



EE-585 – Space Mission Design and Operations

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Ecole Polytechnique Fédérale de Lausanne

Week 04 – 03 Oct 2025

Today's outline

Orbital manoeuvres: Hohmann transfer and plane change

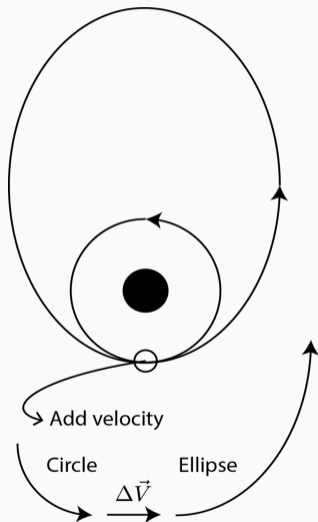
The Earth viewed from space

Orbit determination

Positioning & station-keeping

Orbital manoeuvres: Hohmann transfer and plane change

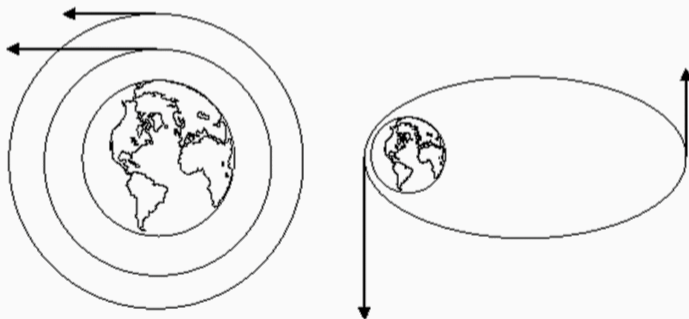
Manoeuvres in-orbit



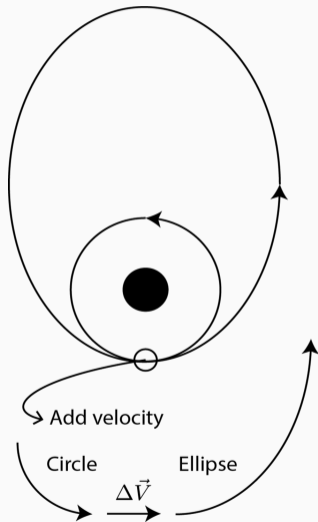
- A manoeuvre in orbit is an application of external force on the spacecraft \rightarrow vectorial change of the velocity vector $\Delta\vec{v}$ causing a change of the orbital elements.
- We will use indifferently the terms manoeuvre, “Delta-V” or “Burn”.
- We will focus first on instantaneous manoeuvres, that is the Δv occurs during a very short time frame Δt , that is $\Delta t \ll T$ the orbital period.
- Continuous manoeuvres can exert a thrust on the spacecraft for long period of time.

Reminder: orbital velocity and altitude

- As orbital altitude increases, $z \nearrow$, orbital period increases, $T \nearrow$, and orbital velocity decreases, $v \searrow$.
- Velocity is greatest at perigee and least at apogee.



Single impulse orbital change



Assume a satellite is on a circular orbit at $z_i = 200$ km. The operator wants to change to an elliptical orbit of $z_i = z_p = 200 \times z_a = 500$ km.

$$\text{Initial velocity } v_i = \sqrt{\frac{\mu}{R_{\oplus} + z_i}} \approx 7780 \text{ m/s}$$

$$\text{Semi-major of the elliptical orbit } a = R_{\oplus} + 350 \text{ km}$$

$$\text{Vis Viva equation } v = \sqrt{\frac{2\mu}{r} - \frac{\mu}{a}}:$$

$$v_f(z_p) = \sqrt{\frac{2\mu}{R_{\oplus} + z_p} - \frac{\mu}{a}} \approx 7870 \text{ m/s}$$

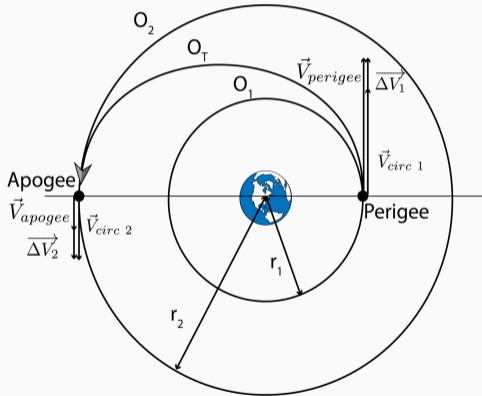
Velocity differential Δv :

$$\Delta v = v_f(z_p) - v_i \approx 90 \text{ m/s}$$

Posigrade Δv to go to a higher orbit.

Hohmann Transfer

The Hohmann transfer is a very common method of transfer from one circular orbit to another, around the same central body. The transfer orbit is tangent to both the initial orbit and the destination orbit.



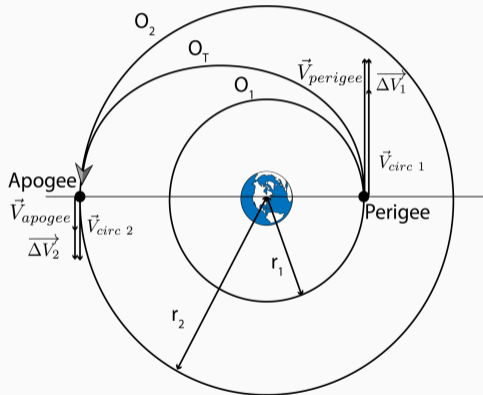
Again, let's start with the Vis Viva equation:

$$\begin{aligned}
 v_{\text{perigee}} &= \sqrt{\frac{2\mu}{r_1} - \frac{2\mu}{r_1 + r_2}} \\
 &= \sqrt{\frac{2\mu(r_1 + r_2) - 2\mu r_1}{r_1(r_1 + r_2)}} \\
 &= \sqrt{\frac{2\mu r_2}{r_1(r_1 + r_2)}}
 \end{aligned}$$

$$\Rightarrow \Delta v_{\text{perigee}} = \sqrt{\frac{2\mu r_2}{r_1(r_1 + r_2)}} - v_{\text{circ},1}$$

Similarly, $v_{\text{apogee}} = \sqrt{\frac{2\mu r_1}{r_2(r_1 + r_2)}} \Rightarrow \Delta v_{\text{apogee}} = v_{\text{circ},2} - v_{\text{apogee}}$

Transit time and energy considerations



The transit time from the lower orbit to the higher circular one is

$$t_{\text{transit}} = T_T/2 = \pi \sqrt{\frac{a_T^3}{\mu}} = \pi \sqrt{\frac{(r_1 + r_2)^3}{8\mu}}$$

The Hohmann transfer is the most efficient transfer with 2 burns.

