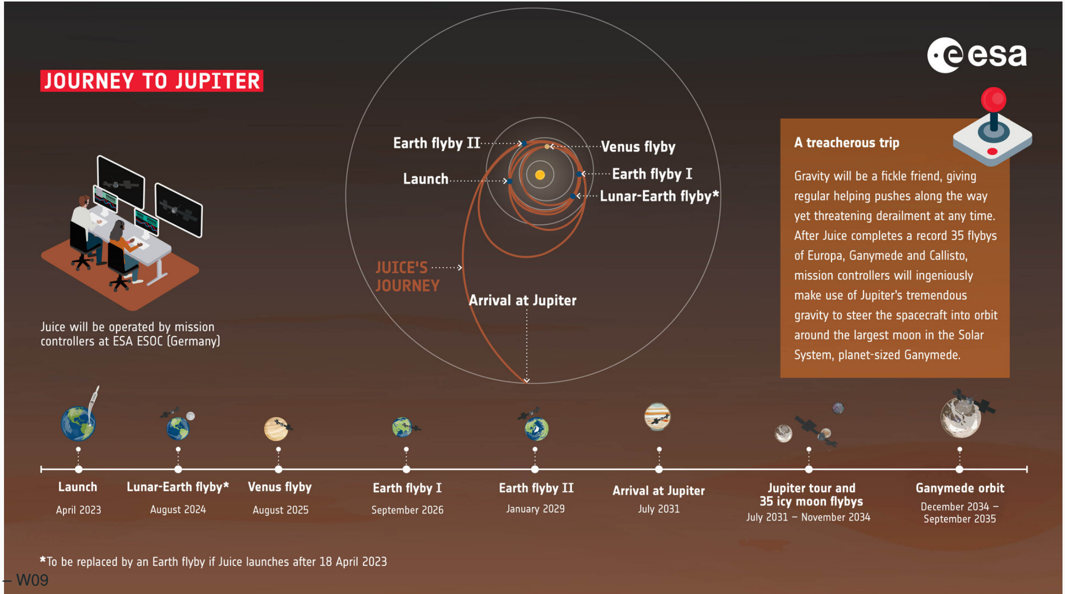
Cassini-Huygens (4 gravity assists)



	Body	Date	ΔV gain
1	Venus	26 Apr 1998	7 km/s
2	Venus	24 Jun 1999	sent C.-H. towards Earth
3	Earth	18 Aug 1999	5.5 km/s
4	Jupiter	30 Dec 2000	2 km/s

Credits: Credits: PD, USGOV

ESA's Juice: 8 years to destination with 4 gravity assists



ESA's Juice: August 2024 Moon-Earth flyby

Credits: ESA/Juice/JMC

The flyby of the Moon increased the heliocentric speed by 0.9 km/s, guiding it towards Earth.

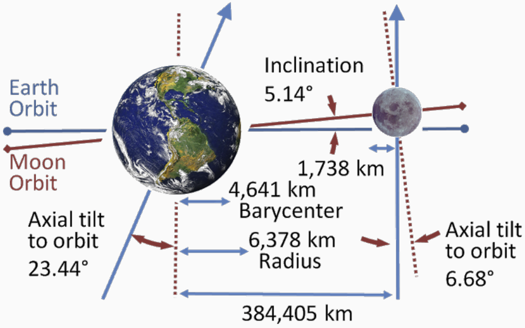
The flyby of Earth reduced the heliocentric speed by 4.8 km/s, guiding Juice onto a new trajectory towards Venus.

Overall, the lunar-Earth flyby deflected Juice by an angle of 100° compared to its pre-flyby path.

In the month before the flyby, spacecraft operators gave Juice slight nudges to put it on exactly the right approach trajectory. Overall, flybys will have saved 100-150 kg of fuel.

Lunar trajectories I: getting to the Moon

The Earth-Moon system



Credits: SeriousScience.org/NASA

The $\oplus$-☽ barycenter orbits the Sun in 1 year (by definition).

$\oplus$-☽ system revolve around the center of mass in 27.31 days, with up to 7 hours variations because of solar perturbations.

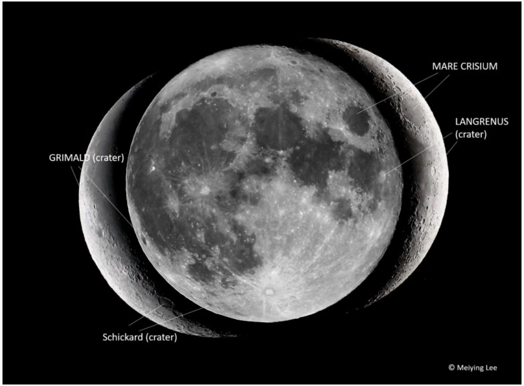
$d_{\oplus☽} \sim 384,400\,\text{km} \sim 40\%$ of Earth's Sphere of influence

Eccentricity $e \approx 0.055$. During a lunar month, the distance to $\oplus$ varies by $\sim 7.5R_{\oplus}$

Inclination of Moon's orbit $i \approx 4.98 - 5.3°$, on average $5.14°$

The inclination w.r.t. Earth's equator varies between $23.44 \pm 5.14 = 18.3° - 28.6°$.

Lunar libration



The Moon period of revolution around the Earth is the same as period of rotation around its axis $\rightarrow$ always same face turned towards Earth (tidal locking).

