Solution to Exercise Session 1

Problem 1 Multiple Choice Questions

A) *A geostationary satellite is orbiting the Earth at an altitude of 36'000 km. Assuming the satellite is stopped instantaneously and starts to fall, at what speed will it reach the top of the Earth's atmosphere, which is 100 km above the Earth's surface?*

(1) 89.2 km/s
(2) 10.2 km/s
(3) 14.8 km/s
(4) 7.4 km/s

Solution. This result is computed using the conservation of mechanical energy in the Earth-centered inertial frame. To be remembered that we are expressing the potential energy and the kinetic energy per unit mass (as shown in the class).

$$\begin{aligned} E_{\text{pot1}} + E_{\text{kin1}} &= E_{\text{pot2}} + E_{\text{kin2}} \\ -\frac{\mu}{R_\oplus + h_1} + 0 &= -\frac{\mu}{R_\oplus + h_2} + \frac{1}{2}v^2 \\ \Rightarrow \quad v &= \sqrt{2\mu\left(\frac{1}{R_\oplus + h_2} - \frac{1}{R_\oplus + h_1}\right)} = \mathbf{10.21\ km/s} \end{aligned}$$

with Earth gravitational parameter $\mu = G \cdot M_\oplus = 3.986 \cdot 10^{14}\ \text{m}^3\text{s}^{-2}$, Earth radius $R_\oplus = 6378$ km, $h_1 = 36'000$ km and $h_2 = 100$ km.

B) *($\star$) Estimate the equilibrium temperature of the Earth (considered without atmosphere) by using the equation of the radiation power from the Sun and that of the self-radiation power from the Earth into space, and solving for the temperature. Use the black body assumption $\alpha/\epsilon = 1$.*

(1) $21°C$
(2) $6°C$
(3) $-21°C$
(4) $0°C$

Solution. The power of the Sun absorbed by Earth is given by :

$$P_a = \alpha S A_n$$

Where $A_n = \pi R_\oplus^2$ is the cross-section (disc) intercepting the sun's radiation. Earth emitted power is approximated using the black-body radiation :

$$P_e = \epsilon \sigma T^4 A_{tot}$$

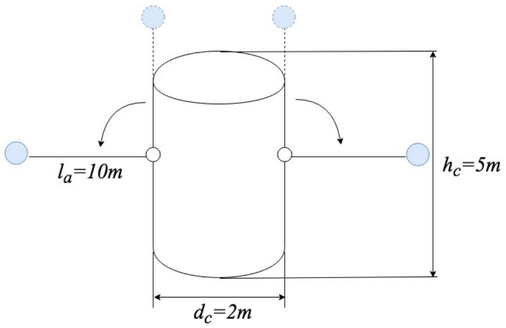
Where A_{tot} is the emitting surface (sphere) : $A_{tot} = 4\pi R_{\oplus}^2$. At equilibrium, we have $P_e = P_a$; therefore we can isolate T :

$$T = \left(\frac{\alpha}{\epsilon}\right)^{1/4} \times \left(\frac{SA_n}{\sigma A_{tot}}\right)^{1/4}$$

$$T = \left(\frac{S}{4\sigma}\right)^{1/4} = \left(\frac{1.367 \times 10^3}{4 \cdot 5.67 \times 10^{-8}}\right)^{1/4}$$
$$= 278.63°K \approx 6°C$$

Note: If we consider that only the fraction of the solar radiation reaching the Earth contributes to heating (total solar radiation minus the albedo), then we find $T = 251.8°K = -21°C$.

C) *The Cassini-Huygens spacecraft was launched in 1997 to explore Saturn. It was roughly of cylindrical shape with two straight arms whose purpose was to deploy antennas far from the main body some time after the launch, at an angle of 90 degrees with the cylindrical axis.*



If before antenna deployment the spacecraft was rotating around its main axis at a rate of 1 *rpm, what was the rotation rate after antenna deployment? Consider the antennas to be point masses and the mass of the deployment arms, negligible.*

Data : Mass of Cassini without the antennas: $M_c = 5600$ kg, height $h_c = 5\ m$, diameter $d_c = 2\ m$. Mass of each antenna $m_a = 50\ kg$, length of each arm $l_a = 10\ m$