Reminder: the specific energy of an orbit is

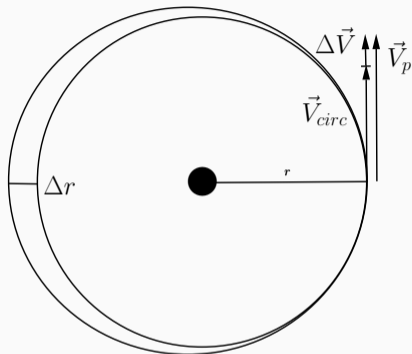
$$\epsilon = -\frac{\mu}{2a}$$

So, the energy of the orbit increases $\epsilon_1 < \epsilon_T < \epsilon_2$ from its initial to final orbit because

$$r_1 < a_T = \frac{r_1 + r_2}{2} < r_2.$$

The energy remains $\epsilon < 0$ (bound to the Earth) but approaches 0.

Hohmann Transfer – Case of small Δv



If Δr is small, e.g. $\Delta r/r < 10^{-3}$, we can linearise the equation for Δv .

The semi-major axis of the elliptical orbit is

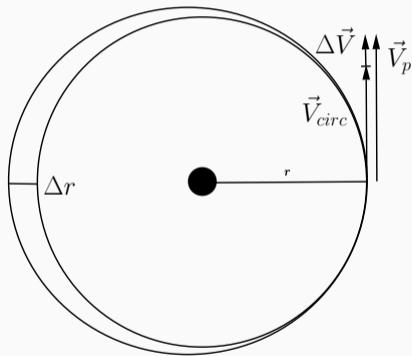
$$a = \frac{r + (r + \Delta r)}{2} = r + \frac{\Delta r}{2}$$

The velocity at perigee of the elliptical orbit is

$$v_p = \sqrt{\frac{2\mu}{r} - \frac{\mu}{a}} = \sqrt{\frac{\mu}{r}} \sqrt{2 - \frac{1}{1 + \frac{\Delta r}{2r}}} \approx \underbrace{\sqrt{\frac{\mu}{r}}}_{=v_{\text{circ}}} \sqrt{2 - \left(1 - \frac{\Delta r}{2r}\right)} \approx v_{\text{circ}} \sqrt{1 + \frac{\Delta r}{2r}} \approx v_{\text{circ}} \left(1 + \frac{\Delta r}{4r}\right)$$

using $\sqrt{1+x} \approx 1 + \frac{x}{2} \quad \forall x | x \ll 1$

Hohmann Transfer – Case of small Δv



Recalling that $v_p \geq v_{circ}$, for a slightly elliptical orbit we have $v_p = v_{circ} + \Delta v$,

$$\frac{v_p}{v_{circ}} = \frac{v_{circ} + \Delta v}{v_{circ}} \approx 1 + \frac{\Delta r}{4r} \implies \boxed{\frac{\Delta r}{r} \approx 4 \frac{\Delta v}{v}}$$

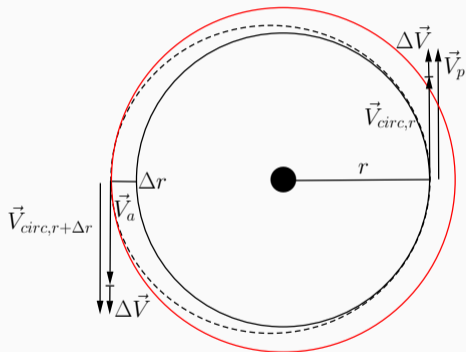
For LEO, where $r \sim R_{\oplus} + 500$ km and $v \sim 7.7$ km/s,

$$\boxed{\Delta r \approx 3.5 \Delta v}$$

LEO approximation

with Δr in km and Δv in m/s.

Full transfer from one circular orbit to another



For the full transfer from black to red orbit is twice the LEO approximation:

$$\Delta v_{\text{total}} \approx 2\Delta v = 2 \times \frac{\Delta r}{r} \times \frac{v}{4} = \frac{1}{2} \frac{v}{r} \Delta r \approx 0.57 \Delta r_{\text{circ}}$$

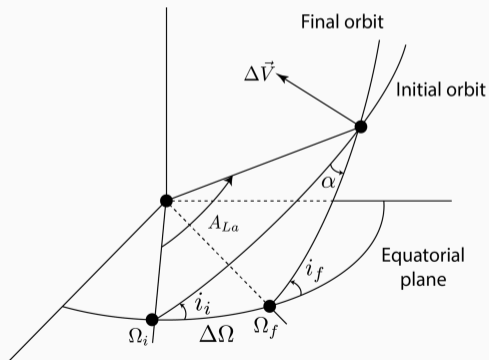
or, rearranging to get Δr_{circ} ,

$$\Delta r_{\text{circ}} \approx \frac{3.5}{2} \Delta v_{\text{total}}$$

with Δr in km and Δv in m/s.

Change of orbital plane

$$\Delta \vec{v} \rightarrow \alpha \rightarrow \Delta \Omega \leftrightarrow \Delta i$$



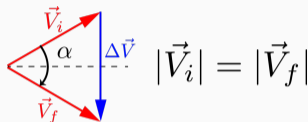
A manoeuvre with an out-of-plane component, typically a $\Delta \vec{v}$ perpendicular to the orbital plane will cause a change of several orbital parameters, in particular Ω (RAAN) and i .