We can see $\approx$ 59% of the Moon's surface $\rightarrow$ "lunar libration". Three causes:

- Inclination of the orbit
- Eccentricity of the orbit
- Non-sphericality of the bodies

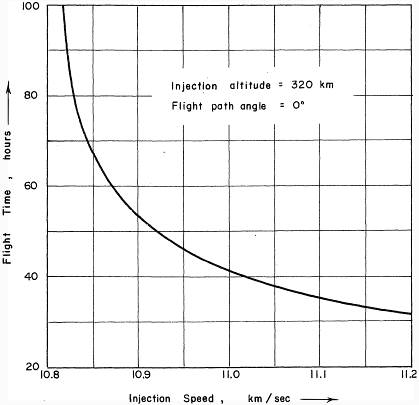
Simple Earth-Moon trajectories

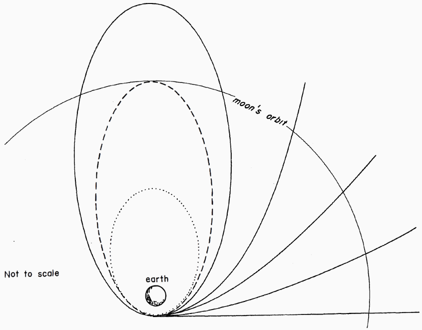
We could take all the perturbations into account and simulate a trajectory. Because of the Moon's motion, plans would heavily depend on the time of departure $\rightarrow$ computationally intensive.

Let's make simplifying assumptions:

- $e_{☽} = 0$, $r_{☽} = 384,400\,\text{km}$
- Spacecraft trajectory is coplanar with Moon's orbit $\rightarrow$ this usually the actually selected geometry to avoid a plane change.
- Neglect the Moon's gravitational field (*we will relax this soon!*)

Moon injection trajectories



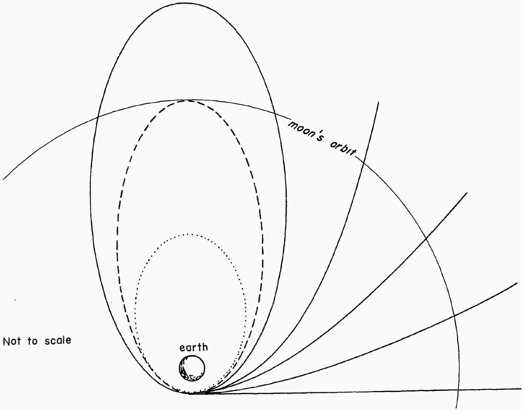


Credits: Bate et al., *Fundamentals of astrodynamics*

Significant reduction in time of flight is possible with only modest increases in injection speed.

Flight time for Apollo lunar landing missions is $\sim$ 72 hours.

Analysis of possible trajectories



Credits: Bate et al., *Fundamentals of astrodynamics*

Minimum injection speed is 10.82 km/s $\Longrightarrow$ $t_{\text{ToF}} \approx 120$ hours, this is the maximum possible flight time (on a Hohmann transfer).

Minimum eccentricity is $e = 0.966$

$v_{\text{apo}} = 0.188$ km/s ($v_{☽} = 1$ km/s $\rightarrow$ impact on the leading = eastern edge). Spacecraft going faster will impact on the side of the Moon facing us.

Patched-conic approximation for lunar trajectories

Moon's gravitational well is significant! $\rightarrow$ let's remove the massless Moon assumption.

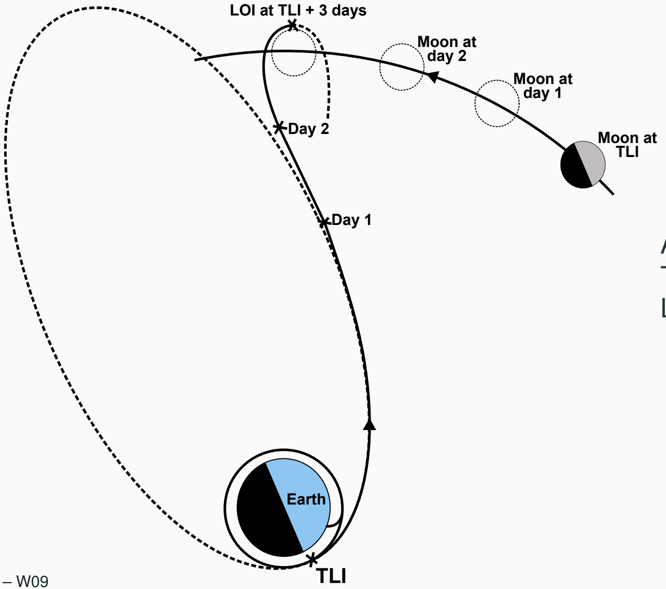
Geocentric motion becomes a selenocentric motion upon entering the Moon's sphere of influence.

As in the interplanetary trajectories ($\rightarrow$ week 8), the transition from geocentric motion to selenocentric motion is a gradual process which takes place over a finite arc of the trajectory during which both Earth and Moon affect the path.

Good enough approximations for preliminary mission analysis.

$R_{\mathrm{SOI},☽} = 66,300\,\mathrm{km} \approx 1/6 d_{\oplus ☽} = 8.5^\circ$ as seen from Earth

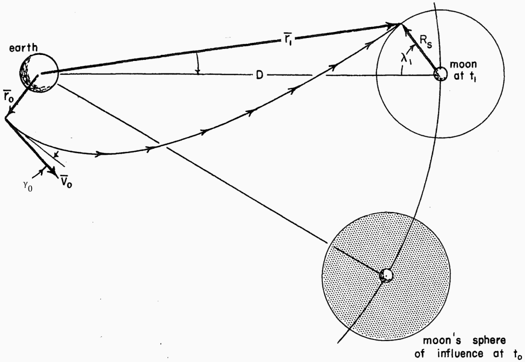
Example: Apollo trajectory from launch to Lunar Orbit Insertion (LOI)



Apollo missions typically made:
TLI burn at 2h45min MET
LOI burn at 3d 3h MET.