(1) 7.61 deg/s

(2) 0.57 deg/s

(3) 1.16 deg/s

(4) 3.84 deg/s

Solution. The angular momentum before the deployment of the arm is equal to the moment to the angular momentum after the deployment : $L_1 = L_2$ where $L_1 = I_c \cdot \omega_1$ and $L_2 = I_{c+a} \cdot \omega_2$.

$$I_c = \frac{1}{2}M_c\left(\frac{d_c}{2}\right)^2 + 2m_a\left(\frac{d_c}{2}\right)^2$$
$$I_{c+a} = \frac{1}{2}M_c\left(\frac{d_c}{2}\right)^2 + 2m_a\left(l_a + \frac{d_c}{2}\right)^2$$
$$\Rightarrow \omega_2 = \omega_1\frac{I_c}{I_{c+a}} \approx 1.16\,\mathrm{deg}/s$$

D) *(⋆) An artificial satellite orbiting the Earth is in an elliptical orbit with a perigee altitude of $h_p = 250$ km and an apogee altitude of $h_a = 800$ km. What is its orbital period?*

(1) 18.0 min
(2) 89.5 min
(3) 95.1 min
(4) 100.9 min

Solution. The orbital period is given by :

$$T = 2\pi\sqrt{\frac{a^3}{\mu_\oplus}} \tag{1}$$

where $a = \frac{R_p+R_a}{2}$. R denotes the distance from the Earth center such that :

$$\begin{aligned} R_p &= h_p + R_\oplus = 6628 \text{ km} \\ R_a &= h_a + R_\oplus = 7178 \text{ km} \end{aligned}$$

hence $a = 6903$ km and therefore $T = 5708$ s $= 95.1$ min

E) *A spacecraft is on a free trajectory in the vicinity of the Earth. From which statement can it be deduced that this spacecraft has sufficient energy to leave the gravitational well of the Earth (i.e., to not be on orbit around the Earth)?*

(1) $\mathbf{E_{tot} \geq 0}$
(2) $E_{tot} < 0$
(3) $E_{tot} \to \infty$
(4) $E_{tot} \to -\infty$

Solution. The total energy at infinity should be zero (or higher). The borderline case of a closed orbit around the Earth corresponds to a parabolic orbit for which $\frac{1}{2}v^2 = \frac{\mu}{r}$, ie. v always equal to the escape velocity at any distance from the Earth's center. If we have $\frac{1}{2}v^2 \leq \frac{\mu}{r}$, or $E_{tot} < 0$, then we have a closed orbit around the Earth.

F) *You are currently aboard the ISS (orbiting at an altitude of 400 km) and ground control tells you that there is another satellite on the same orbital plane but 50 km higher. Assuming you have just spotted it exactly above you at a certain time (conjunction), how long do you have to wait until the next conjunction?*

(1) 13.2 days
(2) 2.1 hours
(3) 15.2 hours
(4) 5.9 days

Solution. First, we need to compute the period of both the ISS and the satellite. Knowing the orbit period is:

$$T = 2\pi\sqrt{\frac{a^3}{\mu_\oplus}}$$

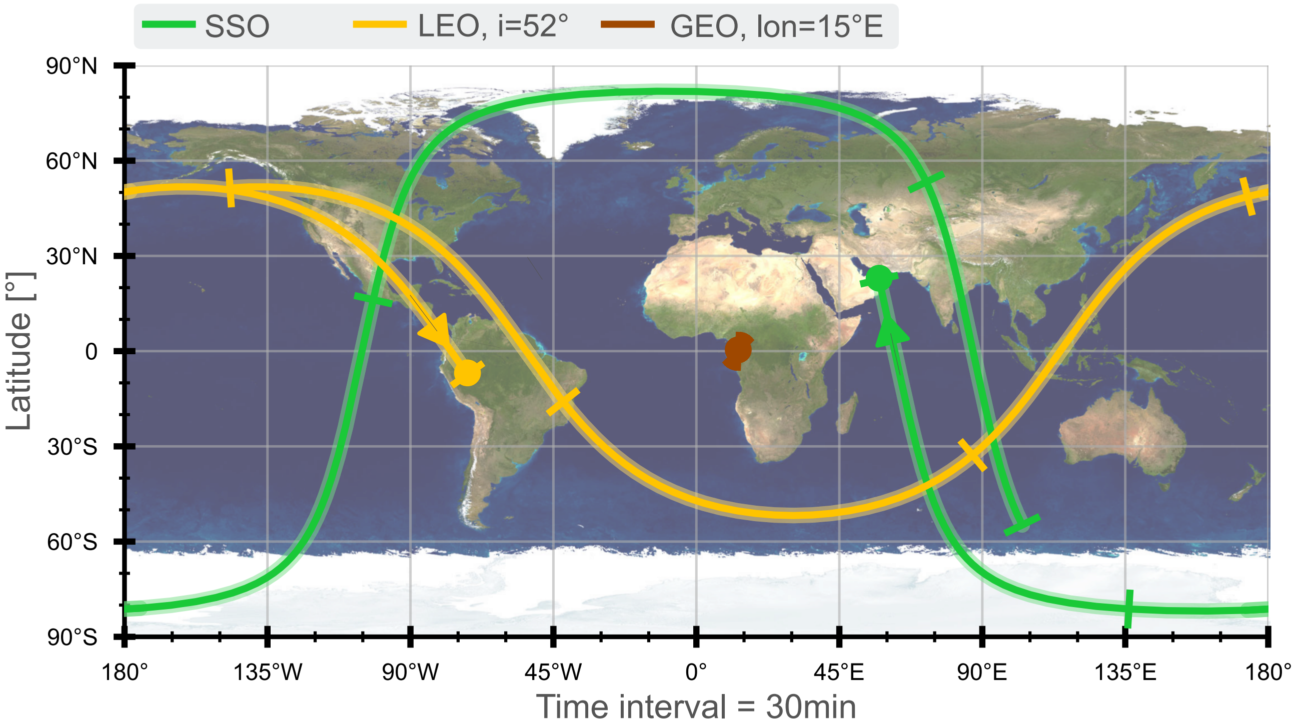
We get $T_{ISS} = 92.56\ [min] = 1.54\ [h]$ and $T_{sat} = 93.58\ [min] = 1.56\ [h]$ assuming $R_\oplus = 6378\ [km]$ and $\mu_\oplus = 3.986 * 10^{14}\ [m^3 s^{-2}]$. To find the next conjunction, we need to determine the synodic period.

$$\frac{1}{T_{syn}} = \frac{1}{T_{ISS}} - \frac{1}{T_{sat}} \implies T_{syn} = 140.70\ [h] = 5.9\ days$$

It means we can see the satellite passing by every 5.9 days.

Problem 2 Ground track

Draw 1 1/4 orbit for a spacecraft in SSO, LEO at $i = 52°$ and a satellite in GEO at a longitude of $15° E$.



The SSO orbit is $i \sim 98°$, so it will reach almost all latitudes. This is a retrograde orbit, from east to west.

A satellite on a $i = 52°$ orbit will circle the Earth in a posigrade way, from west to east, and reach latitude of $\pm 52°$.

Both ground track will move west by $\sim 22.5°$ each ascending node.

The angle between the ground track and the equator at the ascending node is i, that is $52°$ for the LEO satellite and $97°$ for the SSO satellite.

The satellite in GEO will not move ($e = 0, i = 0$) and will remain at a longitude of $15°$ and a latitude of $0°$.

Problem 3 Escape velocity

The Rosetta spacecraft launched by the European Space Agency successfully entered the orbit of the comet 67P/Churyumov– Gerasimenko in August 2014. On 12 November 2014 the Philae lander was released and touched down 7 hours later at a speed of 1 m/s.
The harpoon mechanism that was supposed to secure the lander failed and bounced off the comet. Assuming a purely elastic impact, will the lander leave the comet or return at some point?

Data: Mass of the lander: $m_l = 100$ kg, mass of the comet: $M_c = 3.14 \cdot 10^{12}$ kg, radius of the comet (assume a spherical shape): $R_c = 2$ km.

Solution. Due to the elastic impact and important mass difference between the lander and the comet, we can assume that the post-impact (f) velocity is the same (in amplitude) as the velocity before (i) the impact : $|\vec{v}_f|=|\vec{v}_i|$

The escape velocity is the minimum velocity that an object or spacecraft has to be given to escape a celestial body forever (reaching infinity at zero velocity).