As illustrated here, the $\Delta \vec{v}$ perpendicular to \vec{v} causes a change of direction α in the instantaneous velocity vector. Changes in Ω and i involve spherical trigonometry calculations which will not be detailed here.

An orbital plane change is best performed at equator crossing (simplicity and efficiency).

ΔV needed for a change of orbital plane

The simplest way to perform a plane change is to burn at one of the two crossing points of the initial and final planes.



Δv can be computed by the law of cosines

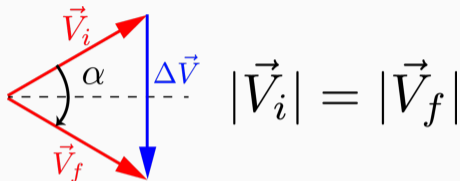
$$\Delta v = \sqrt{v_i^2 + v_f^2 - 2v_i v_f \cos \alpha}$$

if the magnitude of the velocity don't change (i.e. pure rotation of the the orbital plane), no other orbital parameter change:

$$\Delta v = v \sqrt{2(1 - \cos \alpha)} = v \sqrt{2 \cdot 2 \sin^2 \left(\frac{\alpha}{2} \right)}$$

using the trigonometric identity $\cos \alpha = 1 - \sin^2 \frac{\alpha}{2}$

ΔV needed for a change of orbital plane

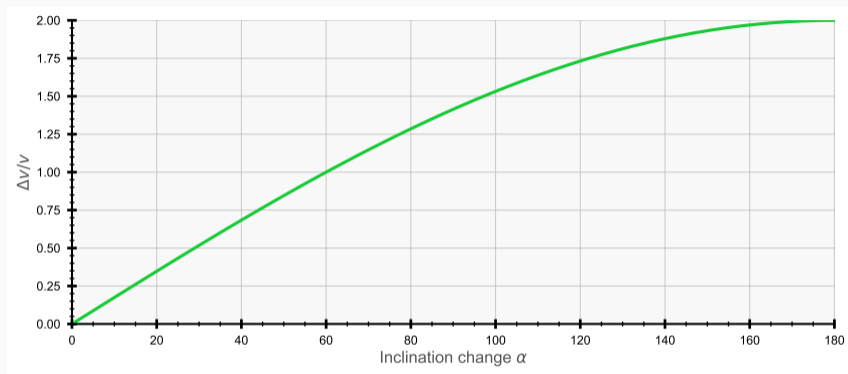


Change of orbital plane best done at equator crossing \rightarrow no change to other orbital parameters

$$\Delta v = 2v_i \sin\left(\frac{\alpha}{2}\right)$$

The specific energy does not change with pure plane change manoeuvres.

ΔV needed for a change of orbital plane

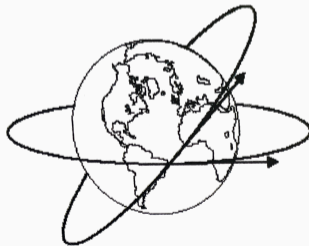


$$\Delta v = 2v_i \sin\left(\frac{\alpha}{2}\right)$$

In LEO, $v \sim 7.7$ km/s \implies plane change with an out-of-plane $\Delta \vec{v}$ is expensive!
 \rightarrow Go high to do your plane change manoeuvre (they cost nothing at infinity)

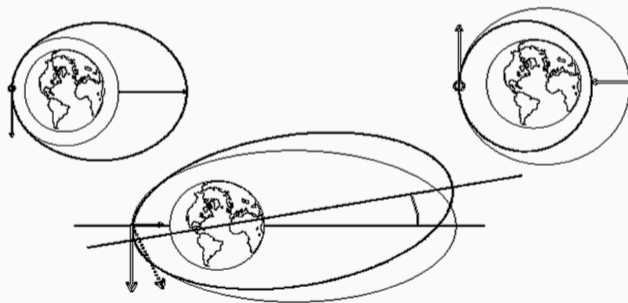
Effects of out-plane burn on the orbit

- Plane change must be performed at intersection of two orbits (at the nodal crossing).
- There are 2 opportunities per orbit.
- Plane change requires a large amount of propellant compared to in-plane manoeuvres.



Effects of in-plane burns on the orbit

- **Posigrade** burns **increase** altitude 180° from the burn point.
- **Retrograde** burns **decrease** altitude 180° from the burn point.
- **Radial** burns **shift** the semi-major axis without significantly altering other orbital parameters.

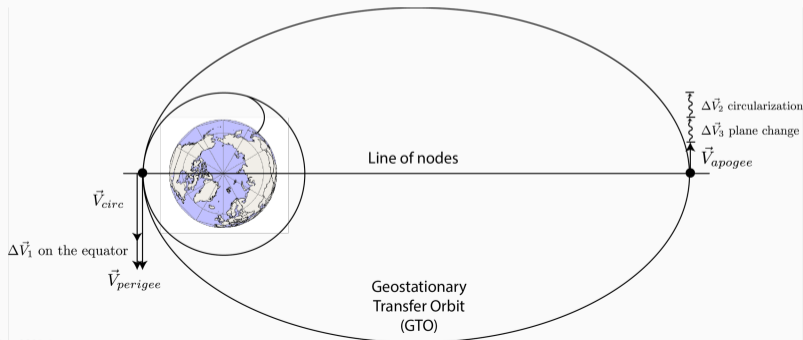


Strategy to reach the geostationary orbit

A satellite bound for GEO is usually launched into an inclined parking orbit in LEO or directly on its transfer orbit. E.g. $i = 7^\circ$ for Kourou in French Guiana and 28.5° from Florida.

Three burns are required to reach the GEO conditions:

1. A first Hohmann burn to get into the transit orbit.
2. A second Hohmann burn to circularise the orbit in GEO.
3. A plane change manoeuvre to bring i to 0.