Credits: W. D. Woods, *How Apollo Flew to the Moon*

Translunar injection point and trajectory



Credits: Bate et al., *Fundamentals of astrodynamics*

There are 4 quantities that completely specify the geocentric phase:

r_0, v_0 state vector of the translunar injection point (TLI)

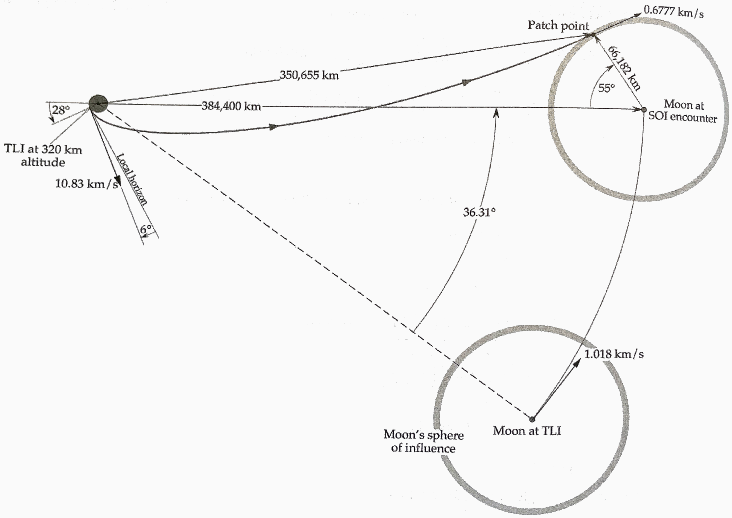
γ_0 the flight path angle at TLI's burn

λ_1 is the point at which geocentric trajectory crosses the Moon's SOI.

The time of flight (ToF) from TLI to SOI can be computed from the eccentric anomalies, but from the previous slides, ToF$\sim$ 40 – 120 hours.

$\rightarrow$ Fast computation (i.e. 2 body problem) needed to iterate over the 4 quantities to find optimal trajectory.

Example: departing from the Earth



Parking orbit at 320 km altitude.

When angle perigee-TLI $\alpha_0 = 28°$, the probe is launched into a translunar trajectory with a flight path angle of $\gamma_0 = 6°$

$\Longrightarrow v_0 = 10.8\,\text{km/s}$

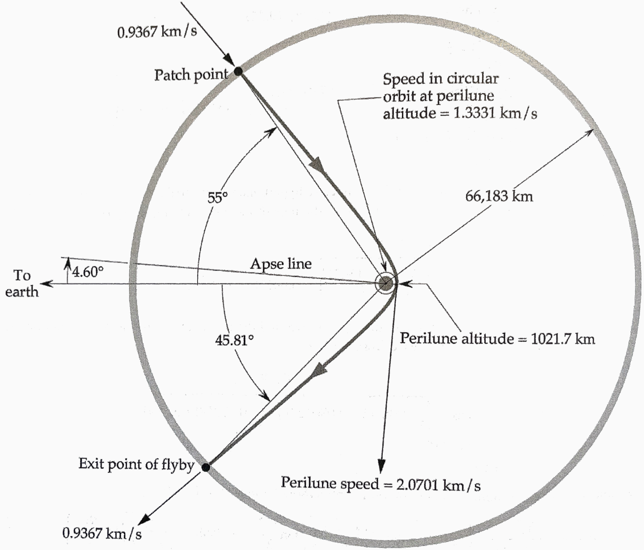
$\Longrightarrow v_1 = 0.7\,\text{km/s}$

$\Longrightarrow t_{\text{ToF}} = t_1 - t_0 = 66.5\,\text{h}$

Moon lead angle at TLI of $\omega_m t_{\text{ToF}} = 36.3°$ (i.e. phase angle)

Credits: Curtis, *Orb. Mech. for Eng. Students*

Example: flyby from patch point



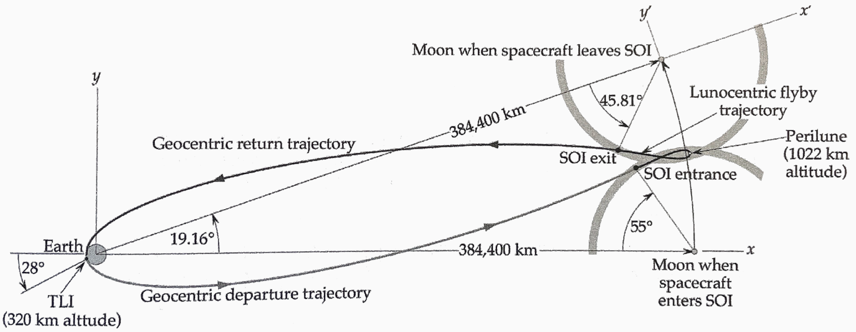
This is the same as an arrival to a planet after an interplanetary trajectory.
ToF to perilune = 17.5 hours

$\vec{v}_2 = \vec{v}_1 - \vec{v}_{☽} \implies v_2 = 0.96\,\text{km/s}$

alt. $z_{p_2} \approx 1020\,\text{km}$

$\Delta v_2 = -0.73\,\text{km/s}$ to circularise

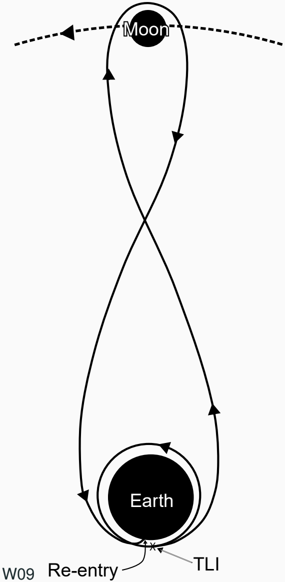
Example: complete coplanar ballistic trajectory



Credits: Curtis, *Orb. Mech. for Eng. Students*

Inertial Earth-fixed reference frame, drawn to scale.
At the exit of the lunar SOI, the geocentric $v_3 = 0.65\,\text{km/s}$,
$e_3 = 0.97 \implies z_3 = 6090\,\text{km} < R_\oplus \rightarrow$ **free-return trajectory: only one Δv at TLI**.