Conservation of Energy:

$$\underbrace{-\frac{\mu}{R_c} + \frac{1}{2}v^2_{e_{(r=R_c)}}}_{At\ the\ surface} = \underbrace{0+0}_{At\ infinity} \implies$$

$$v_{e_{(r=R_c)}} = \sqrt{\frac{2\mu_c}{R_c}} = 0.458m/s$$

where R_c is the radius of the comet.

The velocity of the lander after impact is more than twice as high as the escape velocity which shows that if the impact is perfectly elastic, the lander will escape the comet and never come back.

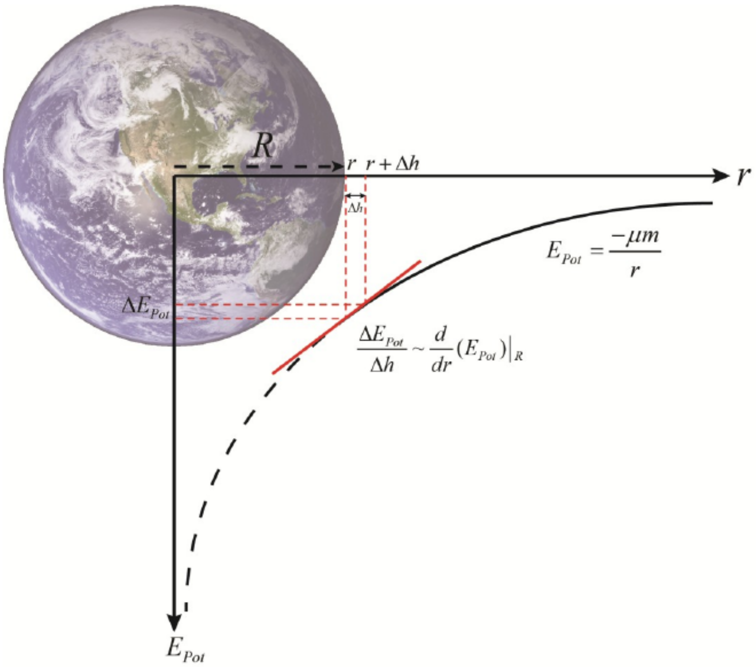
Problem 4 ($\star$) Potential energy close to the surface of the Earth

The general expression for the potential energy of a mass m in Earth's gravitational field is $E_{pot} = -\frac{m\mu}{r}$, r being the distance to the center of the Earth. In the vicinity of the surface of the Earth, the difference in potential energy for a mass m when the height above the ground is changed by Δh is equal to $mg\Delta h$, where g is the gravitational acceleration at the surface of the Earth. Derive this approximate expression from the general expression.

Intuitive solution. The difference in potential energy can be explained as the derivative of general expression of potential energy with respect to r :

$$\frac{\mathrm{d}}{\mathrm{d}r}E_{\mathrm{pot}} = \frac{\mathrm{d}}{\mathrm{d}r}\left(-\frac{m\mu}{r}\right) = \frac{m\mu}{r^2} = mg(r)$$

The gravitational acceleration of the Earth is $g(r) \equiv \frac{\mu}{r^2}$. At Earth's surface, $g(R) = 9.81$ m/s^2. At the vicinity of the Earth's surface, when r varies as Δh ($\Delta h \ll R$), we can approximate the above



derivative:

$$\frac{\mathrm{d}}{\mathrm{d}r}E_{\text{pot}}\bigg|_R = \frac{m\mu}{R^2} \sim \frac{\Delta E_{\text{pot}}}{\Delta h} \implies \Delta E_{\text{pot}} \sim mg(R)\Delta h$$

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Problem 5 Radiation balance

A) *Consider two spherical satellites with radii r and $2r$, respectively. Determine the radiation balance of each object if they are exposed to solar radiation only and compare their temperatures.*

B) *Consider a cylindrical satellite (radius=1 m, height=2 m) that is spin-stabilised, and hence rotating about its longitudinal axis. Assume that it is on an orbit where eclipses are negligible, and that its longitudinal axis of rotation remains perpendicular to the sun rays. The external structure of the satellite is made of steel (AM 350) with a (α/ε) ratio of 1.79. We only consider the Sun's radiation on the satellite and neglect the Earth's albedo and infrarred self-radiation.*

During a space shuttle mission, the science instrument of this satellite has to be replaced, and a space-walk of two crew-members is planned. Will it be safe for the astronauts performing this task to touch the surface of the satellite with their gloves if the "touch – no touch" limit is at 80 °C?