Reminder, for GEO:

$$z \approx 36'000 \text{ km}$$

$$i = 0^\circ$$

$$e = 0$$

Δv savings with a combined manoeuvre (1/2)

When should the plane changed be performed?

There are 3 options:

1. First, make the plane change and then Hohmann transfer.
2. A plane change at apogee and then a circularisation.
3. A combined manoeuvre that changes both the plane and circularises at apogee.

Δv savings with a combined manoeuvre (1/2)

When should the plane change be performed?

There are 3 options:

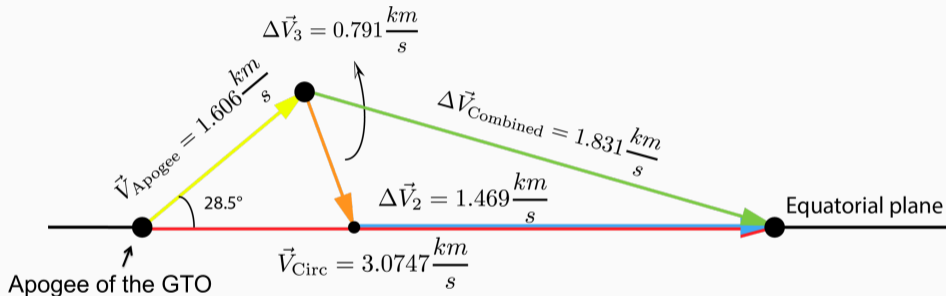
1. First, make the plane change and then Hohmann transfer.
2. A plane change at apogee and then a circularisation.
3. A combined manoeuvre that changes both the plane and circularises at apogee.

→ since the plane change is $\Delta v = 2v_i \sin\left(\frac{\alpha}{2}\right)$ and at apogee, $v_{apo} < v_{peri}$, it is better to change the orbital plane high in the gravitational well, when v is low.

→ Not option 1

Δv savings with a combined manoeuvre (2/2)

Example: combined maneuver at the apogee of the transfer orbit for insertion into GEO for a launch from Kennedy Space Center, Florida (Lat. 28.5°).



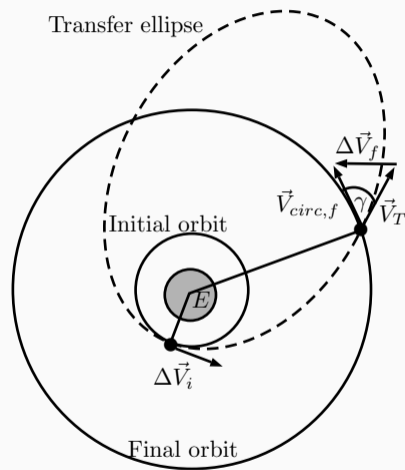
Separate manoeuvres: $|\Delta \vec{v}_{\text{tot, apo}}| = |\Delta \vec{v}_2| + |\Delta \vec{v}_3| = 2.26 \text{ km/s}$

Combined manoeuvres: $|\Delta \vec{v}_{\text{tot, apo}}| = |\Delta \vec{v}_2 + \Delta \vec{v}_3| = 1.83 \text{ km/s}$

$\Delta \vec{v}_2$ 2nd Hohmann manoeuvre, $\Delta \vec{v}_3$ plane change.

→ Option 3: a combined manoeuvre at apogee is more efficient.

One-tangent burn manoeuvres



If the transit time needs to be shorter than the Hohmann transfer or the satellite should intercept the target orbit at a given true anomaly of the second orbit, then one-tangent burn is the solution.

Recipe to analyse the one-tangent manoeuvre:

- Compute e_T and a_T , eccentric anomaly E
- Get the time of flight
- Compute $v_{T,peri}$ of the transfer orbit and $v_{i,circ}$
- Get $\Delta \vec{v}_i$
- Compute the flight path angle γ , v_T , $v_{f,circ}$
- Get $\Delta \vec{v}_f$

The time of flight might be shorter, but $|\Delta \vec{v}_i| + |\Delta \vec{v}_f| > \Delta V_{\text{Hohmann}}$

Oberth effect or the powered flyby effect

The impact of a burn Δv on the specific energy depends on the instantaneous velocity v .

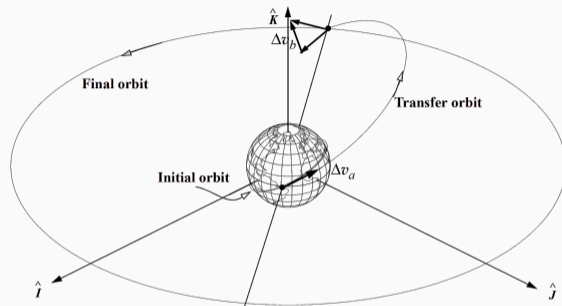
The increase in kinetic energy

$$\Delta KE = (v + \Delta v)^2/2 - v^2/2$$

is largest when v is largest (thus at periapsis, i.e. deep in the gravitational well).

In planetary flybys, this effect can give a very significant boost (! \neq slingshot manoeuvres).

Lambert's problem: finding complex transfer trajectories



Credits: Vallado, *Fundamentals of Astrodynamics and Applications*, 4th edition

To change multiple orbital parameters with a transfer orbit (combined manoeuvre or rotation of line of apsides, RAAN change, . . .), the best is to solve the so-called Lambert problem.

The classical formulation of Lambert's problem finds the orbit between two points (a, b) as a function of the time of flight.

For transfer orbits, the Δv_a and Δv_b can be computed from the expected initial and final velocity vectors.

Lambert's problem can find the minimum-energy solution. It also handles multiple revolution on the transfer orbit.

Many solutions (and algorithms) to the Lambert's problem exist.

A few remarks on orbital manoeuvres

1. A Hohmann transfer is the most efficient 2-burn manoeuvre.
2. Plane change...
 - are costly, avoid if possible.
 - should be made at equator crossing to rotate the plane only.
 - should be made at apogee (you might consider raising the apogee before).
3. Make use of the perturbations to change the parameters of the orbit.
4. On a very non-circular orbit ($e \neq 0$), the most efficient way to change the energy of the orbit is to make a Δv at perigee.