Free-return trajectory



Rotating Earth-Moon fixed reference frame.

Note the slightly asymmetry in the trajectory such that a re-entry is guaranteed upon return to Earth.

This constrains some parameters such as the perilune, to get the right lunar gravity assist.

Credits: Adapted from W. D. Woods, *How Apollo Flew to the Moon*

Lunar trajectories II: orbiting the Moon

Orbits within the sphere of influence

At **low altitude** ($z_{☽} \lesssim 100\,\text{km} \implies T \sim 2.1$ hours), the orbits are **significantly perturbed** by the inhomogeneous mass distribution.

There are **frozen low altitude orbits** (27°, 50°, 76°, and 86°) in which a spacecraft can stay in a low orbit indefinitely, discovered in 2001 (!)

Mountains on Moon can reach ~ 6.1 km height $\rightarrow$ be careful.

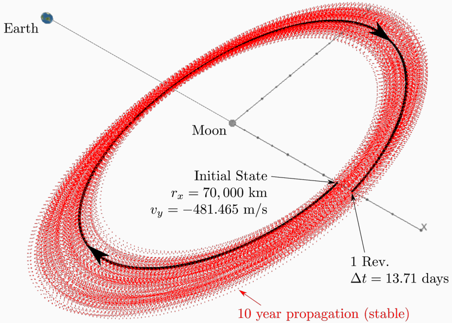
High lunar orbits are **significantly perturbed** by the Earth's gravitational influence (even if within the SOI). Integration of the three-body model integration is necessary.

Orbits in cislunar space: DRO

The Lagrange points (→ week 3) of the Earth-Moon system can provide stable orbits in the lunar vicinity, such as halo orbits and distant retrograde orbits.

Distant Retrograde Orbits (DROs) move in a retrograde direction around the Moon, appearing as large quasi-elliptical inplane orbits. The gravitational perturbations are significant from both bodies.

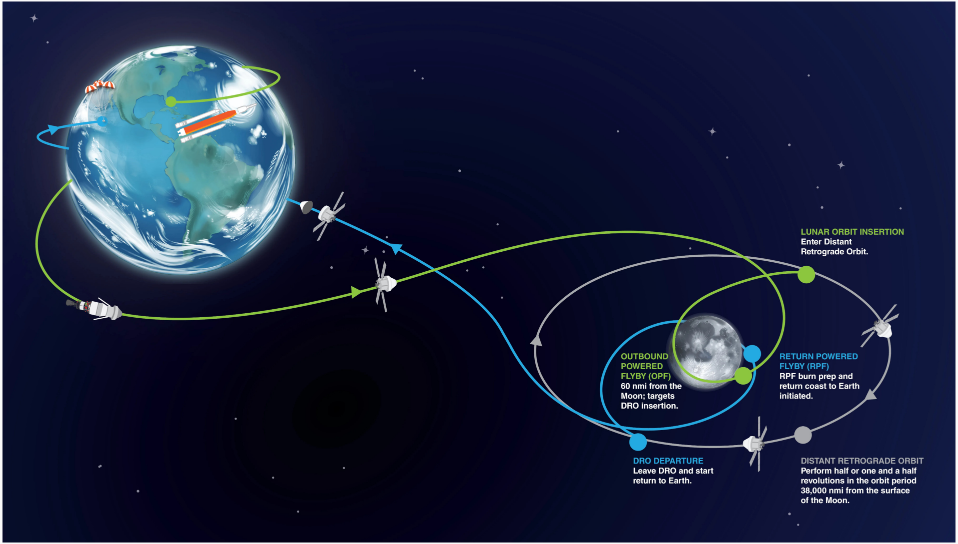
This is the orbit chosen for the Artemis 1 mission (launch date 16 Nov 2022, duration 25.4 days with 6 days in DRO).



Credits: F. Dawn, *Trajectory Design Considerations for Exploration Mission 1*, 2018

In a realistic force model, the orbits are quasi-periodic and the line of nodes (x-axis) crossing distance oscillates over time, but remain very stable. Little Δv is required for station-keeping.

Artemis I mission profile – 22 Nov 2022 - 11 Dec 2022



Credits: NASA

Artemis II mission profile – Not Earlier Than (NET) Feb 2026 - 10 days

Credits: NASA

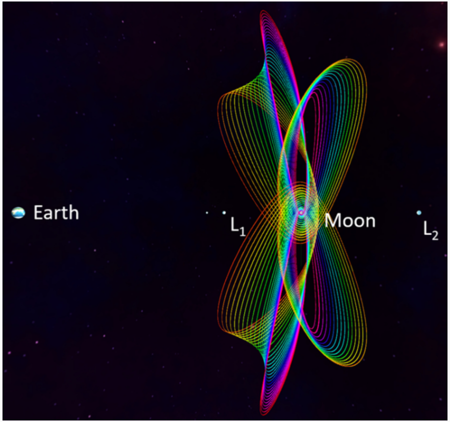
Free-return trajectory. Perilune $z_p = 7,400\,\text{km}$ from surface

Orbits in cislunar space: Near Rectiline Halo Orbits (NRHOs)

Near Rectiline Halo Orbits (NRHOs) are out-of-plane orbits (whereas DROs are inplane).

