

Exercise Session 3

Note Problems marked with a (★) are complimentary exercises and will not be solved in class.

Problem 1 Multiple Choice Questions

- A) (★) A spacecraft is on a parking orbit at 1000 km above the surface of the Earth. At some point, the spacecraft is accelerated to a departure velocity of 11 km/s. What will be the speed on the surface of the sphere of influence and at infinity?
- (1) 0.32, 0 km/s
 - (2) 1.18, 1.12 km/s
 - (3) 3.72, 3.60 km/s
 - (4) 6.15, 3.92 km/s
- B) The MSL spacecraft, which carried Curiosity to Mars, had a mass of 4050 kg at 200 km altitude on a LEO parking orbit. Its orbital velocity was 7.78 km/s. To reach Mars, it had to accelerate to a departure velocity v_d of 11.50 km/s with a specific impulse of 320 s at a mass expulsion rate of 5 kg/s. What was the duration of the burn?
- (1) 20 s
 - (2) 562 s
 - (3) 1832 s
 - (4) 102 s
- C) (★) A similar spacecraft has ion thrusters of specific impulse 3200 s. We want to perform a similar increase of velocity. Would the “burn” be shorter or longer?
- | | | |
|-------------|------|--|
| | | (1) The I_{sp} is much higher |
| (1) Longer | Why? | (2) The thrust is much lower |
| (2) Shorter | | (3) The exhaust velocity is much lower |
| | | (4) The thrust is much higher |
- D) (★) A rocket is in free space, made of propellant only. Its initial mass is m and its thrust constant at a value mg_0 . Its specific impulse is I_{sp} . It burns off completely in a time Δt . What is the value of Δt ?
- (1) Its I_{sp} in seconds
 - (2) 9.81 s
 - (3) 42 s
 - (4) Cannot be determined with available data.

E) The Galileo mission which explored the Jovian system used the gravity assist of Venus during its complicated trip to Jupiter's neighbourhood. Its v_a^∞ at the entrance of the sphere of influence of Venus was perpendicular to the velocity vector of the planet and had an amplitude of 6.2 km/s. The gravity assist manoeuvre resulted in a deflection of Galileo's planetocentric velocity vector by $\delta = +43^\circ$. The heliocentric arrival velocity at Venus was $V_{\text{before}} = 37.4$ km/s. The heliocentric velocity of Venus is $V_v = 35.2$ km/s. Estimate the heliocentric velocity of the spacecraft at the departure from the sphere of influence of Venus.

Hint. Use the law of the cosine $c^2 = a^2 + b^2 - 2ab \cos(\gamma)$ to compute the heliocentric velocity after the slingshot.

- (1) Cannot be determined with available data.
- (2) 33.7 km/s
- (3) 35.2 km/s
- (4) 39.7 km/s

Problem 2 Interplanetary Transfer

We plan the interplanetary transfer of a spacecraft from the Earth to Jupiter. We assume that both orbits are circular and coplanar for simplicity.

- A) Assume that the spacecraft follows a Hohmann transfer trajectory. Determine the departure hyperbolic excess velocity v_d^∞ needed to leave the influence of the Earth on our way to Jupiter and the arrival hyperbolic excess velocity v_a^∞ .
- B) What is the departure velocity v_d to achieve this mission from a circular parking orbit around the Earth at 200 km altitude?
- C) Find the spacecraft's energy (per unit mass) on the hyperbolic Earth escape trajectory.
- D) What is the time needed for the trip from Earth to Jupiter, on an Hohmann transfer trajectory?
- E) Upon arrival in the sphere of influence of Jupiter, the impact parameter is chosen so as to achieve, in the hyperbolic trajectory inside the Jupiter sphere of influence, a closest distance to the center of Jupiter equal to 100,000 km. What is the value of the braking impulse Δv_i so as to inject the spacecraft on a circular orbit around Jupiter at that altitude?
- F) What is the phase angle between the Earth and Jupiter at Earth's departure?
- G) How often does this launch windows occur?

Numerical values:

Radius of Jupiter's orbit $R_{\text{Jupiter}} = 5.204$ AU.

Mass of Jupiter $m_{\text{Jupiter}} = 1.8989 \cdot 10^{27}$ kg, mean radius $r_J = 69,911$ km.