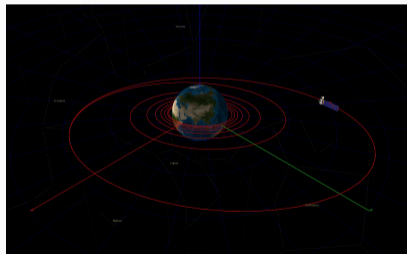
Non-impulsive manoeuvres

So far, we assumed that the duration of the burn Δv was much shorter than the orbital period, i.e. $t_b \ll T_{\text{orb}}$, almost instantaneous.

This assumption holds for most Δv conducted with propulsion subsystems that deliver very high thrust.

During the burn, the equation of motion must be rewritten with an external force \vec{F} :

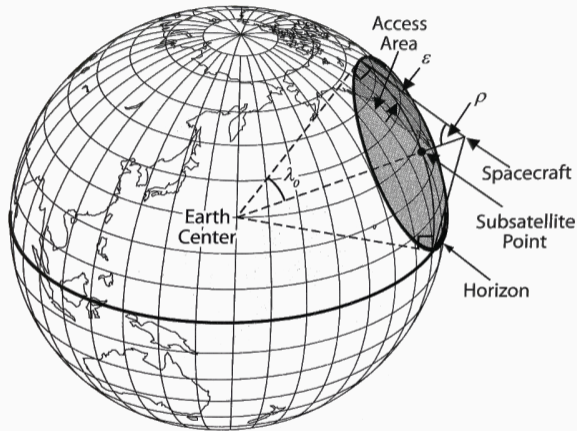
$$\ddot{\vec{r}} = -\mu \frac{\vec{r}}{r^3} + \frac{\vec{F}}{m} = \text{gravity} + \text{thrust}$$



Resulting trajectory is typically a spiral

The Earth viewed from space

Earth viewed from space (1/2)



What is the angular radius, ρ , of the (spherical) Earth as seen from a satellite?

The line from the satellite to Earth's horizon is \perp to Earth's radius, thus

$$\sin \rho = \frac{R_{\oplus}}{R_{\oplus} + z}$$

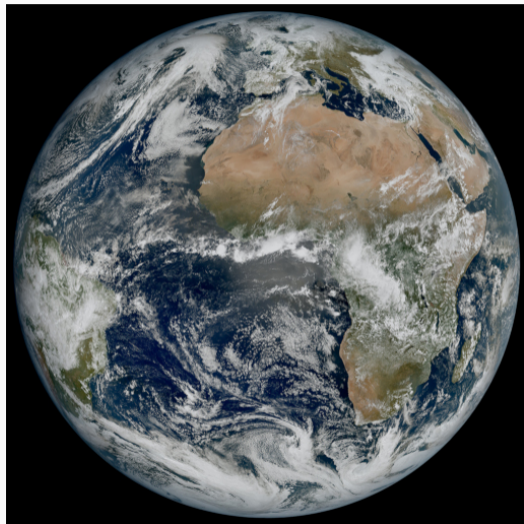
Credits: SMAD

| | | | | | | | | |
|---------------|-----|-----|-----|------|------|--------|--------|----------------|
| Altitude [km] | 400 | 500 | 800 | 1000 | 2000 | 20'000 | 36'000 | Moon (400'000) |
| 2ρ [deg] | 140 | 116 | 126 | 120 | 100 | 28 | 17 | 2.0 |

Earth viewed from space (2/2)



ISS seen from Space Shuttle Endeavour on 20 Feb 2010 Credits: NASA



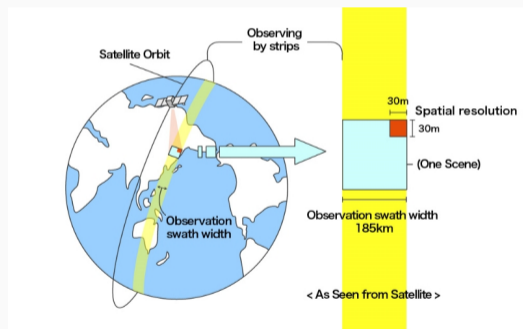
First EUMETSAT MTG i1 image on 31 Mar 2023

Credits: EUMETSAT

Elevation angle ε and swath of images

Typical ε_{\min} to communicate with a satellite is 5° .

For Earth observation, $\varepsilon_{\min} \sim 40 - 50^\circ$.

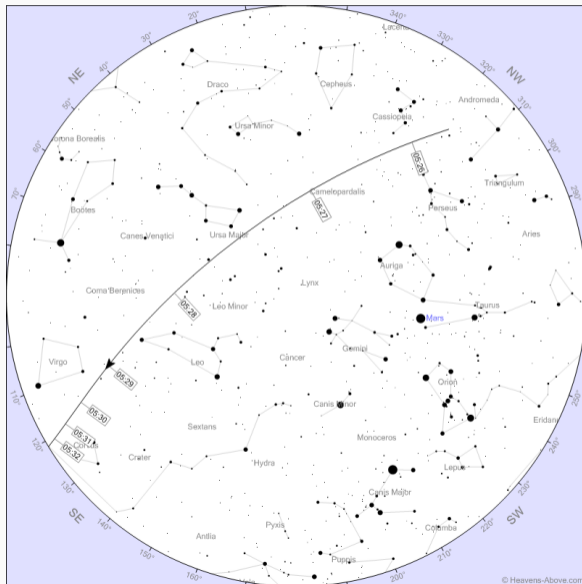


Credits: www.restec.or.jp

The minimum elevation angle, ε_{\min} , defines the maximum swath, that is the maximum area a satellite can observe.

It is often given as a width (in km) of, e.g. the largest possible image.

Apparent motion of satellite: sky plot of a pass



A skyplot shows the trajectory of a satellite across the sky as viewed from an observer on the surface.