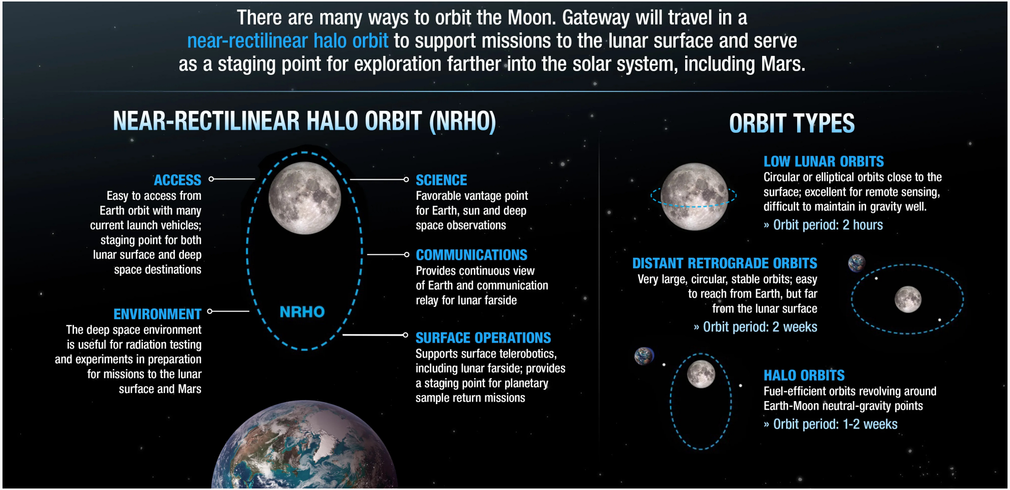
Perilune z_p in the range 1850-17,350 km altitude, period 1-2 weeks.

Low Δv requirements for station keeping (down to $\sim 0.2 - 25$ m/s for 1 year)



Credits: D. Davis et al., *Orb. Maint. and Nav. of Human S/C at cislunar NHROs*, 2017

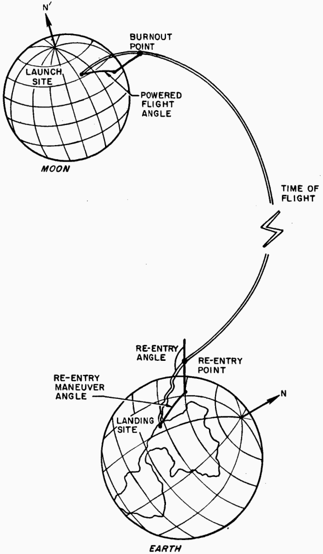
Lunar orbiting station: Gateway (status unclear, likely dropped)



Credits: NASA

Lunar trajectories III: returning to the Earth

Independent parameters to the problem

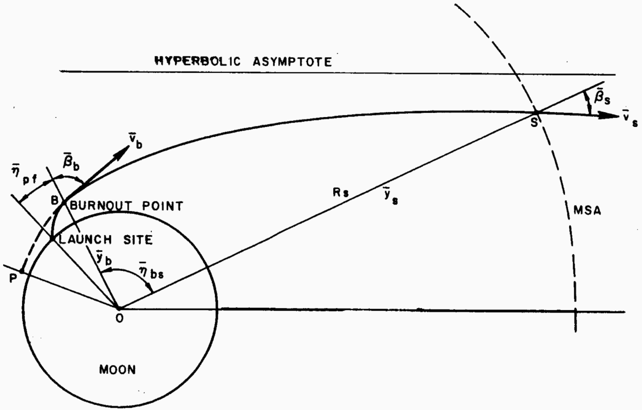


Independent parameters for Moon-to-Earth trajectory:

- launch site
- burnout altitude
- landing site latitude
- re-entry parameters
- combination of day of launch, landing site longitude and the total time of flight.

To solve for all parameters, we need an iterative back-propagation approach: starting with landing and making our way to Moon launch.

This discussion follows P. A. Penzo, *An Analysis Of Moon-To-Earth Trajectories*, 1961

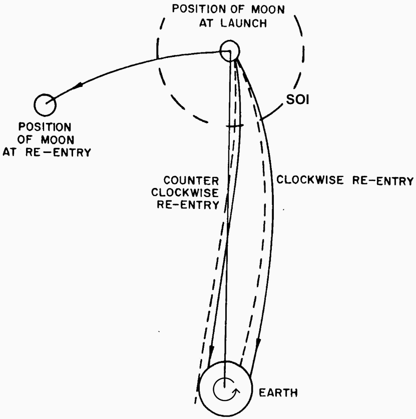


MSA = Moon sphere of action = SOI

There is a powered flight from launch site to burnout point, then ballistic trajectory.

Treatment is similar to the Earth's departure for an interplanetary transfer $\rightarrow$ hyperbolic trajectory.

Errors made using the patched-conic approximation



Dashed line: exact trajectory

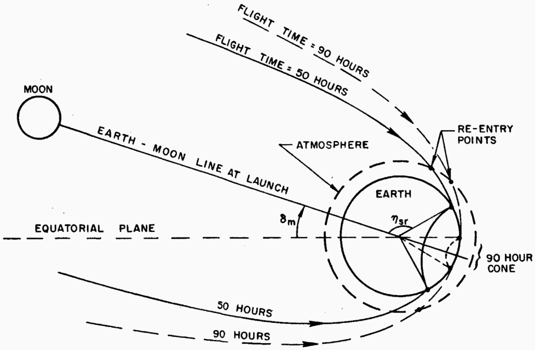
Full line: Patched 2 body approximation

The effect of the Moon on the Earth phase trajectory is 2-3$\times$ larger than the effect of the Earth on the Moon phase.

This choice of parameters makes the patched-conic approx. more accurate: faster trajectories, steeper re-entry angles.

