Exercise Session 4

Note Problems marked with a ($\star$) are complimentary exercises and will not be solved in class.

Problem 1 Multiple Choice Questions

A) You observe a satellite 30 seconds later than you would expect from the Two Line Elements orbital data at your disposal. Which parameter of the trajectory has changed and how?

(1) Semi-major axis a, decrease
(2) Semi-major axis a, increase
(3) Inclination i, increase
(4) It is a measurement error, surely.

B) The satellite that you observed at the previous point was previously on an orbit at an altitude of 400 km and an inclination of 52°. You are told that it manoeuvred 12 orbits before your observation of the previous point. What is the value of the changed parameter of its orbit?

(1) $i = 52.04°$
(2) $z = 398$ km
(3) $z = 402$ km
(4) $z = 408$ km

C) ($\star$) A small satellite is placed in LEO, and is hence in the shadow of the Earth for 40% of its orbit. It has a nominal electric consumption of 500 W. What is the size of the solar arrays necessary to produce the required average electrical power over the full orbit, knowing that the conversion efficiency of the solar arrays is 25%? We assume that during the sunlit portion of the orbit, excess electrical energy is stored in batteries and fully recovered during the eclipse.

(1) 0.8 m^2
(2) 1.5 m^2
(3) 2.5 m^2
(4) 6.7 m^2

D) STS-75 was a Space Shuttle mission (in 1996) whose primary objective was to carry a tethered satellite system (TSS-1R) into orbit and to deploy it on a conducting tether. The Shuttle circled the Earth at an altitude of 296 kilometers, placing the 20 kilometers tether system upwards within the rarefied, electrically charged ionosphere. As the Space Shuttle orbiter/tethered satellite system orbit the Earth, the tether rapidly cuts across the Earth's magnetic field lines.
The interaction creates a voltage across the tether. However, for an electric current to be produced, a complete circuit must be formed. This is accomplished by using an electron generator on the orbiter to return charged particles back into the ionosphere. Knowing that the Earth's magnetic field is equal to $3 \cdot 10^{-5}$ T, what is the produced voltage along the conductor of the tethered system at the full deployed length of 20 kilometers? Assume that the velocity vector, the tether between the Shuttle and the satellite, and the magnetic field lines are all perpendicular to each another.

(1) 5.50 kV
(2) 4.64 kV
(3) 3.66 kV
(4) 4.80 kV

Problem 2 "Houston, we have a problem"

You are in the STS-75, in the same conditions as described in Question 1D).

A) The tether is aligned with the Earth's gravitational field. The Space Shuttle has a mass of 100 tons and the satellite, of 500 kg. The tethered satellite is deployed upwards (away from the Earth). What are the exact altitude of both the Space Shuttle and the tethered satellite when the system is deployed (the tether being considered massless)?

B) The orbit of the Space Shuttle is inclined by $i = 28.5°$. At some point, the tether breaks at the attachment point to the satellite. What will be the resulting orbit for the satellite that has suddenly become free?

C) Will the new orbit of the satellite remain coplanar with the orbit of the Shuttle? If not, in which direction (East or West) will the satellite orbit line of nodes drift vs the Shuttle orbit line of nodes? Why is this deviation taking place?

Problem 3 Falcon 9 Staging

Falcon 9 is a two-stage rocket designed and manufactured by SpaceX, which is a private space transport company headquartered in Hawthorne, California. It was founded in 2002 by the former PayPal entrepreneur Elon Musk. Falcon 9 is capable of carrying up to 10.45 tones of payload to LEO. To optimise the m_i/m_f ratio in the Tsiolkovsky formula, the Falcon 9 propulsion system included two stages (Table 1).

A) If the atmospheric effects and the gravity effect are **not** taken in consideration, what is the total Δv produced by Falcon 9?

B) In comparison, what would be the Δv produced by an equivalent single-stage launch system with $I_{\text{sp}} = 342$ s (Table 2)?

Stage	Dry mass [t]	Wet mass [t]	Propellant
First stage	20	260	LOX/Kerosene (RP-1) $I_{\text{sp}} = 282$ s at sea level $I_{\text{sp}} = 311$ s in vacuum
Second stage	3	53	LOX/Kerosene (RP-1) $I_{\text{sp}} = 342$ s in vacuum
Payload	10		

Table 1

Stage	Dry mass [t]	Wet mass [t]
Single stage	23	313
Payload	10	

Table 2

Problem 4 ($\star$) Orbit precession

A satellite is launched into a circular Sun-Synchronous orbit at an altitude of 1000 km. During the launch, the last propulsion stage failed to deliver the satellite to the correct orbit. The satellite is now into the correct inclination plan for the original 1000 km circular orbit, has an orbit apogee of 1000 km but a perigee of only 400 km.

A) At which rate will this new orbit deviate (node regression) compared to the planned Sun-Synchronous orbit ?

B) A way to recover the synchronicity with the Sun is to perform a boost with the satellite propulsion system to change the orbit inclination. What is the Δv needed to get this failed orbit to a Sun-Synchronous condition? The line of apsides of the degraded orbit is aligned with the Equator.