Time in view of the station depends (heavily) on ϵ_{\min} and how far the observer is from the subsatellite ground track. For LEO, the pass duration is $\sim 7 - 12$ minutes.

Note that the apparent motion across the sky is not uniform.

Orbit determination

Keeping track of a satellite's position

Knowing the state vector of the satellite to a good enough accuracy is crucial for mission operations:

- pass prediction
- observability of targets
- timing of burns
- relative motion with respect to other satellites (rendezvous or avoidance
→ next lecture)

The orbit parameters determined at time t_0 can be propagated to some future (or past) time t_1 , but there are errors in the perturbations and the initial knowledge of the orbit → need a measurement, this is orbit determination.

Initial orbit determination

Initial orbit determination describes the process of estimating an orbit or state based solely on measurements and without a priori information.

You have a set of measurements of the position and its epoch of an object. (Initial) orbit determination algorithms transform the measurements into a representation of the trajectory, i.e. orbital parameters.

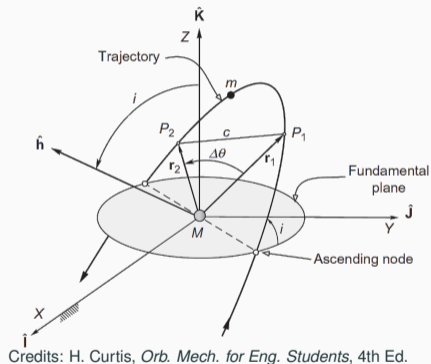
Initial orbit determination is the same for asteroids, planets, satellites.

There are many possible algorithms that solve slightly different problems. It mainly depends on the observables.

1. 2 or 3 position vectors, \vec{r}_i with their epoch
2. 3 angular positions, $(\alpha_i, \delta_i, t_i)$, $i = 1, 2, 3$ (α right ascension, δ declination)

The position of the site must be known to a good accuracy. Not accounting for the Earth's equatorial bulge results in errors on the order of 15 km!

Lambert's problem, revisited



Suppose that 2 observations are made of an objects that yield the geocentric position vector, \vec{r}_1, \vec{r}_2 at t_1, t_2 . What is the orbit?

This is exactly the same problem we encountered to find a transfer trajectory between two arbitrary orbits.

The solution space includes all orbit types, and the elliptical transfers may also include multi-revolution cases.

Ground observatories: radar, satellite laser ranging



Radar data return angle and range data.

Main limitation is distance to objects → best for LEO

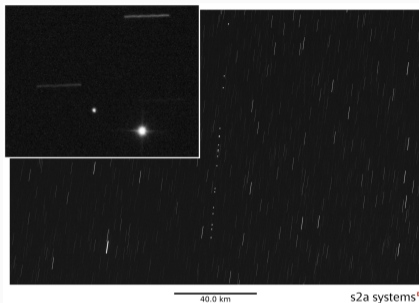
Credits: LeoLabs



SLR is measures the two-way time-of-flight Δt of ultra-short laser pulses emitted from a ground station (GS) to a satellite, which ideally has retro-reflectors, and reflected back to the GS.

Angle and range data, but SLR needs good a priori knowledge. Credits: UNIBE

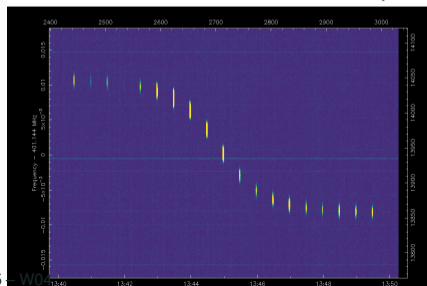
Ground observatories: optical observations and Doppler



Optical images yield angle only data.

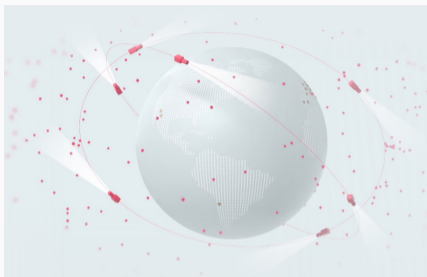
Typical observation technique: sky staring and detecting fast moving objects in a series of images.

Credits: s2a Systems, 1st batch of G60 Chinese SATCOM tracked



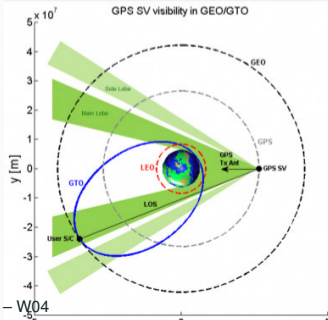
Doppler effect on the frequency of the radio signal received by a ground station. → works only with active satellites.

Spacecraft-based methods: GNSS, space-based observatories



Some satellites can observe other objects to compute the relative position and generate precise orbits. Better observations because no atmosphere.

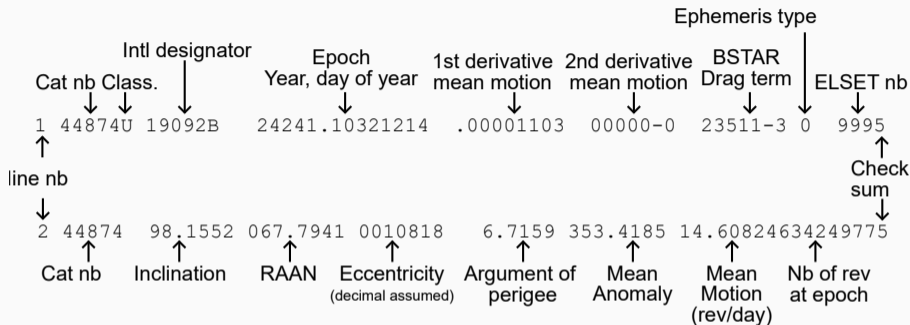
Credits: Vyoma



GNSS signals can be used to triangulate the position (and with a few points speed) of an object. Has been shown to work up to GEO and the Moon (in early 2025).