Actual re-entry point is east of the desired point. This comes from the fact the Moon is moving east (as seen from Earth) $\rightarrow$ the Moon pulls the trajectory eastwards.

Allowable touchdown cones



Cones for fixed re-entry angle and two flight times.

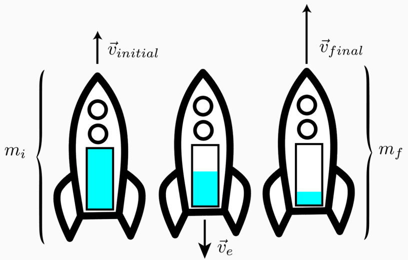
The maximum allowable latitude will be reached for trajectories passing the north pole whereas minimum latitude will be for a trajectory passing over the south pole.

For a given landing site latitude, ToF, re-entry flight path, there are limited possible declinations of the Moon δ_m (which is equivalent to days of the lunar month) $\rightarrow$ trajectories and landing site restricted by Earth-Moon system geometry.

Longitude of landing site depends on launch time, ToF, re-entry angle.

Spacecraft propulsion

Tsiolkovsky equation or rocket equation



$$\Delta v = v_e \ln\left(\frac{m_i}{m_f}\right)$$

where:
Δv is the change of velocity induced by the propulsion system,
v_e is the exhaust velocity of the gas in the propulsion system,
m_i, m_f are the initial and final mass.

Valid in free space. Gravitational field-induced and perturbations-induced Δv will be added to the propulsion-induced Δv.

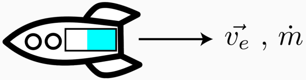
Thrust and acceleration

Thrust of the propulsion system (free space case):

$$F = v_e \dot{m} \quad \text{with } \dot{m} \text{ mass flow and } v_e \text{ exhaust velocity}$$

The resulting acceleration of the spacecraft:

$$\frac{\mathrm{d}v}{\mathrm{d}t} = \frac{F}{m} = v_e \frac{\dot{m}}{m}$$



Integrating between the initial and final conditions:

$$\boxed{\Delta v = v_e \ln\left(\frac{m_i}{m_f}\right)}$$

Tsiolkovsky equation

Thrust and acceleration

Other form of the Tsiolkovsky equation:

$$\boxed{\Delta v = I_{sp} g_0 \ln\left(\frac{m_i}{m_f}\right)}$$

where
g_0 is always Earth's gravity acceleration, 9.81 m/s^2,
I_{sp} is the specific impulse, measured in seconds, that is

$$I_{sp} = \frac{\text{Thrust}}{\text{Sea-level weight flow of consumption}} = \frac{F}{\dot{m} g_0} = \frac{v_e}{g_0}$$

Specific Impulse I_{sp}

I_{sp} is the specific impulse, measured in seconds, that is

$$I_{sp} = \frac{\text{Thrust}}{\text{Sea-level weight flow of consumption}} = \frac{F}{\dot{m} g_0} = \frac{v_e}{g_0}$$

It is an important performance parameter for a thruster/propellant combination.

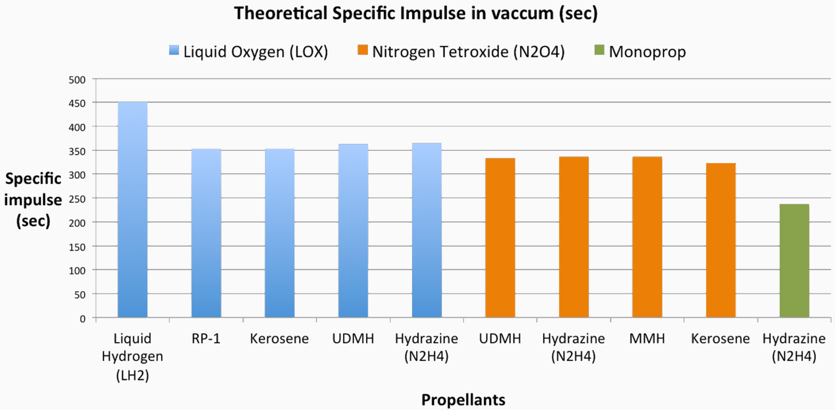
I_{sp} gives how many seconds a given propellant, with a given engine, can accelerate its own initial mass at 1 g_0 (i.e. Earth's surface gravity). The longer it can accelerate its own mass, the more Δv it delivers to the whole system.

For a given thrust F, if $I_{sp} \nearrow$, $\dot{m} \searrow$, i.e. less propellant need to reach the same Δv.

A large $I_{sp} \implies$ large thrust F if and only if the mass flow $\dot{m}$ is large.

By definition, I_{sp} is always linked to $g_0 = 9.81$ m/s^2 as a constant. It will never be the local gravitational acceleration, but the Earth's surface one.

Kind of chemical propulsion



Credits: NASA

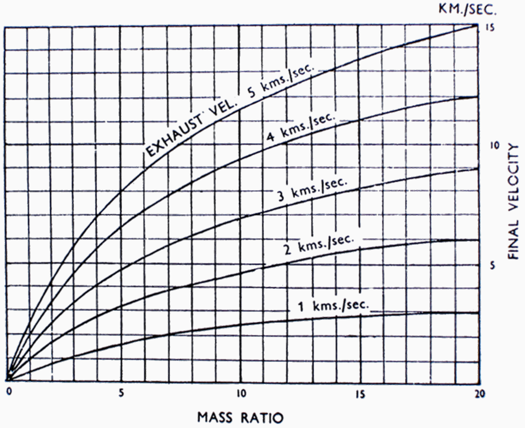
Cold gas propulsion: $I_{sp} = 50 - 75$ s, solid motor: $I_{sp} = 280 - 300$ s

Chemical propulsion is typically used for launch, orbit insertion and large & fast manoeuvres. The highest value I_{sp} is for liquid hydrogen as propellant and liquid oxygen as oxidizer. It will give $I_{sp} = 450$ s for the specific impulse, which means $v_e \approx 4.5$ km/s.