Solution. The satellite's temperature can be determined with the radiation balance and is given by the following formula

$$T = \left(\frac{\alpha}{\varepsilon}\right)^{1/4}\left(\frac{SA_n}{\sigma A_{\text{tot}}}\right)^{1/4}$$

A) As for a spherical satellite $A_n = \pi r^2$ and $A_{\text{tot}} = 4\pi r^2$, this corresponds to

$$T = \left(\frac{\alpha}{\varepsilon}\right)^{1/4} \left(\frac{S\pi r^2}{\sigma 4\pi r^2}\right)^{1/4} = \left(\frac{\alpha}{\varepsilon}\right)^{1/4} \left(\frac{S}{4\sigma}\right)^{1/4}$$

The temperature of a spherical satellite is therefore not influenced by its radius.

B) As the Sun rays remain perpendicular to the rotational axis of the satellite, the surface perpendicular to Sun's direction given by

$$A_n = 2rh$$

where r is the radius of the cylinder and h its height.

The satellite emits over the whole surface of the cylinder, the total surface is therefore

$$A_{\text{tot}} = 2\pi rh + 2\pi r^2$$

Hence the temperature of the satellite is given by :

$$T = \left(\frac{\alpha}{\varepsilon}\right)^{1/4} \left(\frac{SA_n}{\sigma A_{\text{tot}}}\right)^{1/4} = \left(\frac{\alpha}{\varepsilon}\right)^{1/4} \left(\frac{S \cdot 2rh}{\sigma \cdot 2\pi r(h+r)}\right)^{1/4} = \left(\frac{\alpha}{\varepsilon}\right)^{1/4} \left(\frac{S \cdot h}{\sigma \cdot \pi(h+r)}\right)^{1/4}$$

This yields a temperature of the surface of

$$T = (1.79)^{1/4} \left(\frac{1.4 \cdot 10^3 \cdot 2}{5.67 \cdot 10^{-8} \cdot \pi(2+1)}\right)^{1/4} = 311.2 \text{ K}$$

The surface temperature of the satellite is 311.2 K or 38 °C and it is below the critical limit of "touch" – "no touch" and the astronauts can touch its surface without any risk.

Problem 6 ($\star$) Mars and Deimos gravitational wells

Determine the gravitational accelerations on the surface of Mars and one of its two satellites, Deimos, and make a scale drawing of the gravitational wells of both of them, normalized on the Earth's gravitational acceleration.

	Mars	Deimos
Mass M,[kg]	$0.64 \cdot 10^{24}$	$1.48 \cdot 10^{15}$
Mean radius R,[km]	3397	6.2
Mean distance Mars center – Deimos d,[km]	23'460	

Solution. The depth of the gravitational well of Mars normalized on the Earth's gravitational acceleration can be retrieved from:

$$W_{♂} = R_{♂} g_{♂} = R_{♂} g_{♂} \frac{g_0}{g_0} = \frac{R_{♂} g_{♂}}{g_0} g_0$$

where the term $\frac{R_{♂} g_{♂}}{g_0}$ is the normalized gravitational well (see next page).

Determined values	Mars	Deimos
$\mu[m^3s^{-2}]$ $g[ms^{-2}]$	$\mu_♂ = GM_♂ = 4.27 \cdot 10^{13}[m^3s^{-2}]$ $g_♂ = \mu_♂/R_♂^2 = 3.7[ms^{-2}]$	$\mu_{\text{Deimos}} = GM_{\text{Deimos}} = 9.9 \cdot 10^4[m^3s^{-2}]$ $g_{\text{Deimos}} = \mu_{\text{Deimos}}/R_{\text{Deimos}}^2 = 2.5 \cdot 10^{-3}[ms^{-2}]$
Depth of the grav. wells (normalized to g_0), $[km]$	$R_{♂,norm.} = 1281\ [km]$	$R_{Deimos,norm.} = 1.6 \cdot 10^{-3}\ [km]$