Credits: ESA

Two-line element set (ELSET) and other formats



The two line element set (TLE, 2LE, sometimes 3LE with an initial comment line) are the most common way of disseminating the orbital data.

Precision limited by the format and there is a limit on the epoch (YYDDD.ffffff), no indication on the errors.

Other formats: Orbit Mean-Element Message (OMM, json/xml extension of TLE), ephemeris (state vectors), comprehensive with covariances or additional information.

Positioning & station-keeping

Launch and early orbit phase (LEOP)

→ *launch will be discussed in a later lecture*

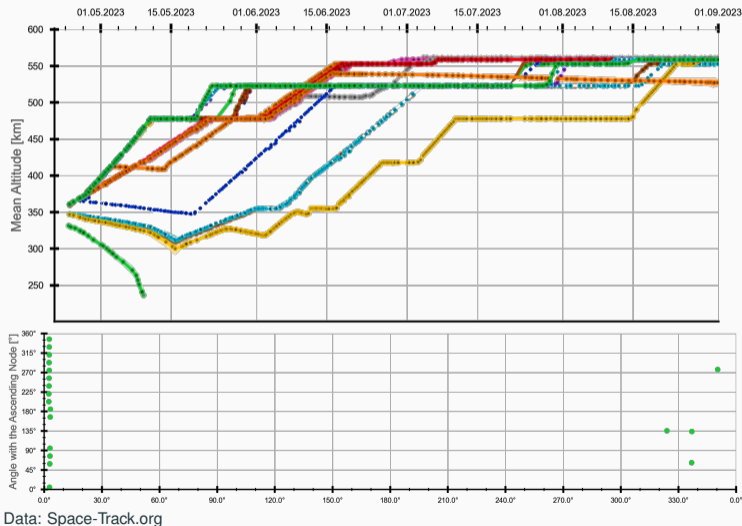
LEOP and commissioning start after separation with the launch vehicle. It contains – but is not limited to – solar panel deployment, system test and checks, “first light”, calibration of instruments and positioning to the final operating orbit.

This is a critical phase of the mission. The operating team works on an extended schedule and there might be additional contacts with the ground stations. LEOP, especially with extended orbital changes, can be long. Typically weeks to months.

Going from LEO to GEO on a GEO-transfer orbit (GTO) is an example of positioning.

End of mission disposal (e.g. orbit raising to graveyard orbit or perigee lowering) are part of a dedicated end of life phase.

Example: Starlink group 6 batch 2



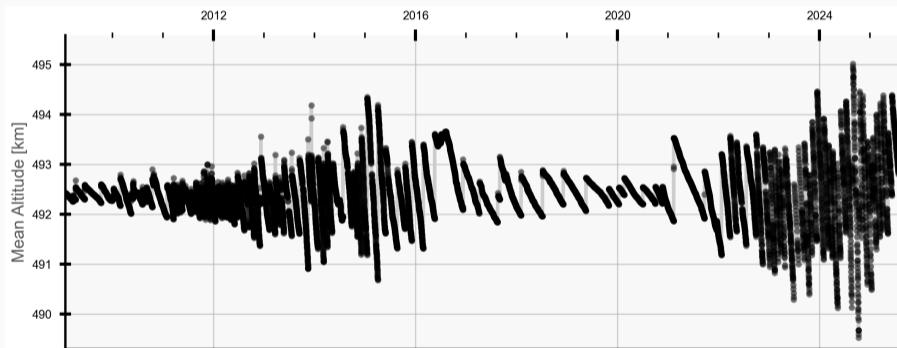
21 satellites launched 19 Apr 2023. Injected at ~ 300 km in a train – all at the same time. Positioning needs to:

- Check that the platform is fine
- Check that the payload is fine
- Raise the orbit to ~ 550 km
- Distribute the satellites in the orbital plane (using a differential RAAN drift)

1 failed early (light green), a second one at the end of the positioning manoeuvre (orange).

Station-keeping in LEO

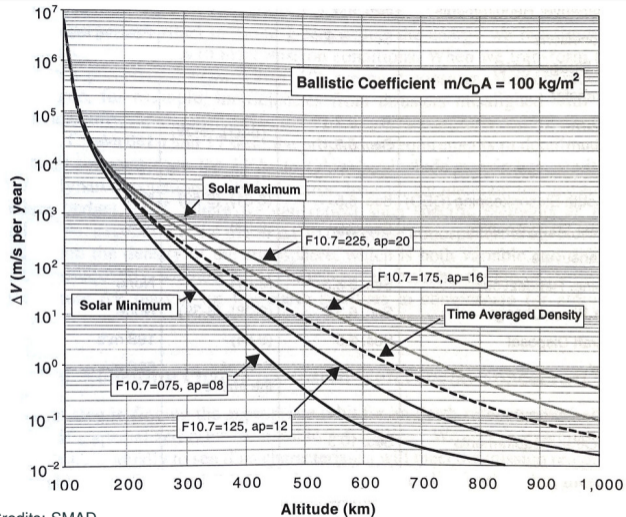
The Δv budget needed for station-keeping – that is maintaining the orbit within the operational parameters – depends on the mission objectives, altitude and the Sun's activity.



WorldView 1 (optical Earth observation) operated by Maxar – Data: Space-Track.org

The example above shows a long-lived satellite that has almost gone through two solar cycles. Both the amplitude, Δa , and the separation between the manoeuvres, Δt , are varied to keep the satellite at $z \approx 492.5$ km.

Typical station-keeping in LEO



The Δv budget is given in m/s/yr to compensate for atmospheric drag.

For other BCs, $\sim \Delta v \cdot 100/BC \rightarrow$ if BC is $10\times$ lower, you need $10\times$ the Δv .

The main problem in LEO is to predict the solar activity.

Expect much longer lifetime if you launch in early quiet activity period.