Final velocity VS Mass ratio

The typical final velocity for a rocket to reach LEO is $v_f \approx 10$ km/s and make up for losses (e.g. gravitational and drag).

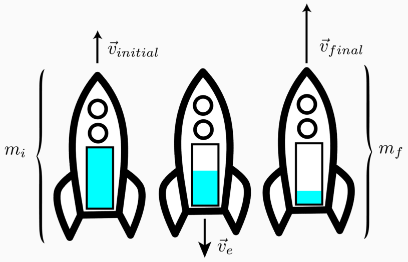
For a LH2/LOX mix ($v_e \approx 4.5$ km/s), the mass ratio is about 10 $\implies$ about 90% of the total mass at lift off is propellant mass.



Credits: *Ascent to Orbit*, Arthur C. Clarke

Mass of propellant needed

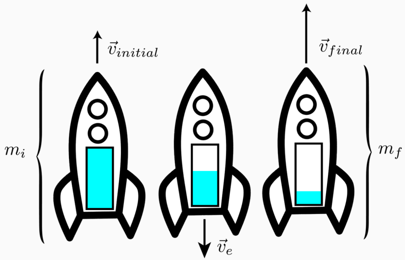
$$\Delta v = I_{sp} g_0 \ln\left(\frac{m_i}{m_f}\right) \implies \begin{cases} m_p = m_i \left[1 - \exp\left(-\frac{\Delta v}{I_{sp} g_0}\right)\right] \\ m_p = m_f \left[\exp\left(\frac{\Delta v}{I_{sp} g_0}\right) - 1\right] \end{cases}$$



where
Δv is the change of velocity induced by the propulsion system,
m_i, m_f are the initial and final mass [kg],
m_p is the propellant mass consumed to produce the given Δv [kg],
g_0 is Earth's gravity acceleration, 9.81 m/s^2,
I_{sp} is the specific impulse [s].

Mass of propellant needed

$$\Delta v = I_{sp} g_0 \ln\left(\frac{m_i}{m_f}\right) \implies \begin{cases} m_p = m_i\left[1 - \exp\left(-\frac{\Delta v}{I_{sp} g_0}\right)\right] \\ m_p = m_f\left[\exp\left(\frac{\Delta v}{I_{sp} g_0}\right) - 1\right] \end{cases}$$



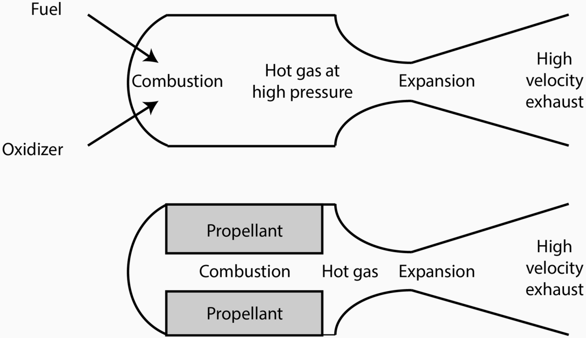
The *wet* mass is the initial mass of the spacecraft + propellant

The *dry* mass is the mass of the spacecraft with no propellant.

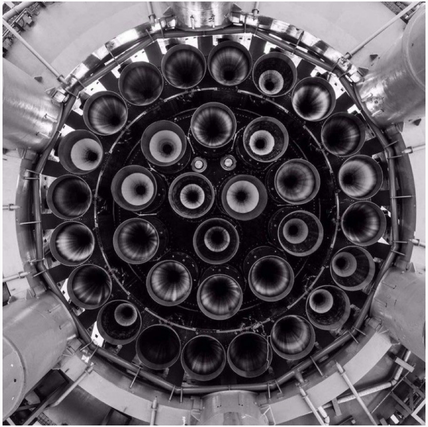
Liquid and solid propellant rocket motors

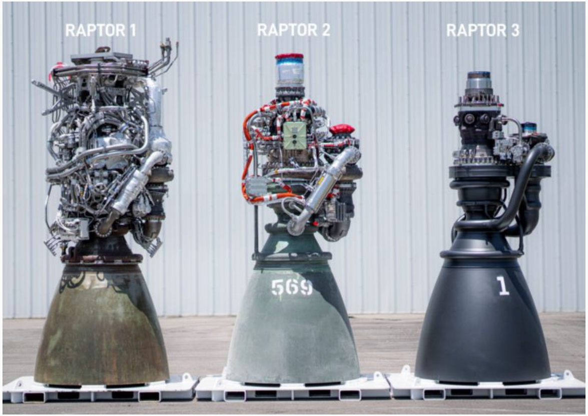
In addition to the central main engine Vulcain, Ariane 6 has 2 or 4 solid propellant boosters with a central cavity in order to increase the surface of the burning propellant generating thrust.

Credits: ESA



Starship – 33 Raptor engines

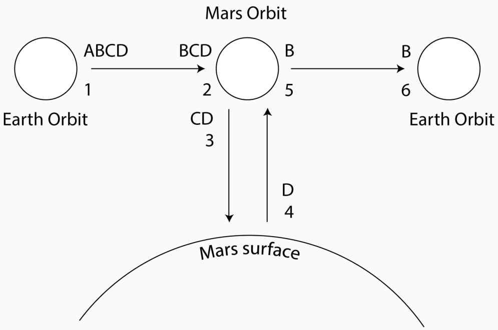




Credits: SpaceX

Cryogenic liquid CH_4 and LOX, a mixture known as methalox. Thrust of Raptor 3 $\approx$ 2.75 MN, $I_{sp} = 327$ s. Design for re-use with minimal maintenance.

Propellant needed for a mission to Mars



Vehicles

A Discarded Booster
B Earth-Mars Transfer Vehicle
C Mars Lander
D Ascent Vehicle

Maneuvers

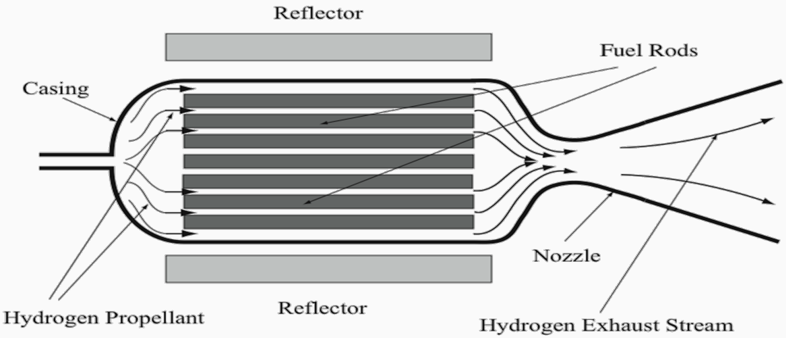
1 LEO to MTO
2 MTO to LMO
3 LMO to Surface
4 Surface to LMO
5 LMO to ETO
6 ETO to LEO

A mission to Mars and back involves (at the very least) 6 Δv manoeuvres.

The propellant for manoeuvre 6, for instance, has to be carried to Mars and back. The payload for manoeuvre 1 includes all the propellant needed for the other manoeuvrers.

Reducing propellant is a major requirement; Producing propellant on the surface of Mars is an option.

Nuclear rocket principle



Hot fuel rods heat hydrogen propellant.

The hot hydrogen expands in the nozzle as in a chemical rocket motor.

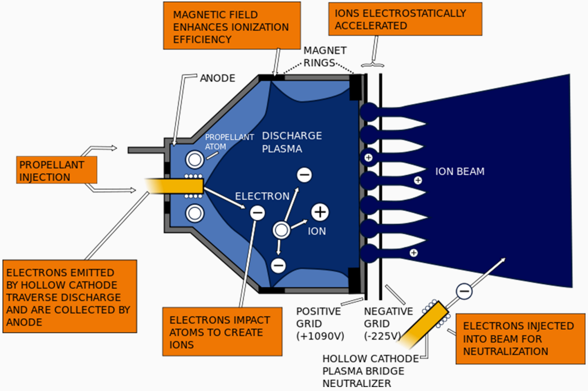
Electric or ion propulsion

Ionization of the propellant material, and acceleration with an electric field.

Higher exhaust velocities than with a liquid-fuelled or solid propellant rocket engine.

There is very high ejection velocity and high efficiency, but low thrust, of the order of a fraction of a Newton, because the mass flow is very small.

Such a system can be used for propulsion in space, but not for leaving the Earth's surface.



Credits: Wikipedia, Chabacano, retrieved from NASA

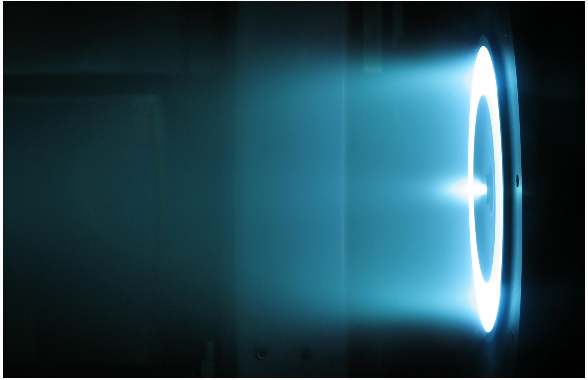
Hall thruster as an example of electric propulsion

Electric propulsion = acceleration of charged particles (ions) by an electric field or electric and magnetic fields.

Typical $I_{sp} = 1000 - 5000$ s $\Leftrightarrow$ exhaust velocities of 10 to 500 km/s.

Thrust values of typically $10^{-2} - 10^{-1}$ N.

This a 6 kW Hall thruster in operation at the NASA Jet Propulsion Laboratory



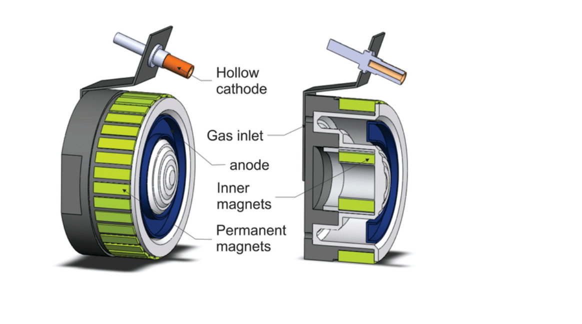
Credits: NASA/JPL

Want to know more about propulsion? $\rightarrow$ *Course ENG-510 / Space propulsion*

Prevalence of electric propulsion

Electric propulsion is a very common choice for station-keeping and even some positioning activities.

Virtually all manoeuvrable small satellites use electric propulsion, mostly for fuel efficiency reasons, however, they must be able to generate large electric fields $\rightarrow$ high demand on the power generation subsystem.



Skematics of a Hall thruster

Credits: Moraes et al., 2014

Example: all 10,000+ Starlink satellites use electric propulsion. Early models used krypton-fueled ion thrusters. SpaceX switched to argon Hall thruster because it is $\sim 100\times$ cheaper (and $1.5 \times I_{sp}$).

Interactive quiz

→ EchoPoll platform

- You can scan a QR code or go to the link
- EchoPoll is the EPFL-recommended solution
- You do not have to register, just skip entering a username and/or email address