Credits: SMAD

Station-keeping in GEO

In GEO, frequent station-keeping manoeuvres are required to maintain the longitudinal slot. There are two main effects:

- lunar & solar perturbations lead to $\Delta i \sim 0.85^\circ/\text{year}$

$\implies \sim 45 \text{ m/s/yr}$ “North-South”

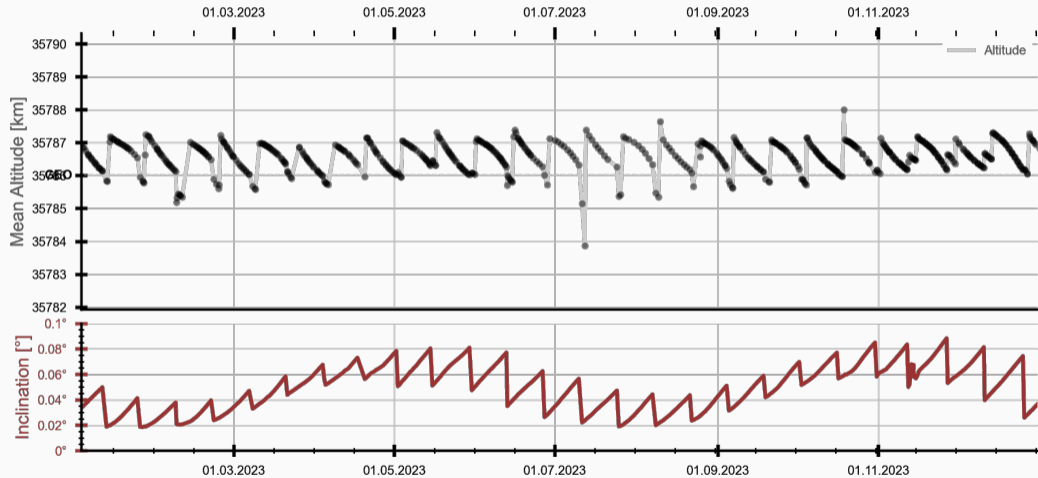
Maximum inclination of $\sim 15^\circ$ reached after 26.5 years.

- Earth’s non-circularity & solar radiation lead to $\Delta a, \Delta e$ (respectively)

$\implies \sim 2 \text{ m/s/yr}$ “East-West”

Depends on longitude because there are 2 stable equilibrium points + 2 unstable equilibrium points. Typically manoeuvre every ~ 2 weeks.

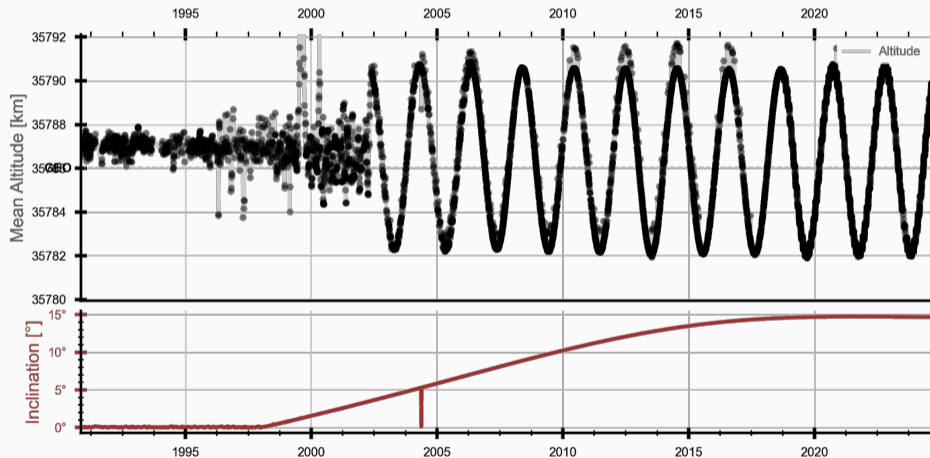
Station-keeping in GEO



Eutelsat 9B (SATCOM) – Data: Space-Track.org

Station-keeping in GEO

End of operations (mid 2002) and no disposal in graveyard orbit.



INSAT 1D (SATCOM, weather) – Data: Space-Track.org

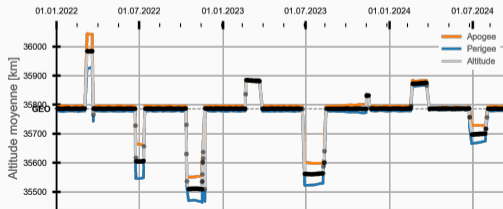
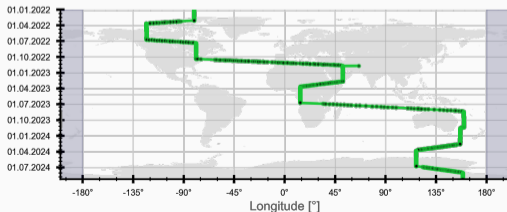
Satellite re-positioning in GEO

To relocate a satellite from one GEO slot to another, the semi-major axis can be lowered → **relocation eastwards** or increased → **westards** during a week- to month-long coasting time.

The change in longitude in $^{\circ}/d$, \dot{L} , is

$$\dot{L} = 360^{\circ} \left(1 - \frac{T}{T_S} \right) = 360^{\circ} \left(1 - \frac{2\pi\sqrt{a^3/\mu}}{T_S} \right)$$

with $T_S = 86164.0905$ s, the sidereal day.



A typical Δv budget must account for the following activities:

- positioning
- station-keeping/operational activities ($\Delta a, \Delta e, \Delta i, \Delta \Omega, \dots$)
- if applicable: mission phase transition (e.g. if there are different operational altitudes)
- if applicable: attitude control (\rightarrow see subsequent lecture)
- disposal (re-entry, orbit raising to disposal orbit)
- margins!

Additional Δv means additional storage and mass \rightarrow linked to the mass and structure budgets.

The satellite lifetime can be constrained by Δv reserves or reliability of or consumables for the payload.

→ EchoPoll platform

- You can scan a QR code or go to the link
- EchoPoll is the EPFL-recommended solution
- You do not have to register, just skip entering a username and/or email address